Algebra I

Course No. 1200310

Bureau of Exceptional Education and Student Services
Florida Department of Education

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Exceptional Student Education

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Unit 1: Are These Numbers Real?

This unit recalls the relationships between sets of real numbers and the rules involved when working with them.

Unit Focus

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Algebra Body of Knowledge

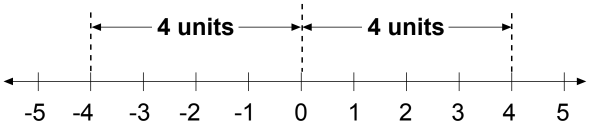
Standard 10: Mathematical Reasoning and Problem Solving

- MA.912.A.10.1
  Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing- and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.

- MA.912.A.10.3
  Decide whether a given statement is always, sometimes, or never true (statements involving linear or quadratic expressions, equations, or inequalities, rational or radical expressions, or logarithmic or exponential functions).

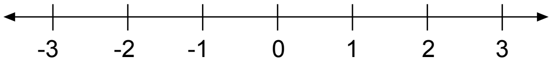
Vocabulary

Use the vocabulary words and definitions below as a reference for this unit.

absolute value	a number's distance from zero (0) on a number line; distance expressed as a positive value *Example*: The absolute value of both 4, written |4|, and negative 4, written |-4|, equals 4. 
addend	any number being added *Example*: In 14 + 6 = 20, the addends are 14 and 6.
additive identity	the number zero (0); when zero (0) is added to another number the sum is the number itself *Example*: 5 + 0 = 5
additive inverses	a number and its opposite whose sum is zero (0); also called *opposites* *Example*: In the equation 3 + (-3) = 0, the additive inverses are 3 and -3.
algebraic expression	an expression containing numbers and variables ($7x$) and operations that involve numbers and variables ($2x + y$ or $3a^2 - 4b + 2$); however, they do not contain equality (=) or inequality symbols (<, >, ≤, ≥, or ≠)
associative property	the way in which three or more numbers are grouped for addition or multiplication does *not* change their sum or product, respectively *Examples*: (5 + 6) + 9 = 5 + (6 + 9) *or* (2 x 3) x 8 = 2 x (3 x 8)

braces { }	grouping symbols used to express sets
commutative property	the order in which two numbers are added or multiplied does *not* change their sum or product, respectively *Examples*: 2 + 3 = 3 + 2 *or* 4 x 7 = 7 x 4
counting numbers (natural numbers)	the numbers in the set {1, 2, 3, 4, 5, …}
cube (power)	the third power of a number *Example*: 4^3 = 4 x 4 x 4 = 64; 64 is the cube of 4
decimal number	any number written with a decimal point in the number *Examples*: A decimal number falls between two whole numbers, such as 1.5, which falls between 1 and 2. Decimal numbers smaller than 1 are sometimes called *decimal fractions*, such as five-tenths, or $\frac{5}{10}$, which is written 0.5.
difference	a number that is the result of subtraction *Example*: In 16 – 9 = 7, the difference is 7.
digit	any one of the 10 symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9
element or **member**	one of the objects in a set
empty set or **null set (ø)**	a set with no elements or members
equation	a mathematical sentence stating that the two expressions have the same value *Example*: $2x = 10$

even integer	any integer divisible by 2; any integer with the digit 0, 2, 4, 6, or 8 in the units place; any integer in the set {… , -4, -2, 0, 2, 4, …}
exponent (exponential form)	the number of times the base occurs as a factor *Example*: 2^3 is the exponential form of 2 x 2 x 2. The numeral two (2) is called the *base*, and the numeral three (3) is called the *exponent*.
expression	a mathematical phrase or part of a number sentence that combines numbers, operation signs, and sometimes variables *Examples*: $4r^2$; $3x + 2y$; $\sqrt{25}$ An expression does *not* contain equal (=) or inequality (<, >, ≤, ≥, or ≠) signs.
finite set	a set in which a whole number can be used to represent its number of elements; a set that has bounds and is limited
fraction	any part of a whole *Example*: One-half written in fractional form is $\frac{1}{2}$.
grouping symbols	parentheses ( ), braces { }, brackets [ ], and fraction bars indicating grouping of terms in an expression
infinite set	a set that is not finite; a set that has no boundaries and no limits
integers	the numbers in the set {… , -4, -3, -2, -1, 0, 1, 2, 3, 4, …}
irrational number	a real number that cannot be expressed as a ratio of two integers *Example*: $\sqrt{2}$

member or **element**	one of the objects in a set
multiples	the numbers that result from multiplying a given whole number by the set of whole numbers *Example*: The multiples of 15 are 0, 15, 30, 45, 60, 75, etc.
natural numbers (counting numbers)	the numbers in the set {1, 2, 3, 4, 5, …}
negative integers	integers less than zero
negative numbers	numbers less than zero
null set (ø) or **empty set**	a set with no elements or members
number line	a line on which ordered numbers can be written or visualized 
odd integer	any integer *not* divisible by 2; any integer with the digit 1, 3, 5, 7, or 9 in the units place; any integer in the set {… , -5, -3, -1, 1, 3, 5, …}
opposites	two numbers whose sum is zero; also called *additive inverses* *Examples*: $\underset{\uparrow}{-5} + \underset{\uparrow}{5} = 0$ *or* $\underset{\uparrow}{\frac{2}{3}} + \left(\underset{\uparrow}{-\frac{2}{3}}\right) = 0$ opposites opposites

order of operations	the order of performing computations in parentheses first, then exponents or powers, followed by multiplication and/or division (as read from left to right), then addition and/or subtraction (as read from left to right); also called *algebraic order of operations* *Example*: $5 + (12 - 2) \div 2 - 3 \times 2 =$ $5 + 10 \div 2 - 3 \times 2 =$ $5 + 5 - 6 =$ $10 - 6 =$ 4
pattern (relationship)	a predictable or prescribed sequence of numbers, objects, etc.; may be described or presented using manipulatives, tables, graphics (pictures or drawings), or algebraic rules (functions) *Example*: 2, 5, 8, 11 … is a pattern. Each number in this sequence is three more than the preceding number. Any number in this sequence can be described by the algebraic rule, $3n - 1$, by using the set of counting numbers for n.
pi (π)	the symbol designating the ratio of the circumference of a circle to its diameter; an irrational number with common approximations of either 3.14 or $\frac{22}{7}$
positive integers	integers greater than zero
positive numbers	numbers greater than zero
power (of a number)	an exponent; the number that tells how many times a number is used as a factor *Example*: In 2^3, 3 is the power.

product	the result of multiplying numbers together *Example*: In 6 x 8 = 48, the product is 48.
quotient	the result of dividing two numbers *Example*: In 42 ÷ 7 = 6, the quotient is 6.
ratio	the comparison of two quantities *Example*: The ratio of a and b is a:b or $\frac{a}{b}$, where $b \neq 0$.
rational number	a number that can be expressed as a ratio $\frac{a}{b}$, where a and b are integers and $b \neq 0$
real numbers	the set of all rational and irrational numbers
repeating decimal	a decimal in which one digit or a series of digits repeat endlessly *Examples*: 0.3333333… or $0.\overline{3}$ 24.6666666… or $24.\overline{6}$ 5.27272727… or $5.\overline{27}$ 6.2835835… or $6.2\overline{835}$
root	an equal factor of a number *Examples*: In $\sqrt{144} = 12$, the square root is 12. In $\sqrt[3]{125} = 5$, the cube root is 5.
set	a collection of distinct objects or numbers
simplify an expression	to perform as many of the indicated operations as possible

solve	to find all numbers that make an equation or inequality true
square (of a number)	the result when a number is multiplied by itself or used as a factor twice *Example*: 25 is the square of 5.
sum	the result of adding numbers together *Example*: In 6 + 8 = 14, the sum is 14.
terminating decimal	a decimal that contains a finite (limited) number of digits *Example*: $\frac{3}{8} = 0.375$ $\frac{2}{5} = 0.4$
value (of a variable)	any of the numbers represented by the variable
variable	any symbol, usually a letter, which could represent a number
Venn diagram	a diagram which shows the relationships between sets
whole numbers	the numbers in the set {0, 1, 2, 3, 4, …}

Unit 1: Are These Numbers Real?

Introduction

The focus of Algebra I is to introduce and strengthen algebraic skills. These skills are necessary for further study and success in mathematics. Algebra I fosters

- an understanding of the real number system
- an understanding of different sets of numbers
- an understanding of various ways of representing numbers.

Many topics in this unit will be found again in later units. There is an emphasis on problem solving and real-world applications.

Lesson One Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

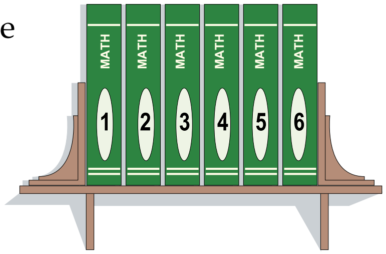
Standard 10: Mathematical Reasoning and Problem Solving

- MA.912.A.10.1
  Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing- and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.

- MA.912.A.10.3
  Decide whether a given statement is always, sometimes, or never true (statements involving linear or quadratic expressions, equations, or inequalities, rational or radical expressions, or logarithmic or exponential functions).

The Set of Real Numbers

A **set** is a collection. It can be a collection of DVDs, books, baseball cards, or even numbers. Each item in the *set* is called an **element** or **member** of the set. In algebra, we are most often interested in sets of numbers.



A set *can be a collection of books or numbers.*

The first set of numbers you learned when you were younger was the set of **counting numbers**, which are also called the **natural numbers**. These are the **positive numbers** you count with (1, 2, 3, 4, 5, …). Because this set has *no final number*, we call it an **infinite set**. A set that has a *specific number of elements* is called a **finite set**.

Symbols are used to represent sets. **Braces { }** are the symbols we use to show that we are talking about a set.

A set with *no elements* or members is called a **null set (ø)** or **empty set**. It is often denoted by an *empty set* of *braces* { }.

The set of *counting numbers* looks like {1, 2, 3, …}.

Remember: The counting numbers can also be called the *natural numbers*, naturally!

The set of natural number **multiples** of 10 is {10, 20, 30, …}.

The set of integers looks like {…, -4, -3, -2, -1, 0, 1, 2, 3, 4, …}.

The set of **integers** that are *multiples* of 10 is {… , -30, -20, -10, 10, 20, 30, …}.

As you became bored with simply counting, you learned to add and subtract numbers. This led to a new set of numbers, the **whole numbers**.

The *whole numbers* are the counting numbers *and* zero {0, 1, 2, 3, …}.

Remember getting negative answers? Those **negative numbers** made another set of numbers necessary. The *integers* are the counting numbers, their **opposites** (also called **additive inverses**), and zero.

The integers can be expressed (or written) as {… , -3, -2, -1, 0, 1, 2, 3, …}.

Even integers are integers divisible by 2. The integers {… , -4, -2, 0, 2, 4, …} form the set of *even integers*.

Remember: Every even integer ends with the **digit** 0, 2, 4, 6, or 8 in its ones (or units) place.

Odd integers are integers that are *not* divisible by 2. The integers {… , -5, -3, -1, 1, 3, 5, …} form the set of *odd integers*.

Remember: Every odd integer ends with the *digit* 1, 3, 5, 7, or 9 in its units place.

Note: There are *no* **fractions** or **decimals** listed in the set of integers above.

When you learned to divide and got answers that were *integers*, *decimals*, or *fractions*, your answers were all from the set of **rational numbers**.

Rational numbers can be expressed as fractions that can then be converted to **terminating decimals** (with a *finite* number of digits) or **repeating decimals** (with an *infinitely* repeating sequence of digits). For example, $-\frac{3}{5} = -0.6$, $\frac{6}{2} = 3$, $-\frac{8}{4} = -2$ and $\frac{1}{3} = 0.333\ldots$ or $0.\overline{3}$.

As you learned more about mathematics, you found that some numbers are **irrational numbers**. *Irrational numbers* are numbers that cannot be written as a **ratio**, or a comparison of two quantities because their decimals never repeat a **pattern** and never end.

Irrational numbers like **π (pi)** and $\sqrt{5}$ have non-*terminating*, non-*repeating decimals*.

If you put all of the rational numbers and all of the irrational numbers together in a set, you get the set of **real numbers**.

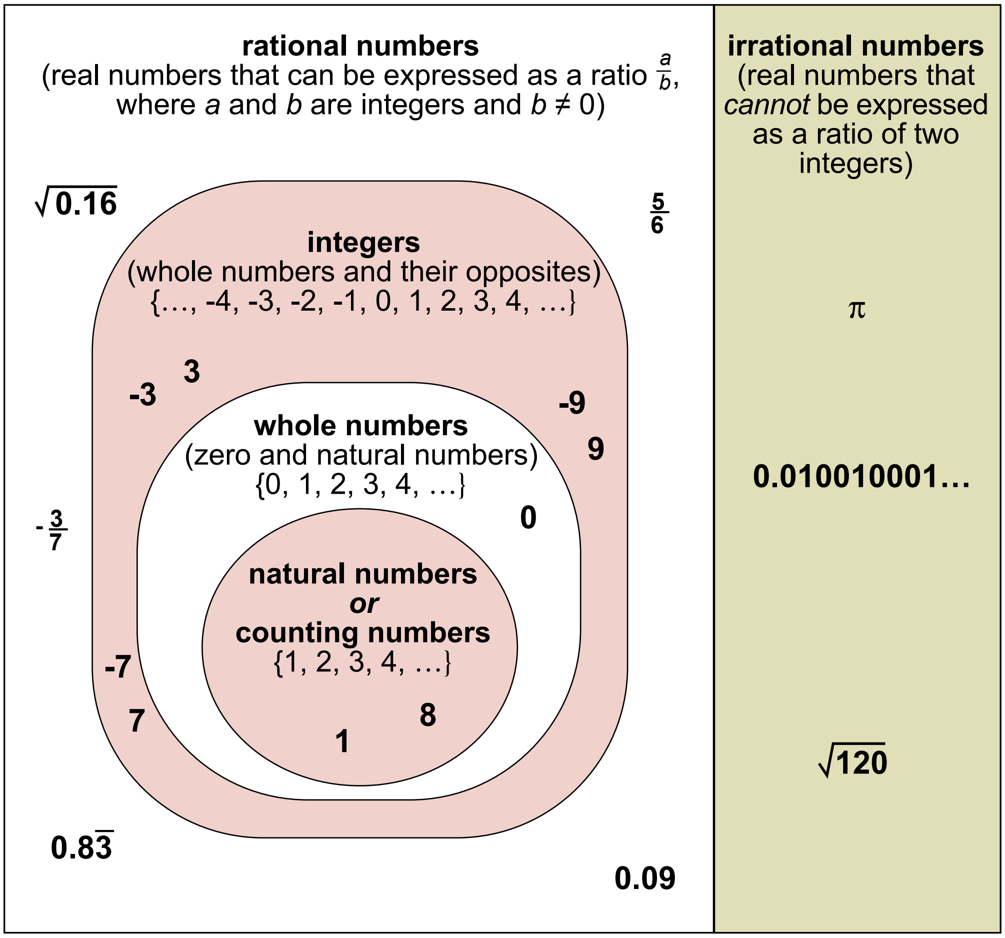
The set of *real numbers* is often symbolized with a capital R.

A diagram showing the *relationships* among all the sets mentioned is shown on the following page.

Remember: Real numbers include all rational numbers and all irrational numbers.

The diagram below is called a **Venn diagram**. A *Venn diagram* shows the relationships between different sets. In this case, the sets are types of numbers.

The Set of Real Numbers



Practice

Match each definition with the correct term. Write the letter on the line provided.

_______ 1. the numbers in the set {… , -4, -3, -2, -1, 0, 1, 2, 3, 4, …}

_______ 2. the numbers in the set {1, 2, 3, 4, 5, …}

_______ 3. a number that can be expressed as a ratio $\frac{a}{b}$, where a and b are integers and $b \neq 0$

_______ 4. a set in which a whole number can be used to represent its number of elements; a set that has bounds and is limited

_______ 5. the numbers in the set {0, 1, 2, 3, 4, …}

_______ 6. a real number that cannot be expressed as a ratio of two integers

_______ 7. any integer *not* divisible by 2

_______ 8. a set that is *not* finite; a set that has *no* boundaries and *no* limits

_______ 9. any integer divisible by 2

_______ 10. the numbers that result from multiplying a given whole number by the set of whole numbers

_______ 11. the set of all rational and irrational numbers

_______ 12. the symbol designating the ratio of the circumference of a circle to its diameter

A. even integer

B. finite set

C. infinite set

D. integers

E. irrational number

F. multiples

G. natural numbers (counting numbers)

H. odd integer

I. pi (π)

J. rational number

K. real numbers

L. whole numbers

Practice

Match each **description** *with the correct* **set***. Write the letter on the line provided.*

________ 1. {2, 3, 4, 5, 6}

________ 2. {0, 1, 2, 3}

________ 3. {3, 6, 9, 12, …}

________ 4. {-2, 0, 2}

________ 5. {6, 12, 18, …}

________ 6. {1, 2, 3, 4, 5}

________ 7. {-3, -1, 1, 3}

________ 8. {… , -18, -12, -6, 0, 6, 12, 18, …}

A. {counting numbers between 1 and 7}

B. {even integers between -3 and 4}

C. {first five counting numbers}

D. {first four whole numbers}

E. {integers that are multiples of 6}

F. {natural-number multiples of 3}

G. {odd integers between -4 and 5}

H. {whole number multiples of 6}

Practice

Write **finite** *if the* **set** *has bounds and is limited. Write* **infinite** *if the* **set** *has* no *boundaries and is* not *limited.*

________________________ 1. {whole numbers less than 1,000,000}

________________________ 2. {natural numbers with four digits}

________________________ 3. {whole numbers with 0 as the last numeral}

________________________ 4. {real numbers between 6 and 8}

________________________ 5. {counting numbers between 2 and 10}

________________________ 6. {first five counting numbers}

________________________ 7. {natural-number multiples of 5}

________________________ 8. {integers less than 1,000,000}

________________________ 9. {counting numbers with three digits}

________________________ 10. {whole numbers with 5 as the last numeral}

Practice

Write **True** *if the statement is correct. Write* **False** *if the statement is* not *correct.*

___________ 1. 7 is a rational number.

___________ 2. $\frac{5}{3}$ is a real number.

___________ 3. -9 is a whole number.

___________ 4. 0 is a counting number.

___________ 5. $\sqrt{4}$ is irrational.

___________ 6. $\sqrt{7}$ is a rational number.

___________ 7. $\frac{10}{3}$ is a whole number.

___________ 8. -9 is a natural number.

___________ 9. 0 is an even integer.

__________ 10. π is a real number.

Practice

Use the list below to write the correct term for each definition on the line provided.

additive inverses
element or **member**
negative numbers
null set (ø) or **empty set**
positive numbers
repeating decimal
terminating decimal

________________________ 1. a set with no elements or members

________________________ 2. a decimal that contains a finite (limited) number of digits

________________________ 3. a decimal in which one digit or a series of digits repeat endlessly

________________________ 4. a number and its opposite whose sum is zero (0)

________________________ 5. numbers less than zero

________________________ 6. numbers greater than zero

________________________ 7. one of the objects in a set

Lesson Two Purpose

Reading Process Strand

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- LA.910.1.6.1
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- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.

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- MA.912.A.10.1
  Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing- and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.

- MA.912.A.10.3
  Decide whether a given statement is always, sometimes, or never true (statements involving linear or quadratic expressions, equations, or inequalities, rational or radical expressions, or logarithmic or exponential functions).

The Order of Operations

Algebra can be thought of as a game. When you know the rules, you have a much better chance of winning! In addition to knowing how to add, subtract, multiply, and divide integers, fractions, and decimals, you must also use the **order of operations** correctly.

Although you have previously studied the rules for *order of operations*, here is a quick review.

Rules for Order of Operations

Always start on the *left* and move *to* the *right*.

1. Do operations inside *grouping symbols* first. ( ) , [ ], **or** $\frac{x}{y}$
2. Then do all *powers* (exponents) **or** *roots*. x^2 **or** $\sqrt{x}$
3. Next do *multiplication* **or** *division*— as they occur from left to right. • **or** ÷
4. Finally, do *addition* **or** *subtraction*— as they occur from left to right. **+ or –**

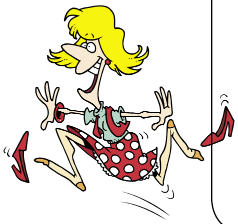
Remember: The fraction bar is considered a **grouping symbol**.

Example: $\frac{3x^2 + 8}{2} = (3x^2 + 8) \div 2$

Note: In an **expression** where more than one set of *grouping symbols* occurs, work within the innermost set of symbols first, then work your way outward.

The order of operations makes sure everyone doing the problem correctly will get the same answer.

Some people remember these rules by using this mnemonic device to help their memory.



Please **P**ardon **M**y **D**ear **A**unt **S**ally*

Please	**P**arentheses (grouping symbols)
Pardon....................	**P**owers
My **D**ear..................	**M**ultiplication or **D**ivision
Aunt **S**ally...............	**A**ddition or **S**ubtraction

*Also known as **P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally—**P**arentheses, **E**xponents, **M**ultiplication or **D**ivision, **A**ddition or **S**ubtraction.

Remember: You do multiplication **or** division—*as they occur* from *left to right*, and then addition **or** subtraction—*as they occur* from *left to right*.

Study the following.

25 – 3 • 2 =

There are no grouping symbols. There are no **powers (exponents)** or **roots**. We look for multiplication or division and find multiplication. We multiply. We look for addition or subtraction and find subtraction. We subtract.

25 – 3 • 2 =
25 – 6 =
19

Study the following.

12 ÷ 3 + 6 ÷ 2 =

There are no grouping symbols. There are no *powers* or *roots*. We look for multiplication or division and find division. We divide. We look for addition or subtraction and find addition. We add.

12 ÷ 3 + 6 ÷ 2 =
4 + 3 =
7

If the rules were ignored, one might divide 12 by 3 and get 4, then add 4 and 6 to get 10, then divide 10 by 2 to get 5—which is the wrong answer. Agreement is needed—using the agreed-upon *order of operations*.

Study the following.

$30 - 3^3 =$

There are no grouping symbols. We look for powers and roots and find powers, 3^3. We calculate this. We look for multiplication or division and find none. We look for addition or subtraction and find subtraction. We subtract.

$$\begin{aligned}&30 - 3^3 =\\&30 - 27 =\\&\quad 3\end{aligned}$$

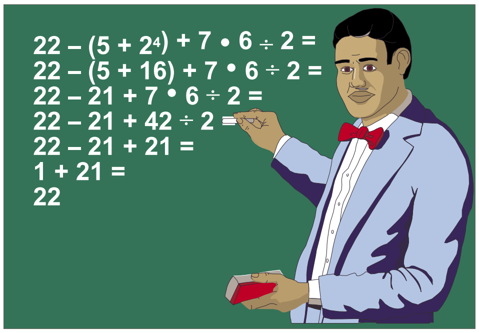
Study the following.

$22 - (5 + 2^4) + 7 \bullet 6 \div 2 =$

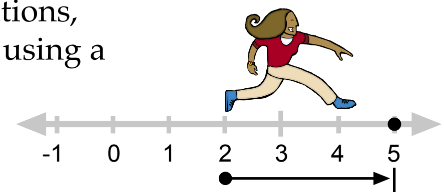
We look for grouping symbols and see them. We must do what is inside the parentheses first. We find addition and a power. We do the power first and then the addition. There are no roots. We look for multiplication or division and find both. We do them in the order they occur, left to right, so the multiplication occurs first. We look for addition or subtraction and find both. We do them in the order they occur, left to right, so the subtraction occurs first.

Please	**P**arentheses
Pardon	**P**owers
My	**M**ultiplication *or*
Dear	**D**ivision
Aunt	**A**ddition *or*
Sally	**S**ubtraction

$$\begin{aligned}&22 - (5 + 2^4) + 7 \bullet 6 \div 2 =\\&22 - (5 + 16) + 7 \bullet 6 \div 2 =\\&22 - 21 + 7 \bullet 6 \div 2 =\\&22 - 21 + 42 \div 2 =\\&22 - 21 + 21 =\\&1 + 21 =\\&22\end{aligned}$$



Adding Numbers by Using a Number Line



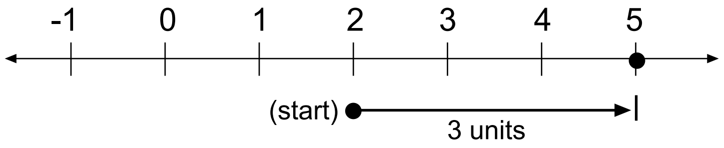
After reviewing the rules for order of operations, let's get a visual feel for adding integers by using a **number line**.

Example 1

Add 2 + 3

1. Start at 2.
2. Move 3 units to the right in the *positive* direction.
3. Finish at 5.

So, 2 + 3 = 5.

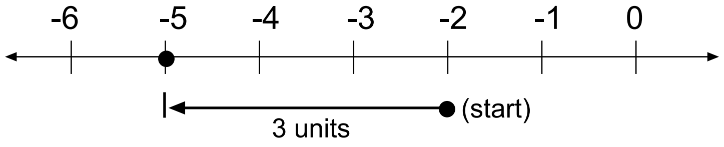


Example 2

Add -2 + (-3)

1. Start at -2.
2. Move 3 units to the left in the *negative* direction.
3. Finish at -5.

So, -2 + (-3) = -5.

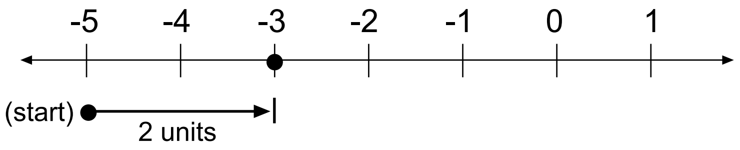


Example 3

Add -5 + 2

1. Start at -5.
2. Move 2 units to the right in a *positive* direction.
3. Finish at -3.

   So, -5 + 2 = -3.

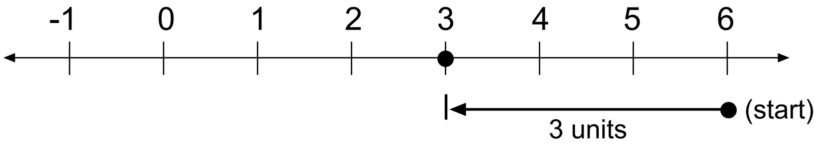


Example 4

Add 6 + (-3)

1. Start at 6.
2. Move 3 units to the left in a *negative* direction.
3. Finish at 3.

   So, 6 + (-3) = 3.



Addition Table

Look for *patterns* in the Addition Table below.

Addition Table

+	**4**	**3**	**2**	**1**	**0**	**-1**	**-2**	**-3**	**-4**
4	8	7	6	5	4	3	2	1	0
3	7	6	5	4	3	2	1	0	-1
2	6	5	4	3	2	1	0	-1	-2
1	5	4	3	2	1	0	-1	-2	-3
0	4	3	2	1	0	-1	-2	-3	-4
-1	3	2	1	0	-1	-2	-3	-4	-5
-2	2	1	0	-1	-2	-3	-4	-5	-6
-3	1	0	-1	-2	-3	-4	-5	-6	-7
-4	0	-1	-2	-3	-4	-5	-6	-7	-8

← **addends**
(any numbers being added)

sums
(the result of adding
numbers together)

↑ **addends** **sums**

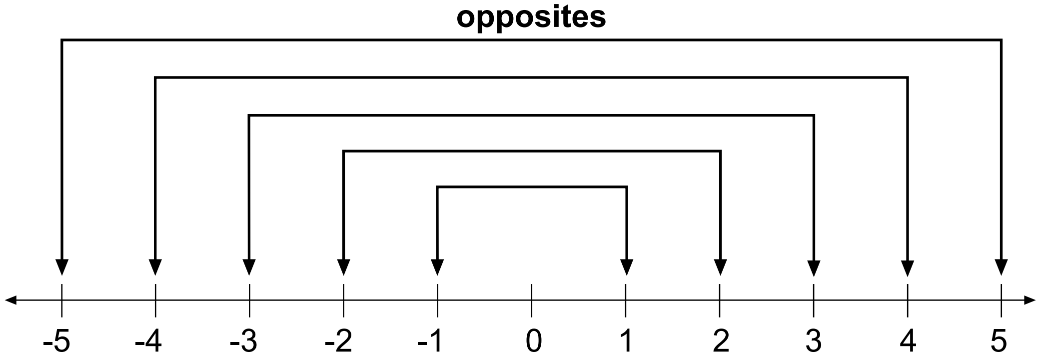
- Look at the *positive sums* in the table. Note the *addends* that result in a positive sum.
- Look at the *negative sums* in the table. Note the *addends* that result in a negative sum.
- Look at the sums that are *zero*. Note the *addends* that result in a sum of zero.
- **Additive Identity Property**—when zero is added to any number, the sum is the number. Note that this property is true for addition of integers.
- **Commutative Property of Addition**—the order in which numbers are added does *not* change the sum. Note that this property is true for addition of integers.
- **Associative Property of Addition**—the way numbers are grouped when added does *not* change the sum. Note that this property is true for addition of integers.

Opposites and Absolute Value

Although we can visualize the process of adding by using a number line, there are faster ways to add. To accomplish this, we must know two things: *opposites* or *additive inverses* and **absolute value**.

Opposites or Additives Inverses

5 and -5 are called *opposites*. Opposites are two numbers whose points on the number line are the same distance from 0 but in opposite directions.



The opposite of 0 is 0.

Every **positive integer** can be paired with a **negative integer**. These pairs are called *opposites*. For example, the opposite of 4 is -4 and the opposite of -5 is 5.

The opposite of 4 can be written -(4), so -(4) equals -4.

$$-(4) = -4$$

The opposite of -5 can be written -(-5), so -(-5) equals 5.

$$-(-5) = 5$$

Two numbers are opposites or *additive inverses* of each other if their sum is zero.

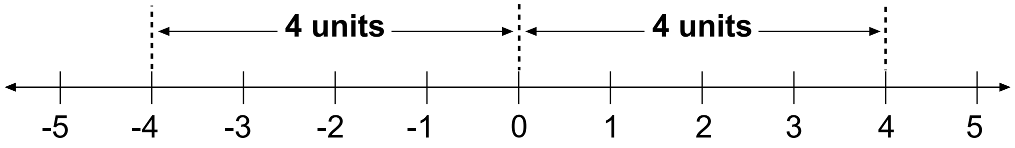
For example: $4 + -4 = 0$
$-5 + 5 = 0$

Absolute Value

The *absolute value* of a number is the distance the number is from the *origin* or zero (0) on a number line. The symbol | | placed on either side of a number is used to show absolute value.

Look at the number line below. -4 and 4 are different numbers. However, they are the same distance in number of units from 0. Both have the same *absolute value* of 4. Absolute value is *always* positive because distance is always positive—you cannot go a negative distance. The absolute value of a number tells the number's *distance* from 0, not its *direction*.

$|-4| = |4| = 4$ The absolute value of a number is *always* positive.



However, the number 0 is neither positive nor negative. The absolute value 0 is 0.

$|-4|$ denotes the absolute value of -4.

$|-4| = 4$

$|4|$ denotes the absolute value of 4.

$|4| = 4$

The absolute value of 10 is 10. We can use the following notation.

$|10| = 10$

The absolute value of -10 is also 10. We can use the following notation.

$|-10| = 10$

Both 10 and -10 are 10 units away from the origin. So, the absolute value of both numbers is 10.

The absolute value of 0 is 0.

$|0| = 0$

The *opposite* of the absolute value of a number is *negative*.

$-|8| = -8$

Now that we have this terminology under our belt, we can introduce two rules for adding numbers which will enable us to add quickly.

Adding Positive and Negative Integers

There are specific rules for adding positive and negative numbers.

1. **If the two integers have the *same sign*, *add* their absolute values, and *keep* the *sign*.**

   Example
   -5 + (-7)

Think: Both integers have the same signs and the signs are negative. Add their absolute values.

| -5 | = 5

| -7 | = 7

5 + 7 = 12

Keep the sign. The sign will be negative because both signs were negative. Therefore, the answer is -12.

-5 + -7 = -12

2. **If the two integers have *opposite signs*, *subtract* the absolute values. The answer has the *sign* of the integer with the *greater* absolute value.**

Example
-8 + 3

Think: Signs are opposite. Subtract the absolute values.

| -8 | = 8

| 3 | = 3

8 – 3 = 5

The sign will be negative because -8 has the greater absolute value. Therefore, the answer is -5.

-8 + 3 = -5

Example
-6 + 8

Think: Signs are opposite. Subtract the absolute values.

| -6 | = 6

| 8 | = 8

8 – 6 = 2

The sign will be positive because 8 has a greater absolute value. Therefore, the answer is 2.

-6 + 8 = 2

Example
5 + (-7)

Think: Signs are opposite. Subtract the absolute values.

| 5 | = 5

| -7 | = 7

7 – 5 = 2

The sign will be negative because -7 has the greater absolute value. Therefore, the answer is -2.

5 + -7 = -2

Rules for Adding Integers

• The sum of two positive integers is *positive*.	(+) + (+) = +	
• The sum of two negative integers is *negative*.	(-) + (-) = -	
• The sum of a positive integer and a negative integer takes the *sign of the number with the greater absolute value*.	(-) + (+) = (+) + (-) =	} use sign of number with greater absolute value
• The sum of a positive integer and a negative integer is zero if numbers have the *same absolute value*.	(*a*) + (-*a*) = 0 (-*a*) + (*a*) = 0	

Check Yourself Using a Calculator When Adding Positive and Negative Integers

Use a **calculator** with a $\boxed{+/-}$ **sign-change** key.

For example, for -16 + 4, you would enter (for most calculators) 16 $\boxed{+/-}$ $\boxed{+}$ 4 $\boxed{=}$ and get the answer -12.

Subtracting Integers

In the last section, we saw that 8 plus -3 equals 5.

$$8 + (-3) = 5$$

We know that 8 minus 3 equals 5.

$$8 - \ \ 3 \ \ = 5$$

Below are similar examples.

$$10 + (-7) = 3 \qquad\qquad 12 + (-4) = 8$$

$$10 - \ \ 7 \ \ = 3 \qquad\qquad 12 - \ \ 4 \ \ = 8$$

These three examples show that there is a connection between adding and subtracting. As a matter of fact, we can make any subtraction problem into an addition problem and any addition problem into a subtraction problem.

This idea leads us to the following definition.

Definition of Subtraction
$\boldsymbol{a - b = a + (-b)}$

Examples $8 - 10 = 8 + (-10) = -2$

$12 - 20 = 12 + (-20) = -8$

$-2 - 3 = -2 + (-3) = -5$

Even if we have $8 - (-8)$, this becomes

8 plus the opposite of -8, which equals 8.

$8 + [-(-8)] =$

$8 + \ \ \ 8 \ \ \ = 16$

And -9 – (-3), this becomes

-9 plus the opposite of -3, which equals 3.

-9 + -(-3)

-9 + 3 = -6

Shortcut Two negatives become one positive!

10 – (-3) becomes 10 plus 3.

10 + 3 = 13

And -10 – (-3) becomes -10 plus 3.

-10 + 3 = -7

Generalization for Subtracting Integers

Subtracting an integer is the same as adding its opposite.

$$a - b = a + (-b)$$

✓ Check Yourself Using a Calculator When Subtracting Negative Integers

Use a **calculator** with a $\boxed{+/-}$ **sign-change** key.

For example, for 18 – (-32), you would enter
18 $\boxed{-}$ 32 $\boxed{+/-}$ $\boxed{=}$ and get the answer 50.

Practice

Answer the following.

1. On May 22, 2004, in Ft. Worth, Texas, Annika Sorenstam became the first woman in 58 years to play on the PGA Tour. *Par* for the eighteen holes was 3 for four holes, 4 for twelve holes, and 5 for two holes, yielding a total par of 70 on the course. Sorenstam's scores on Day One in relation to par are provided in the table below. Determine her total for Day One.

Note: Par is the standard number of strokes a good golfer is expected to take for a certain hole on a given golf course. On this course, 70 is par. Therefore, add the total number of strokes in relation to par.

Annika Sorenstam's Golf Scores for Day One

Hole	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Total Score Relative to Par	0	0	0	0	+1	0	0	0	+1	0	0	0	-1	0	0	0	0	0

Answer: ____________

2. Sorenstam's scores on Day One qualified her to continue to play on Day Two. However, her scores on Day Two did *not* qualify her to continue to play in the tournament. Sorenstam's scores in relation to par are provided in the table below. Determine her total for Day Two.

Annika Sorenstam's Golf Scores for Day Two

Hole	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Total Score Relative to Par	0	-1	0	0	+1	+1	0	+1	0	+1	0	+1	0	0	0	0	0	0

Answer: ____________

3. When an unknown integer is added to 12, the sum is *less than* -2.
Give three examples of what the unknown number might be.

Answer: __________

____

____

Complete the following statements.

4. a. The sum of *two positive numbers* is __________ (always, sometimes, never) positive.

b. The sum of *two negative numbers* is __________ (always, sometimes, never) positive.

c. The sum of *a number and its opposite* is __________ (always, sometimes, never) positive.

d. The sum of *a positive number and a negative number* is __________ (always, sometimes, never) positive.

5. *Complete the following statements.*

   a. When a *positive* integer is subtracted from a *positive* integer,

      the result is ____________ (always, sometimes, never) positive.

   b. When a *negative* integer is subtracted from a *negative* integer,

      the result is ____________ (always, sometimes, never) positive.

   c. When a *negative* integer is subtracted from a *positive* integer,

      the result is ____________ (always, sometimes, never) positive.

   d. When a *positive* integer is subtracted from a *negative* integer,

      the result is ____________ (always, sometimes, never) positive.

Practice

Simplify *the following.*

1. 6 – 4

2. 5 – (-3)

3. -14 + 5

4. -12 – (-2)

5. -57 + 3

6. 18 – 24

7. 21 + (-3)

8. 26 – (-26)

9. -37 + 17

10. -37 – (-17)

Practice

Simplify *the following* **expressions**. *Show* **essential** *steps.*

Example: 5 – (8 + 3)
5 – (8 + 3) =
5 – 11 = -6

1. 9 – (5 – 2 + 6)

2. (7 – 3) + (-5 + 3)

3. (5 + 32 – 36) + (12 + 5 – 10)

4. (-26 + 15 – 13) – (4 – 16 + 43)

5. (-15 + 3 – 7) – (26 – 14 + 10)

✓ **Check yourself:** The *sum* of the correct answers from numbers 1-5 above is -86.

Multiplying Integers

What *patterns* do you notice?

$$\begin{aligned} 3(4) &= 12 \\ 3(-4) &= -12 \end{aligned}$$

$$\begin{aligned} 2 \bullet 4 &= 8 \\ 2 \bullet -4 &= -8 \end{aligned}$$

$$\begin{aligned} 1(4) &= 4 \\ 1(-4) &= -4 \end{aligned}$$

$$\begin{aligned} 0 \bullet 4 &= 0 \\ 0 \bullet -4 &= 0 \end{aligned}$$

$$\begin{aligned} -1(4) &= -4 \\ -1(-4) &= 4 \end{aligned}$$

$$\begin{aligned} -2 \bullet 4 &= -8 \\ -2 \bullet -4 &= 8 \end{aligned}$$

$$\begin{aligned} -3(4) &= -12 \\ -3(-4) &= 12 \end{aligned}$$

Ask yourself:

- What is the sign of the **product** of two positive integers?
  $3(4) = 12$ $2 \bullet 4 = 8$ *positive*
- What is the sign of the *product* of two negative integers?
  $-1(-4) = 4$ $-2 \bullet -4 = 8$ *positive*
- What is the sign of the product of a positive integer and a negative integer or a negative integer and a positive integer?
  $3(-4) = -12$ $-2 \bullet 4 = -8$ *negative*
- What is the sign of the product of any integer and 0?
  $0 \bullet 4 = 0$ $0 \bullet -4 = 0$ *neither*; zero is neither positive nor negative

You can see that the sign of a *product* depends on the signs of the numbers being multiplied. Therefore, you can use the following rules to multiply integers.

Rules for Multiplying Integers

• The product of two positive integers is *positive*.	(+)(+) = +
• The product of two negative integers is *positive*.	(-)(-) = +
• The product of two integers with different signs is *negative*.	(+)(-) = - (-)(+) = -
• The product of any integer and 0 is *0*.	(a)(0) = 0 (-a)(0) = 0

Check Yourself Using a Calculator When Multiplying Integers

Use a **calculator** with a $\boxed{+/-}$ **sign-change** key.

For example, for -13 • -7, you would enter
13 $\boxed{+/-}$ $\boxed{x}$ 7 $\boxed{+/-}$ $\boxed{=}$ and get the answer 91.

Practice

Simplify *the following. Do as many mentally as you can.*

1. 5 x 6

5. 3 x (-18)

2. 6 x (-7)

6. 2 x 4

3. -4 x 8

7. -20 x (-20)

4. -5 x (-20)

8. -6 x (-6)

9. The temperature was 83 degrees at 9:00 PM and dropped an average of 1.5 degrees per hour for the next 9 hours. What was the temperature at 6:00 AM?

Answer: ____________ degrees

Dividing Integers

Think:

1. What would you multiply 6 by to get 42?
   6 • ? = 42
   Answer: 7 because 6 • 7 = 42

2. What would you multiply -6 by to get -54?
   -6 • ? = -54
   Answer: 9 because -6 • 9 = -54

3. What would you multiply -15 by to get 0?
   -15 • ? = 0
   Answer: 0 because -15 • 0 = 0

Remember: A **quotient** is the result of dividing two numbers.

Example

42 divided by 7 results in a *quotient* of 6.

$$42 \div 7 = \underset{\substack{\uparrow\\ \text{quotient}}}{6}$$

To find the quotient of 12 and 4 we write:

$$4\overline{)12} \quad \text{or} \quad 12 \div 4 \quad \text{or} \quad \frac{12}{4}$$

Each problem above is read "12 *divided by* 4." In each form, the quotient is 3.

$$\text{divisor} \rightarrow 4\overset{\substack{\text{quotient}\\ \downarrow\\ 3}}{\overline{)\underset{\substack{\uparrow\\ \text{dividend}}}{12}}} \qquad \text{or} \qquad \underset{\substack{\uparrow\\ \text{dividend}}}{12} \div \overset{\substack{\text{divisor}\\ \downarrow}}{4} = \underset{\substack{\uparrow\\ \text{quotient}}}{3} \qquad \text{or} \qquad \frac{\overset{\substack{\text{dividend}\\ \downarrow}}{12}}{\underset{\substack{\uparrow\\ \text{divisor}}}{4}} = 3 \leftarrow \text{quotient}$$

In $\frac{12}{4}$, the bar separating 12 and 4 is called a *fraction bar*. Just as *subtraction* is the *inverse of addition*, *division* is the *inverse of multiplication*. This means that *division* can be *checked by multiplication*.

$$\begin{array}{r}3\\4\overline{)12}\end{array} \qquad \textit{because} \qquad 3 \bullet 4 = 12$$

Division of integers is *related to* multiplication of integers. The sign rules for division can be discovered by writing a related multiplication problem.

For example,

$\frac{6}{2} = 3$ because $3 \bullet 2 = 6$

$\frac{-6}{2} = -3$ because $-3 \bullet 2 = -6$

$\frac{-6}{-2} = 3$ because $3 \bullet -2 = -6$

$\frac{6}{-2} = -3$ because $-3 \bullet -2 = 6$

Below are the rules used to divide integers.

Rules for Dividing Integers

- The quotient of two positive integers is *positive*. (+) ÷ (+) = +
- The quotient of two negative integers is *positive*. (-) ÷ (-) = +
- The quotient of two integers with different signs is *negative*. (+) ÷ (-) = -
(-) ÷ (+) = -
- The quotient of 0 divided by any nonzero integer is *0*. 0 ÷ *a* = 0

Note the special division properties of 0.

$0 \div 9 = 0$ $\qquad$ $0 \div -9 = 0$

$\frac{0}{5} = 0$ $\qquad$ $\frac{0}{-5} = 0$

$\begin{array}{r}0\\15\overline{)\ 0}\end{array}$ $\qquad$ $\begin{array}{r}0\\-15\overline{)\ 0}\end{array}$

Remember: Division by 0 is *undefined*. The quotient of any number and 0 is not a number.

We say that $\frac{9}{0}$, $\frac{5}{0}$, $\frac{15}{0}$, $\frac{-9}{0}$, $\frac{-5}{0}$, and $\frac{-15}{0}$ are *undefined*.

Likewise, $\frac{0}{0}$ is undefined.

For example, try to divide 134 by 0. To divide, think of the related multiplication problem.

$? \times 0 = 134$

Any number times 0 is 0—so mathematicians say that division by 0 is undefined.

Note: On most calculators, if you divide by 0, you will get an *error* indicator.

Check Yourself Using a Calculator When Dividing Integers

Use a **calculator** with a $\boxed{+/-}$ **sign-change** key.

For example, for $\frac{-54}{9}$, you would enter
54 $\boxed{+/-}$ $\boxed{\div}$ 9 $\boxed{=}$ and get the answer -6.

Practice

Simplify *the following. Do as many mentally as you can.*

1. 35 ÷ 5

2. 49 ÷ (-7)

3. 225 ÷ (-15)

4. -121 ÷ 11

5. 169 ÷ (-13)

6. -400 ÷ 25

7. -625 ÷ (-25)

8. 1,000 ÷ (-10)

9. -1,000 ÷ 100

10. -10,000 ÷ (-100)

11. The temperature of 69 degrees dropped to 44 degrees at an average rate of 6.25 degrees per hour. How many hours did the total drop of 25 degrees require?

    Answer: ____________ hours

Practice

Simplify *the following. Show* **essential steps**.

Example: $\dfrac{2(-3 \bullet 6)}{-4}$

$$\frac{2(-3 \bullet 6)}{-4} =$$

$$\frac{2(-18)}{-4} =$$

$$\frac{-36}{-4} =$$

$$9$$

1. $\dfrac{(6)(-5)(3)}{9}$

2. $(-3)(5)(\frac{4}{3})(-2)$

3. $(\frac{1}{2})(-4)(0)(5)$

4. $\dfrac{-3(4)(-2)(5)}{(-16)}$

5. $\dfrac{6(\frac{4}{7})(-\frac{3}{2})(-2)}{-(\frac{3}{7})}$

6. $\left[\dfrac{7-(-3)}{5-3}\right]\left[\dfrac{4+(-8)}{3-5}\right]$

7. $\left[\dfrac{12+(-2)}{3+(-8)}\right]\left[\dfrac{6+(-15)}{8-5}\right]$

8. $\dfrac{3(3+2)-3 \bullet 3+2}{3 \bullet 2+2(2-1)}$

Practice

Use the given **value** *of each* **variable** *to* **evaluate each expression**. *Show* **essential steps**.

Example: Evaluate $5\left(\frac{F-32}{9}\right)$

$\boxed{F = 212}$

Replace F with 212 and simplify.

$$5\left(\frac{212-32}{9}\right) =$$

$$5\left(\frac{180}{9}\right) =$$

$$5(20) =$$

$$100$$

1. $\boxed{E = 18 \quad e = 2 \quad R = 6}$

   $\frac{E-e}{R}$

2. $\boxed{P = 1{,}000 \quad r = 0.04 \quad t = 5}$

   $P + Prt$

3. $\boxed{r = 8 \quad h = 6}$

   $2r(r + h)$

Practice

Simplify *the following. Show* **essential steps**.

Example: $(4 + 1)^2 - \frac{4 \bullet 3^2}{6}$

$$(4 + 1)^2 - \frac{4 \bullet 3^2}{6} =$$

$$5^2 - \frac{4 \bullet 9}{6} =$$

$$25 - \frac{36}{6} =$$

$$25 - 6 =$$

$$19$$

1. $\frac{8 \bullet 2^2}{4^2} + (3 \bullet 1)^2$

2. $\frac{5^2 \bullet 3^2}{4} - (2 + 1)^2$

3. $\frac{3^2 \bullet 2^2}{7 - 2^2} + \frac{(-3)(2)^2}{6 - 3}$

4. $\dfrac{7^2 - 6^2}{10 + 3} + \dfrac{8^2 \bullet (-2)}{(-2)^4}$

5. $\dfrac{(-5)^2 - 3^2}{4 - 6} + \dfrac{-(3)^2 \bullet 2}{5 + 1}$

Use the given **value** *of each* **variable** *to* **evaluate the following expressions**. *Show* **essential steps**.

$\boldsymbol{x = 3}$	$\boldsymbol{y = -2}$

6. $\dfrac{-xy^2}{6} + 2x^2y$

7. $(x + y)^2 + (x - y)^2$

Practice

Match each definition with the correct term. Write the letter on the line provided.

_______ 1. the order in which two numbers are added or multiplied does *not* change their sum or product, respectively

_______ 2. the order of performing computations in parentheses first, then exponents or powers, followed by multiplication and/or division (as read from left to right), then addition and/or subtraction (as read from left to right)

_______ 3. a number's distance from zero (0) on a number line; distance expressed as a positive value

_______ 4. the number of times the base occurs as a factor

_______ 5. any symbol, usually a letter, which could represent a number

_______ 6. the way in which three or more numbers are grouped for addition or multiplication does *not* change their sum or product, respectively

_______ 7. the number zero (0); when zero (0) is added to another number the sum is the number itself

A. absolute value

B. additive identity

C. associative property

D. commutative property

E. exponent

F. order of operations

G. variable

Lesson Three Purpose

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Standard 10: Mathematical Reasoning and Problem Solving

- MA.912.A.10.3
  Decide whether a given statement is always, sometimes, or never true (statements involving linear or quadratic expressions, equations, or inequalities, rational or radical expressions, or logarithmic or exponential functions).

Algebraic Expressions

A mathematical expression with a letter in it is called an **algebraic expression**. The letter represents an unknown or mystery number. The letter used can be any letter in the alphabet.

For example: $7n$ means 7 times some number, n.

We use *algebraic expressions* to help us **solve equations**. Before we can use them, we must be able to translate them. Look at the following expressions translated into algebraic expressions.

- eight *more than* a number is expressed as

  $r + 8$

- sixteen *less than* a number is written as

  $y - 16$

- the *product* of a number and 12 looks like

  $12x$

- the **difference** between 19 and e is written as

  $19 - e$

- 4 *less than* 6 times a number means

  $6d - 4$

- the *quotient* of 18 and a number is

  $18 \div y$ or $\frac{18}{y}$

- four **cubed** is written as

  4^3

- three **squared** is written as

  3^2

Practice

Translate *the following* **expressions** *into* **algebraic expressions**.

1. four *times* a number

   ____

2. a number *times* four

   ____

3. eleven *more than* a number

   ____

4. eleven *increased* by a number

   ____

5. the *quotient* of 15 and a number

   ____

6. the *quotient* of a number and 15

   ____

7. seven *squared*

   ____

8. eight *cubed*

   ____

9. three *more than* twice a number

   ____

10. twice a number *less* three

____

11. three *less than* twice a number

____

12. twice the *sum* of a number and 21

____

13. one-half the *square* of a number

____

14. 22 *increased* by 4 *times* the square of a number

____

Practice

Translate *the following* **algebraic expressions** *into* **words**.

1. $6y$

____

2. $c - 5$

____

3. $5 - c$

____

4. $s + 21$

____

5. $21 + s$

____

6. $10r^2$

____

7. $3d + 7$

____

8. $8x - 11$

____

9. $6(v + 9)$

____

10. $\frac{1}{2}(5 + x^3)$

____

Lesson Four Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Algebra Body of Knowledge

Standard 10: Mathematical Reasoning and Problem Solving

- MA.912.A.10.1
  Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing- and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.

Working with Absolute Value

As discussed earlier in this unit, the *absolute value* of a number is actually the distance that number is from zero on a number line. Because distance is *always* positive, the result when taking the absolute value of a number is *always* positive.

The symbols for absolute value | | can also act as *grouping symbols*. Perform any operations within the grouping symbols first, just as you would within parentheses.

Look at these examples. Notice the digits are the same in each pair, but the answers are different due to the placement of the absolute value marks.

$|-7| + |5| = 7 + 5 = 12$

$|-7 + 5| = |-2| = 2$

$|6| - |-10| = 6 - 10 = -4$

$|6 - -10| = |6 + 10| = |16| = 16$

Practice

Answer the following. **Perform any operations *within* the grouping symbols first.**

1. | -23 + 37 |

2. | 21 – 44 |

3. | 16 + 4 | – | 32 |

4. | 16 + 4 | – | -32 |

5. 22 – | -10 | + | 56 |

✓ **Check yourself:** The *sum* of the answers from numbers 1-5 is | -81 |.

Practice

Use the given **value** *for each* **variable** *to* **evaluate the following expressions**. **Perform any operations *within* the grouping symbols first.**

$a = -5$	$b = 7$	$c = -9$

1. $|a| + |b| - |c|$

2. $|a + b| - |c|$

3. $|c - a| - |b|$

4. $|b + c| + |a|$

5. $|c - b| + |a|$

Use the given **value** *for each* **variable** *to* **evaluate the following expressions**. **Perform any operations *within* the grouping symbols first.**

$a = -5$	$b = 7$	$c = -9$

6. $| a + c | - | -c |$

7. $| a + b + c | - | c - b |$

8. $| a | + | b | + | c |$

9. $a - | b | - | c |$

10. $a + | -b | - | c |$

Practice

Use the given **value** *for each* **variable** *to* **evaluate the following expressions**. **Perform any operations *within* the grouping symbols first.**

$\boldsymbol{a = 6}$	$\boldsymbol{b = -7}$	$\boldsymbol{c = -8}$

1. $|a| + |b| - |c|$

2. $|a + b| - |c|$

3. $|c - a| - |b|$

4. $|a + b + c| - |c - b|$

5. $|a| + |b| + |c|$

6. $a - |b| - |c|$

7. $a + |-b| - |c|$

Answer the following. **Perform any operations *within* the grouping symbols first.**

8. | -33 + 57 |

9. | 16 – 34 |

10. | 26 + 4 | – | 36 |

11. | 26 + 4 | – | -36 |

12. 22 – | 20 | + | 32 |

Practice

Use the list below to complete the following statements.

element or **member**
even
grouping symbols
irrational
finite
odd
rational
real numbers
variable

1. The color green is a(n) ____________________ of the set of colors in the rainbow.

2. A(n) ____________________ a real number that *cannot* be expressed as a ratio of two integers.

3. { } and [ ] are examples of ____________________ .

4. Rational numbers and irrational numbers together make up the set of ____________________

5. Any symbol, usually a letter, which could represent a number in a mathematical expression is a ____________________ .

6. A ____________________ is a number can be expressed as a ratio $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

7. Any integer *not* divisible by 2 is called a(n) ____________________ integer.

8. Any integer divisible by 2 is called a(n) ____________________ integer.

9. A set that has bounds and is limited and a whole number can represent its number of elements is a ____________________ set.

Unit Review

Specify the following sets by **listing the elements** *of each.*

1. {whole numbers less than 8} ______________________________

2. {odd counting numbers less than 12} ________________________

3. {even integers between -5 and 6}___________________________

Write **finite** *if the set has bounds and is limited. Write* **infinite** *if the set has* no *boundaries and is* not *limited.*

_____________________ 4. {the colors in a crayon box}

_____________________ 5. {rational numbers}

_____________________ 6. {negative integers}

Write **True** *if the statement is correct. Write* **False** *if the statement is* not *correct.*

__________ 7. π is rational.

__________ 8. 0 is a whole number.

__________ 9. -9 is a counting number.

Complete the following statements.

10. The sum of *a positive number and a negative number* is ____________ (always, sometimes, never) positive.

11. The difference between *a negative number and its opposite* is ____________ (always, sometimes, never) zero.

Simplify *the following. Show* **essential steps**.

Remember: Order of operations—**P**lease **P**ardon **M**y **D**ear **A**unt **S**ally. (Also known as **P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally.)

12. $\dfrac{(5)(-2)(7)}{10}$

13. $\dfrac{(-6)(4) - (8)(2)}{9 - 4}$

14. $\left[\dfrac{16 - (-4)}{10 - 6}\right]\left[\dfrac{19 + (-8)}{(-2)(3)}\right]$

Use the given **value** *of each* **variable** *to* **evaluate each expression**. *Show* **essential steps**.

15. $P = 100 \quad r = 0.02 \quad t = 6$

Prt

16. $r = 6 \quad h = 8$

$2r(r + h)$

17. $x = -2 \quad y = 3$

$$\frac{-xy^2}{6} + 2xy^2$$

Simplify *the following*. *Show* **essential steps**.

18. $\dfrac{5^2 + (2^2 - 1)^3}{3^2 - 5}$

19. $\dfrac{3^2 \bullet 2(4)}{3^2 - 2^2 + 1} + \dfrac{5^2 + 7}{2^3}$

Translate *the following* **expressions** *into* **algebraic expressions**.

20. eight *more than* a number

____

21. 16 *less than* 2 times a number

____

22. four *more than* the sum of 13 and the square of a number

____

Translate *the following* **algebraic expressions** *into* **words**.

23. $8c - 5$

____

24. $4(x^3 + 7)$

____

25. $13(x + 9)$

____

Answer the following. **Perform any operations *within* the grouping symbols first.**

26. $| 13 - 24 |$

27. $| -19 + 17 | - | 41 + 8 |$

28. $36 - | 14 - 10 | + | 3 - 15 |$

Use the given **value** *for each* **variable** *to* **evaluate the following expressions***.* **Perform any operations *within* the grouping symbols first. Show essential steps.**

a = -6	**b = -2**	**c = 4**

29. $| a + b | - | c - a |$

30. $| b + c | + | -a - b |$

Unit 2: Algebraic Thinking

This unit emphasizes strategies used to solve equations and understand and solve inequalities.

Unit Focus

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Standard 3: Linear Equations and Inequalities

- MA.912.A.3.1
  Solve linear equations in one variable that include simplifying algebraic expressions.
- MA.912.A.3.2
  Identify and apply the distributive, associative, and commutative properties of real numbers and the properties of equality.

- MA.912.A.3.3
  Solve literal equations for a specified variable.

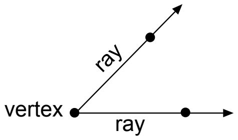
- MA.912.A.3.4
  Solve and graph simple and compound inequalities in one variable and be able to justify each step in a solution.

Standard 10: Mathematical Reasoning and Problem Solving

- MA.912.A.10.1
  Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing- and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.

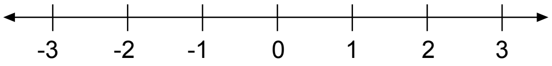
Vocabulary

Use the vocabulary words and definitions below as a reference for this unit.

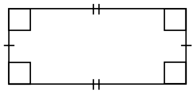
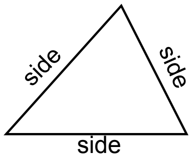
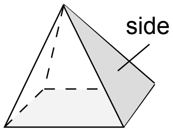
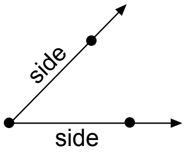
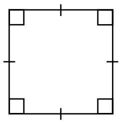
additive identity	the number zero (0); when zero (0) is added to another number the sum is the number itself *Example*: 5 + 0 = 5
additive inverses	a number and its opposite whose sum is zero (0); also called *opposites* *Example*: In the equation 3 + (-3) = 0, the additive inverses are 3 and -3.
angle (∠)	two rays extending from a common endpoint called the vertex; measures of angles are described in degrees (°) 
area (*A*)	the measure, in square units, of the inside region of a closed two-dimensional figure; the number of square units needed to cover a surface *Example*: A rectangle with sides of 4 units by 6 units has an area of 24 square units.
associative property	the way in which three or more numbers are grouped for addition or multiplication does *not* change their sum or product, respectively *Examples*: (5 + 6) + 9 = 5 + (6 + 9) *or* (2 x 3) x 8 = 2 x (3 x 8)
commutative property	the order in which any two numbers are added or multiplied does *not* change their sum or product, respectively *Examples*: 2 + 3 = 3 + 2 *or* 4 x 7 = 7 x 4

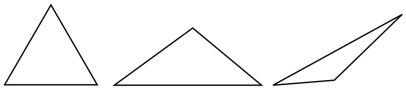
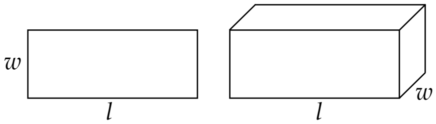
consecutive	in order *Example*: 6, 7, 8 are consecutive whole numbers and 4, 6, 8 are consecutive even numbers.
cube (power)	the third power of a number *Example*: $4^3 = 4 \times 4 \times 4 = 64$; 64 is the cube of 4
cubic units	units for measuring volume
decrease	to make less
degree (°)	common unit used in measuring angles
difference	a number that is the result of subtraction *Example*: In $16 - 9 = 7$, the difference is 7.
distributive property	the product of a number and the sum or difference of two numbers is equal to the sum or difference of the two products *Examples*: $x(a + b) = ax + bx$ $5(10 + 8) = 5 \bullet 10 + 5 \bullet 8$
equation	a mathematical sentence stating that the two expressions have the same value *Example*: $2x = 10$
equivalent (forms of a number)	the same number expressed in different forms *Example*: $\frac{3}{4}$, 0.75, and 75%
even integer	any integer divisible by 2; any integer with the digit 0, 2, 4, 6, or 8 in the units place; any integer in the set {… , -4, -2, 0, 2, 4, …}

expression	a mathematical phrase or part of a number sentence that combines numbers, operation signs, and sometimes variables *Examples*: $4r^2$; $3x + 2y$; An expression does not contain equal (=) or inequality (<, >, ≤, ≥, or ≠) signs.
formula	a way of expressing a relationship using variables or symbols that represent numbers
graph of a number	the point on a number line paired with the number
increase	to make greater
inequality	a sentence that states one expression is greater than (>), greater than or equal to (≥), less than (<), less than or equal to (≤), or not equal to (≠) another expression *Examples*: $a \neq 5$ *or* $x < 7$ *or* $2y + 3 \geq 11$
integers	the numbers in the set {… , -4, -3, -2, -1, 0, 1, 2, 3, 4, …}
inverse operation	an action that undoes a previously applied action *Example*: Subtraction is the inverse operation of addition.
irrational number	a real number that cannot be expressed as a ratio of two integers *Example*: $\sqrt{2}$
length (*l*)	a one-dimensional measure that is the measurable property of line segments

like terms	terms that have the same variables and the same corresponding exponents *Example*: In $5x^2 + 3x^2 + 6$, the like terms are $5x^2$ and $3x^2$.
measure (m) of an angle (∠)	the number of degrees (°) of an angle
multiplicative identity	the number one (1); the product of a number and the multiplicative identity is the number itself *Example*: $5 \text{ x } 1 = 5$
multiplicative inverse	any two numbers with a product of 1; also called *reciprocals* *Example*: 4 and $\frac{1}{4}$; zero (0) has no multiplicative inverse
multiplicative property of -1	the product of any number and -1 is the opposite or additive inverse of the number *Example*: $-1(a) = -a$ and $a(-1) = -a$
multiplicative property of zero	for any number a, $a \bullet 0 = 0$ and $0 \bullet a = 0$
negative numbers	numbers less than zero
number line	a line on which numbers can be written or visualized 
odd integer	any integer not divisible by 2; any integer with the digit 1, 3, 5, 7, or 9 in the units place; any integer in the set {… , -5, -3, -1, 1, 3, 5, …}

order of operations	the order of performing computations in parentheses first, then exponents or powers, followed by multiplication and/or division (as read from left to right), then addition and/or subtraction (as read from left to right); also called *algebraic order of operations* *Example*: $5 + (12 - 2) \div 2 - 3 \text{ x } 2 =$ $5 + 10 \div 2 - 3 \text{ x } 2 =$ $5 + 5 - 6 =$ $10 - 6 =$ 4
perimeter (*P*)	the distance around a figure
positive numbers	numbers greater than zero
power (of a number)	an exponent; the number that tells how many times a number is used as a factor *Example*: In 2^3, 3 is the power.
product	the result of multiplying numbers together *Example*: In $6 \text{ x } 8 = 48$, the product is 48.
quotient	the result of dividing two numbers *Example*: In $42 \div 7 = 6$, the quotient is 6.
ratio	the comparison of two quantities *Example*: The ratio of a and b is $a{:}b$ or $\frac{a}{b}$, where $b \neq 0$.
rational number	a number that can be expressed as a ratio $\frac{a}{b}$, where a and b are integers and $b \neq 0$
real numbers	the set of all rational and irrational numbers

reciprocals	any two numbers with a product of 1; also called *multiplicative inverse* *Examples:* 4 and $\frac{1}{4}$ are reciprocals because $\frac{4}{1}$ x $\frac{1}{4}$ = 1; $\frac{3}{4}$ and $\frac{4}{3}$ are reciprocals because $\frac{3}{4}$ x $\frac{4}{3}$ = 1; zero (0) has no multiplicative inverse
rectangle	a parallelogram with four right angles 
side	the edge of a polygon, the face of a polyhedron, or one of the rays that make up an angle *Example*: A triangle has three sides.  *edge of a polygon*  *face of a polyhedron*  *ray of an angle*
simplify an expression	to perform as many of the indicated operations as possible
solution	any value for a variable that makes an equation or inequality a true statement *Example*: In $y = 8 + 9$ $y = 17$ 17 is the solution.
solve	to find all numbers that make an equation or inequality true
square	a rectangle with four sides the same length 
square (of a number)	the result when a number is multiplied by itself or used as a factor twice *Example*: 25 is the square of 5.

square units	units for measuring area; the measure of the amount of an area that covers a surface
substitute	to replace a variable with a numeral *Example*: $8(a) + 3$ $8(5) + 3$
substitution property of equality	for any numbers a and b, if $a = b$, then a may be replaced by b
sum	the result of adding numbers together *Example*: In $6 + 8 = 14$, the sum is 14.
symmetric property of equality	for any numbers a and b, if $a = b$, then $b = a$
table (or chart)	a data display that organizes information about a topic into categories
triangle	a polygon with three sides 
variable	any symbol, usually a letter, which could represent a number
width (w)	a one-dimensional measure of something side to side 

Unit 2: Algebraic Thinking

Introduction

Algebraic thinking provides tools for looking at situations. You can state, simplify, and show relationships through algebraic thinking. When combining algebraic symbols with algebraic thinking, you can record information or ideas and gain insights into solving problems.

In this lesson you will use what you have learned to solve equations and inequalities on a more advanced level.

Lesson One Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Algebra Body of Knowledge

Standard 3: Linear Equations and Inequalities

- MA.912.A.3.1
  Solve linear equations in one variable that include simplifying algebraic expressions.

- MA.912.A.3.2
  Identify and apply the distributive, associative, and commutative properties of real numbers and the properties of equality.

- MA.912.A.3.4
  Solve and graph simple and compound inequalities in one variable and be able to justify each step in a solution.

Standard 10: Mathematical Reasoning and Problem Solving

- MA.912.A.10.1
  Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing- and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.

Solving Equations

A mathematical sentence that contains an equal sign (=) is called an **equation**. An *equation* is a mathematical sentence stating that the two **expressions** have the same value. An *expression* is a mathematical phrase, or part of a number sentence that contains numbers, operation signs, and sometimes **variables**.

We also learned the rules to add and subtract and to multiply and divide **positive numbers** and **negative numbers**.

Rules for Adding and Subtracting Positive and Negative Integers			
(+) + (+) =	+	(+) – (+) =	positive if first number is greater, otherwise it is negative
(–) + (–) =	–	(–) – (–) =	positive if first number is greater, otherwise it is negative
(+) + (–) =	use sign of **integer** with greater absolute value	(+) – (–) =	+
(–) + (+) =		(–) – (+) =	–

Rules for Multiplying and Dividing Positive and Negative Integers			
(+) • (+) =	+	(+) ÷ (+) =	+
(–) • (–) =	+	(–) ÷ (–) =	+
(+) • (–) =	–	(+) ÷ (–) =	–
(–) • (+) =	–	(–) ÷ (+) =	–

To **solve** the equation is to find the number that we can **substitute** for the *variable* to make the equation true.

Study these examples. Each equation has been *solved* and then checked by substituting the answer for the variable in the original equation. If the answer makes the equation a true sentence, it is called the **solution** of the equation.

Solve:

$$\begin{aligned} n + 14 &= -2 \\ n + 14 - 14 &= -2 - 14 \\ n &= -2 + -14 \\ n &= -16 \end{aligned}$$

Check:

$$\begin{aligned} n + 14 &= -2 \\ -16 + 14 &= -2 \\ -2 &= -2 \quad \textit{It checks!} \end{aligned}$$

Solve:

$$\begin{aligned} y - (-6) &= 2 \\ y + 6 - 6 &= 2 - 6 \\ y &= 2 + -6 \\ y &= -4 \end{aligned}$$

Check:

$$\begin{aligned} y - (-6) &= 2 \\ -4 - (-6) &= 2 \\ -4 + 6 &= 2 \\ 2 &= 2 \quad \textit{It checks!} \end{aligned}$$

Solve:

$$\begin{aligned} -6x &= -66 \\ \frac{-6x}{-6} &= \frac{-66}{-6} \\ x &= 11 \end{aligned}$$

Check:

$$\begin{aligned} -6x &= -66 \\ -6(11) &= -66 \\ -66 &= -66 \quad \textit{It checks!} \end{aligned}$$

Solve:

$$\begin{aligned} \frac{y}{-10} &= 5 \\ (-10)\,\frac{y}{-10} &= 5(-10) \\ y &= -50 \end{aligned}$$

Check:

$$\begin{aligned} \frac{y}{-10} &= 5 \\ \frac{-50}{-10} &= 5 \\ 5 &= 5 \quad \textit{It checks!} \end{aligned}$$

Practice

Solve *each equation and* **check**. *Show* **essential steps**.

1. $y + 12 = 2$

2. $a - (-2) = 2$

3. $r + 15 = -25$

4. $0 = y + -46$

5. $15y = -30$

6. $\frac{y}{15} = -2$

7. $\frac{x}{5} = -9$

8. $-9y = 270$

9. $m - 9 = -8$

10. $3 + x = -3$

11. $\frac{n}{-5} = -2$

12. $-55 = -5a$

13. $12 = -6 + x$

14. $t - 20 = -15$

Interpreting Words and Phrases

Words and phrases can suggest relationships between numbers and mathematical operations. In Unit 1 we learned how words and phrases can be translated into mathematical expressions. Appendix B also contains a list of mathematical symbols and their meanings.

Relationships between numbers can be indicated by words such as **consecutive**, *preceding*, *before*, and *next*. Also, the same mathematical expression can be used to translate many different word expressions.

Below are some of the words and phrases we associate with the four mathematical operations and with powers of a number.

Mathematical Symbols and Words

+	**–**	**x**	**÷**	**power**
add	subtract	multiply	divide	power
sum	difference	product	quotient	square
plus	minus	times		cube
total	remainder	of		
more than	less than	twice		
increased by	decreased by	doubled		

Practice

Write an **equation** *and* **solve** *the problem.*

Example: Sixteen less than a number *n* is 48. What is the number?

Remember: The word *is* means *is equal to* and translates to an = sign.

$$\underbrace{16 \text{ less than a number } n}_{n-16} \underset{\downarrow}{=} \underset{\downarrow}{48}$$

$$\begin{aligned} n - 16 &= 48 \\ n - 16 + 16 &= 48 + 16 \\ n &= 64 \end{aligned}$$

Note: To write 16 less than *n*, you write *n* – 16.

So 64 – 16 = 48 *or* 16 less than 64 is 48.

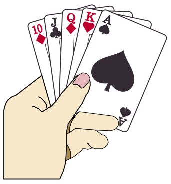
1. A number increased by 9 equals -7. What is the number? (Let *d* = the number).

2. A number times -12 equals -72. What is the number? (Let *x* = the number.)

3. A number decreased by 5 equals -9. What is the number? (Let *y* = the number.)

4. A number divided by 7 equals -25. What is the number? (Let n = the number.)

5. In a card game, Ann made 30 points on her first hand. After the second hand, her total score was 20 points. What was her score on the second hand?



6. A scuba diver is at the -30 foot level. How many feet will she have to rise to be at the -20 foot level?

Solving Two-Step Equations

When solving an equation, you want to get the *variable* by itself on one side of the equal sign. You do this by *undoing* all the operations that were done on the variable. In general, undo the addition or subtraction first. Then undo the multiplication or division.

Study the following examples.

A. Solve:

$$\begin{aligned} 2y + 2 &= 30 \\ 2y + 2 - 2 &= 30 - 2 && \longleftarrow \text{subtract 2 from each side} \\ \frac{2y}{2} &= \frac{28}{2} && \longleftarrow \text{divide each side by 2} \\ y &= 14 \end{aligned}$$

Check:

$$\begin{aligned} 2y + 2 &= 30 \\ 2(14) + 2 &= 30 && \longleftarrow \text{replace } y \text{ with 14} \\ 28 + 2 &= 30 \\ 30 &= 30 && \textit{It checks!} \end{aligned}$$

B. Solve:

$$\begin{aligned} 2x - 7 &= -29 \\ 2x - 7 + 7 &= -29 + 7 && \longleftarrow \text{add 7 to each side} \\ \frac{2x}{2} &= \frac{-22}{2} && \longleftarrow \text{divide each side by 2} \\ x &= -11 \end{aligned}$$

Check:

$$\begin{aligned} 2x - 7 &= -29 \\ 2(-11) - 7 &= -29 && \longleftarrow \text{replace } x \text{ with -11} \\ -22 - 7 &= -29 \\ -29 &= -29 && \textit{It checks!} \end{aligned}$$

C. Solve:

$$\begin{aligned} \tfrac{n}{7} + 18 &= 20 \\ \tfrac{n}{7} + 18 - 18 &= 20 - 18 && \longleftarrow \text{subtract 18 from each side} \\ (\cancel{7})\tfrac{n}{\cancel{7}} &= 2(7) && \longleftarrow \text{multiply each side by 7} \\ n &= 14 && \longleftarrow \text{simplify both sides} \end{aligned}$$

Check:

$$\begin{aligned} \tfrac{n}{7} + 18 &= 20 \\ \tfrac{14}{7} + 18 &= 20 && \longleftarrow \text{replace } n \text{ with 14} \\ 2 + 18 &= 20 \\ 20 &= 20 && \textit{It checks!} \end{aligned}$$

D. Solve:

$$\begin{aligned} \tfrac{t}{-2} + 4 &= -10 \\ \tfrac{t}{-2} + 4 - 4 &= -10 - 4 && \longleftarrow \text{subtract 4 from each side} \\ \frac{\cancel{(-2)}\, t}{\cancel{-2}} &= -14(-2) && \longleftarrow \text{multiply each side by -2} \\ t &= 28 && \longleftarrow \text{simplify both sides} \end{aligned}$$

Check:

$$\begin{aligned} \tfrac{t}{-2} + 4 &= -10 \\ \tfrac{28}{-2} + 4 &= -10 && \longleftarrow \text{replace } t \text{ with 28} \\ -14 + 4 &= -10 \\ -10 &= -10 && \textit{It checks!} \end{aligned}$$

Practice

Solve *each equation and* **check**. *Show* **essential steps**.

1. $4x + 8 = 16$

2. $4y – 6 = 10$

3. $5n + 3 = -17$

4. $2y – 6 = -18$

5. $-8y – 21 = 75$

6. $\frac{a}{8} – 17 = 13$

7. $13 + \frac{x}{-3} = -4$

8. $\frac{n}{8} + 1 = 4$

9. $-3b + 5 = 20$

10. $6 = \frac{x}{4} - 14$

11. $-7y + 9 = -47$

12. $\frac{n}{-6} - 17 = -8$

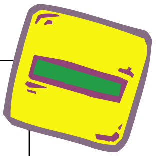
Use the list below to decide which **equation** *to use to solve each problem. Then* **solve** *the problem.*

Equation A: $\frac{n}{4} + 2 = 10$

Equation B: $\frac{n}{4} - 2 = 10$

Equation C: $4n + 2 = 10$

Equation D: $4n - 2 = 10$



13. Two more than the product of 4 and Ann's age is 10.

Equation: ______________________

How old is Ann? n = ___________

14. If you multiply Sean's age by 4 and then subtract 2, you get 10.

Equation: ______________________

What is Sean's age? n = ___________

15. If you divide Joe's age by 4 and then add 2, you get 10.

Equation: ______________________

What is Joe's age? n = ___________

16. Divide Jenny's age by 4, then subtract 2, and get 10.

Equation: ______________________

What is Jenny's age? n = ___________

Circle the letter of the correct answer.

17. The sentence that means the same as the equation $\frac{1}{3}y + 8 = 45$ is ___________ .

a. Eight *more than* one-third of y is 45.
b. One-third of y is eight *more than* 45.
c. y is eight *less than* one-third of 45.
d. y is eight *more than* one-third of 45.

Special Cases

Reciprocals: Two Numbers Whose Product is 1

Note: $5 \bullet \frac{1}{5} = 1$ and $\frac{5}{5} = 1$

When you multiply 5 by $\frac{1}{5}$ and divide 5 by 5, both equations yield 1.

We see that 5 is the **reciprocal** of $\frac{1}{5}$ and $\frac{1}{5}$ is the *reciprocal* of 5. Every number but zero has a reciprocal. (Division by zero is undefined.) Two numbers are reciprocals if their product is 1.

Below are some examples of numbers and their reciprocals.

Number	Reciprocal
$-\frac{1}{4}$	-4
1	1
$\frac{-2}{3}$	$\frac{-3}{2}$
$\frac{7}{8}$	$\frac{8}{7}$
-2	$-\frac{1}{2}$
$\frac{1}{7}$	7
x	$\frac{1}{x}$

Multiplication Property of Reciprocals

any nonzero number times its reciprocal is 1

$$x \bullet \frac{1}{x} = 1$$

If $x \neq 0$

Remember: When two numbers are reciprocals of each other, they are also called **multiplicative inverses** of each other.

Study the following two examples.

Method 1: Division Method

$$\begin{aligned} 5x - 6 &= 9 \\ 5x - 6 + 6 &= 9 + 6 \\ 5x &= 15 \\ \tfrac{5x}{5} &= \tfrac{15}{5} \\ x &= 3 \end{aligned}$$

Method 2: Reciprocal Method

$$\begin{aligned} 5x - 6 &= 9 \\ 5x - 6 + 6 &= 9 + 6 \\ 5x &= 15 \\ \tfrac{1}{5} \bullet 5x &\quad \tfrac{1}{5} \bullet 15 \\ x &= 3 \end{aligned}$$

Both methods work well. However, the *reciprocal method* is probably easier in the next two examples, which have fractions.

$$\begin{aligned} -\tfrac{1}{5}x - 1 &= 9 \\ -\tfrac{1}{5}x - 1 + 1 &= 9 + 1 \\ -\tfrac{1}{5}x &= 10 \\ \mathbf{-5} \bullet -\tfrac{1}{5}x &= \mathbf{-5} \bullet 10 \quad \longleftarrow \text{ multiply by reciprocal of } -\tfrac{1}{5} \text{ which is } -5 \\ x &= -50 \end{aligned}$$

Here is another equation with fractions.

$$\begin{aligned} -\tfrac{3}{4}x + 12 &= 36 \\ -\tfrac{3}{4}x + 12 - 12 &= 36 - 12 \\ -\tfrac{3}{4}x &= 24 \\ -\tfrac{\mathbf{4}}{\mathbf{3}} \bullet -\tfrac{3}{4}x &= -\tfrac{\mathbf{4}}{\mathbf{3}} \bullet 24 \quad \longleftarrow \text{ multiply by reciprocal of } -\tfrac{3}{4} \text{ which is } -\tfrac{4}{3} \\ 1 \bullet x &= -32 \\ x &= -32 \end{aligned}$$

Multiplying by -1

Here is another equation which sometimes gives people trouble.

$5 - x = -10$

Remember: $5 - x$ is not the same thing as $x - 5$. To solve this equation we need to make the following observation.

Property of Multiplying by -1

-1 times a number equals the opposite of that number

$\mathbf{-1 \bullet x = -x}$

This property is also called the **multiplicative property of -1**, which says the *product* of any number and -1 is the opposite or **additive inverse** of the number.

See the following examples.

$-1 \bullet 5 = -5$
$-1 \bullet (-6) = 6$

Now let's go back to $5 - x = -10$ using the property of multiplying by -1. We can rewrite the equation as follows.

$$\begin{aligned} 5 - 1x &= -10 \\ 5 - 1x \mathbf{- 5} &= -10 \mathbf{- 5} &\quad \longleftarrow \text{ subtract 5 from both sides to} \\ -1x &= -15 &\quad \text{isolate the variable} \\ \frac{-1x}{-1} &= \frac{-15}{-1} \\ x &= 15 \end{aligned}$$

This example requires great care with the positive numbers and negative signs.

$$\begin{aligned} 11 - \tfrac{1}{9}x &= -45 \\ 11 - \tfrac{1}{9}x \mathbf{- 11} &= -45 \mathbf{- 11} &\quad \longleftarrow \text{ subtract 11 from both sides} \\ -\tfrac{1}{9}x &= -56 &\quad \text{to isolate the variable} \\ \mathbf{-9} \bullet -\tfrac{1}{9}x &= \mathbf{-9} \bullet -56 &\quad \longleftarrow \text{ multiply by reciprocal of } -\tfrac{1}{9} \text{ which is } -9 \\ x &= 504 \end{aligned}$$

Consider the following example.

Remember: *Decreased by* means *subtract*, *product* means *multiply*, and *is* translates to the = sign.

Five decreased by the product of 7 and *x* is -6. Solve for *x*.

Five decreased by the product of 7 and *x* is -6.

$$\begin{aligned} 5 \quad - \quad 7x &= -6 \\ 5 - 7x \mathbf{- 5} &= -6 \mathbf{- 5} &\quad \longleftarrow \text{ subtract 5 from both sides to} \\ -7x &= -11 &\quad \text{isolate the variable} \\ \frac{-7x}{-7} &= \frac{-11}{-7} \\ x &= \frac{11}{7} \text{ or } 1\tfrac{4}{7} \end{aligned}$$

Practice

Write the **reciprocals** *of the following. If none exist, write* **none**.

1. 10

2. -6

3. $\frac{5}{6}$

4. $-\frac{9}{10}$

5. 0

Solve *the following. Show* **essential steps**.

6. $\frac{1}{5}x + 3 = 9$

7. $\frac{1}{4}x - 7 = 2$

8. $-\frac{1}{2}x - 7 = 23$

9. $10 - 6x = 11$

10. $15 - x = 10$

11. $\frac{1}{8}x + 4 = -6$

12. $-6 - x = 10$

14. $4 - \frac{3}{7}x = 10$

13. $2 + \frac{5}{6}x = -8$

Check yourself: *Use the list of* **scrambled answers** *below and check your answers to problems 6-14.*

-80 -60 -16 -14 -12 $-\frac{1}{6}$ 5 30 36

Answer the following.

15. The difference between 12 and $2x$ is -8. Solve for x.

Practice

Match each definition with the correct term. Write the letter on the line provided.

_______ 1. to find all numbers that make an equation or inequality true

_______ 2. numbers less than zero

_______ 3. a mathematical phrase or part of a number sentence that combines numbers, operation signs, and sometimes variables

_______ 4. any value for a variable that makes an equation or inequality a true statement

_______ 5. any symbol, usually a letter, which could represent a number

_______ 6. to replace a variable with a numeral

_______ 7. a mathematical sentence stating that the two expressions have the same value

_______ 8. numbers greater than zero

_______ 9. any two numbers with a product of 1; also called *multiplicative inverse*

A. equation

B. expression

C. negative numbers

D. positive numbers

E. reciprocals

F. solution

G. solve

H. substitute

I. variable

Practice

Use the list below to write the correct term for each definition on the line provided.

additive inverses
decrease
difference
increase
multiplicative inverses
multiplicative property of -1
product

________________________ 1. any two numbers with a product of 1; also called *reciprocals*

________________________ 2. the result of multiplying numbers together

________________________ 3. to make greater

________________________ 4. to make less

________________________ 5. the product of any number and -1 is the opposite or additive inverse of the number

________________________ 6. a number that is the result of subtraction

________________________ 7. a number and its opposite whose sum is zero (0); also called *opposites*

Lesson Two Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Algebra Body of Knowledge

Standard 3: Linear Equations and Inequalities

- MA.912.A.3.1
  Solve linear equations in one variable that include simplifying algebraic expressions.
- MA.912.A.3.2
  Identify and apply the distributive, associative, and commutative properties of real numbers and the properties of equality.

- MA.912.A.3.4
  Solve and graph simple and compound inequalities in one variable and be able to justify each step in a solution.

Standard 10: Mathematical Reasoning and Problem Solving

- MA.912.A.10.1
  Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing- and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.

The Distributive Property

Consider 4(2 + 6). The rules for **order of operations** would have us add inside the parentheses first.

$$\begin{aligned} 4(2+6) &= \\ 4(8) &= \\ 32 & \end{aligned}$$

Remember: Rules for the order of operations

Always start on the *left* and move *to* the *right*.

1. Do operations inside *grouping symbols* first.
2. Then do all *powers* (exponents) **or** *roots*.
3. Next do *multiplication* **or** *division*—as they occur from left to right.
4. Finally, do *addition* **or** *subtraction*—as they occur from left to right.

However, there is a second way to do the problem.

$$\begin{aligned}4(2+6) &= \\ 4(2)+4(6) &= \\ 8+24 &= \\ 32 &\end{aligned}$$

In the second way, the 4 is *distributed* over the addition. This second way of doing the problem illustrates the **distributive property**.

The Distributive Property

For any numbers a, b, and c,
$a(b + c) = ab + ac$

Also, it works for subtraction:
$a(b - c) = ab - ac$

This property is most useful in simplifying expressions that contain variables, such as $2(x + 4)$.

To **simplify an expression** we must perform as many of the indicated operations as possible. However, in the expression $2(x + 4)$, we can't add first, unless we know what number x represents. The *distributive property* allows us to rewrite the equation:

$$\begin{aligned}2(x+4) &= \\ 2x+2(4) &= \\ 2x+8 &\end{aligned}$$

The distributive property allows you to multiply each term *inside* a set of parentheses by a factor *outside* the parentheses. We say multiplication is *distributive over* addition and subtraction.

$$\begin{aligned}5(3+1) &= (5 \bullet 3) + (5 \bullet 1) \\ 5(4) &= 15 + 5 \\ 20 &= 20\end{aligned} \qquad \begin{aligned}5(3-1) &= (5 \bullet 3) - (5 \bullet 1) \\ 5(2) &= 15 - 5 \\ 10 &= 10\end{aligned}$$

Not all operations are distributive. You cannot distribute division over addition.

$$\begin{aligned} 14 \div (5 + 2) &\neq 14 \div 5 + 14 \div 2 \\ 14 \div 7 &\neq 2.8 + 7 \\ 2 &\neq 9.8 \end{aligned}$$

Study the chart below.

Properties

Addition	**Multiplication**
Commutative: $a + b = b + a$ Associative: $(a + b) + c = a + (b + c)$ Identity: 0 is the identity. $a + 0 = a$ and $0 + a = a$	Commutative: $ab = ba$ Associative: $(ab)c = a(bc)$ Identity: 1 is the identity. $a \cdot 1 = a$ and $1 \cdot a = a$
Addition	**Subtraction**
Distributive: $a(b + c) = ab + ac$ and $(b + c)a = ba + ca$	Distributive: $a(b - c) = ab - ac$ and $(b - c)a = ba - ca$

These properties deal with the following:

order—**commutative property** of addition and commutative property of multiplication

grouping—**associative property** of addition and associative property of multiplication

identity—**additive identity** property and **multiplicative identity** property

zero—**multiplicative property of zero**

distributive—**distributive property** of multiplication over addition and over subtraction

Notice in the distributive property that it does *not* matter whether $\boldsymbol{a}$ is placed on the *right* or the *left* of the expression in parentheses.

$$\boldsymbol{a}(b + c) = (b + c)\boldsymbol{a} \text{ or } \boldsymbol{a}(b - c) = (b - c)\boldsymbol{a}$$

The **symmetric property of equality** (if $a = b$, then $b = a$) says that if one quantity equals a second quantity, then the second quantity also equals the first quantity. We use the **substitution property of equality** when replacing a variable with a number or when two quantities are equal and one quantity can be replaced by the other. Study the chart and examples below that describe properties of equality.

Properties of Equality

Reflexive:	$a = a$
Symmetric:	If $a = b$, then $b = a$.
Transitive:	If $a = b$ and $b = c$, then $a = c$.
Substitution:	If $a = b$, then a may be replaced by b.

Examples of Properties of Equality

Reflexive:	$8 - e = 8 - e$
Symmetric:	If $5 + 2 = 7$, then $7 = 5 + 2$.
Transitive:	If $9 - 2 = 4 + 3$ and $4 + 3 = 7$, then $9 - 2 = 7$.
Substitution:	If $x = 8$, then $x \div 4 = 8 \div 4$. x is replaced by 8.
	or
	If $9 + 3 = 12$, then $9 + 3$ may be replaced by 12.

Study the following examples of how to simplify expressions. Refer to the charts above and on the previous page as needed.

$5(6x + 3) + 8$	
$5(6x + 3) + 8 =$	← use the distributive property to
$5(6x) + 5(3) + 8 =$	distribute 5 over 6*x* and 3
$30x + 15 + 8 =$	← use the associative property to
$30x + 23$	associate 15 and 8

and

$6 + 2(4x - 3)$	
$6 + 2(4x - 3) =$	← use order of operations to multiply before adding, then
$6 + 2(4x) + 2(-3) =$	← distribute 2 over 4*x* and -3
$6 + 8x + -6 =$	← use the associative property to associate 6 and -6
$8x + 0 =$	← use the identity property of addition
$8x$	

Practice

Simplify *by using the* **distributive property**. *Show* **essential steps**.

1. $10(x + 9)$

2. $16(z - 3)$

3. $a(b + 5)$

4. $5(x + 3) + 9$

5. $4(x - 5) + 20$

6. $5(3 + x) - 9$

7. $4(3x + 7) - 2$

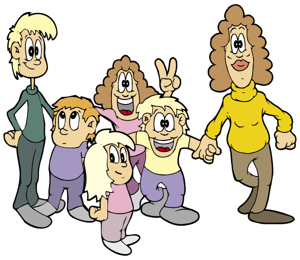
8. $-6(x + 3) + 18$

9. $30 + 2(x + 8)$

10. $x(x + 3)$

11. $a(b + 10)$

Circle the letter of the correct answer.



12. Mrs. Smith has 5 children. Every fall she buys each child a new book bag for $20, a new notebook for $3.50, and other school supplies for $15. Which expression is a correct representation for the amount she spends?

    a. 5(20 + 3.50 + 15)

    b. 5 + (20 + 3.50 + 15)

    c. 5(20 • 3.50 • 15)

    d. 5 • 20 + 3.50 + 15

Number the **order of operations** *in the correct* **order**. *Write the numbers 1-4 on the line provided.*

_______ 13. addition *or* subtraction

_______ 14. powers (exponents)

_______ 15. parentheses

_______ 16. multiplication *or* division

Simplifying Expressions

Here's how to use the distributive property and the definition of subtraction to simplify the following expressions.

Example 1:

Simplify

$-7a - 3a$

$$\begin{aligned} -7a - 3a &= -7a + -3a \\ &= (-7 + -3)a \quad \longleftarrow \text{ use the distributive property} \\ &= -10a \end{aligned}$$

Example 2:

Simplify

$10c - c$

$$\begin{aligned} 10c - c &= 10c - 1c \\ &= 10c + -1c \\ &= (10 + -1)c \quad \longleftarrow \text{ use the distributive property} \\ &= 9c \end{aligned}$$

The expressions $-7a - 3a$ and $-10a$ are called **equivalent** *expressions*. The expressions $10c - c$ and $9c$ are also called *equivalent* expressions. Equivalent expressions express the same number. An expression is in simplest form when it is replaced by an equivalent expression having no **like terms** and no parentheses.

Study these examples.

$$\begin{aligned} -5x + 4x &= (-5 + 4)x \\ &= -x \end{aligned}$$

$$\begin{aligned} 5y - 5y &= 5y + -5y \\ &= (5 + -5)y \\ &= 0y \quad \longleftarrow \text{ multiplicative property of zero} \\ &= 0 \end{aligned}$$

The multiplicative property of 0 says for any number a,
$a \bullet 0 = 0 \bullet a = 0$.

The following shortcut is frequently used to simplify expressions.

First

- rewrite each subtraction as adding the opposite
- then combine *like terms* (terms that have the same variable) by adding.

Simplify

like terms

$2a + 3 - 6a$

$2a + 3 - 6a = 2a + 3 + \text{-}6a$ ← rewrite – 6*a* as + -6*a*

$= \text{-}4a + 3$ ← combine like terms by adding

Simplify

like terms

$8b + 7 - b - 6$

$8b + 7 - b - 6 = 8b + 7 + \text{-}1b + \text{-}6$ ← rewrite – *b* as + -1*b* and – 6 as + -6

$= 7b + 1$ ← combine like terms by adding

like terms

Simplify

$7x + 5 + 3x = 10x + 5$ ← combine like terms

like terms

Practice

Simplify *by combining* **like terms**. *Show* **essential steps**.

1. $5n + 3n$

2. $6n - n$

3. $8y - 8y$

4. $7n + 3n - 6$

5. $-7n - 3n - 6$

6. $6n - 3 + 7$

7. $4x + 11x$

8. $4x - 11x$

9. $-4x - 11x$

10. $10y - 4y + 7$

11. $10y + 4y - 7$

12. $10y - 4 - 7$

13. $8c - 12 - 6c$

14. $8c - 12c - 6$

15. $-10y - y - 15$

16. $-10y + y - 15$

17. $15x - 15x + 6$

18. $15x - 15 + 8x$

19. $20n - 6 - n + 8$

20. $20n - 6n - 1 + 8$

21. $12c - 15 - 12c - 17$

22. $12c - 15c - 12 - 18$

Practice

Match each definition with the correct term. Write the letter on the line provided.

_______ 1. $a(b + c) = ab + ac$

_______ 2. $a + b = b + a$

_______ 3. $(a + b) + c = a + (b + c)$

A. associative property

B. commutative property

C. distributive property

_______ 4. $a \bullet 1 = a$

_______ 5. $a + 0 = a$

_______ 6. $a \bullet 0 = 0$

A. additive identity

B. multiplicative identity

C. multiplicative property of zero

_______ 7. if $a = b$, then $b = a$

_______ 8. if $a = b$, then a may be replaced by b

A. substitution property of equality

B. symmetric property of equality

_______ 9. terms that have the same variables and the same corresponding exponents

_______ 10. to perform as many of the indicated expressions as possible

_______ 11. the order of performing computations in parentheses first, then exponents or powers, followed by multiplication and/or division (as read from left to right), then addition and/or subtraction (as read from left to right)

A. like terms

B. order of operations

C. simplify an expression

Equations with Like Terms

Consider the following equation.

$$2x + 3x + 4 = 19$$

Look at both sides of the equation and see if either side can be simplified.

Always simplify first
by combining like terms.

$$\begin{aligned}
2x + 3x + 4 &= 19 \\
5x + 4 &= 19 && \longleftarrow \text{add like terms} \\
5x + 4 - 4 &= 19 - 4 && \longleftarrow \text{subtract 4 from each side} \\
5x &= 15 \\
\frac{5x}{5} &= \frac{15}{5} && \longleftarrow \text{divide each side by 5} \\
x &= 3
\end{aligned}$$

Always mentally check your answer by *substituting* the solution for the variable in the original equation.

Substitute 3 for x in the equation.

$$\begin{aligned}
2x + 3x + 4 &= 19 \\
2(3) + 3(3) + 4 &= 19 \\
6 + 9 + 4 &= 19 \\
19 &= 19 && \textit{It checks!}
\end{aligned}$$

Consider this example.

The product of *x* and 7 plus the product of *x* and 3 is 45.

Remember: To work a problem like this one, we need to remember two things. The word *product* means *multiply* and the word *is* always translates to =.

$$\text{The }\underbrace{\text{product of } x \text{ and } 7}\text{ plus the }\underbrace{\text{product of } x \text{ and } 3}\text{ is } 45.$$

$$\begin{aligned}
7x + 3x &= 45 \\
10x &= 45 && \longleftarrow \text{ add like terms} \\
\frac{10x}{10} &= \frac{45}{10} && \longleftarrow \text{ divide both sides by 10} \\
x &= 4.5
\end{aligned}$$

Check by substituting 4.5 for *x* in the original equation.

$$\begin{aligned}
7x + 3x &= 45 \\
7(4.5) + 3(4.5) &= 45 \\
31.5 + 13.5 &= 45 \\
45 &= 45 && \textit{It checks!}
\end{aligned}$$

Here is another example which appears to be more challenging.

$$\begin{aligned}
3x - 2 - x + 10 &= -12 \\
3x - 2 - 1x + 10 &= -12 && \longleftarrow \text{ remember: } 1 \cdot x = x \\
3x - 1x - 2 + 10 &= -12 && \longleftarrow \text{ add like terms} \\
2x + 8 &= -12 \\
2x + 8 - 8 &= -12 - 8 && \longleftarrow \text{ subtract 8 from both sides} \\
2x &= -20 \\
\frac{2x}{2} &= \frac{-20}{2} && \longleftarrow \text{ divide both sides by 2} \\
x &= -10
\end{aligned}$$

Check by substituting -10 into the original equation.

$$\begin{aligned}
3x - 2 - x + 10 &= -12 \\
3(-10) - 2 - (-10) + 10 &= -12 \\
-30 - 2 + 10 + 10 &= -12 \\
-32 + 20 &= -12 \\
-12 &= -12 && \textit{It checks!}
\end{aligned}$$

Practice

Solve *these equations by first* **simplifying** *each side. Show* **essential steps**.

1. $4x + 6x = -30$
2. $-2x + 10x - 6x = -12$
3. $12m - 6m + 4 = -32$
4. $3 = 4x + x - 2$
5. $3y - y - 8 = 30$
6. $x + 10 - 2x = -2$
7. $13x + 105 - 8x = 0$
8. $2x + 10 + 3x - 8 = -13$

✓ **Check yourself:** Add all your answers for problems 1-8. Did you get a *sum of* -7? If yes, complete the practice. If no, correct your work before continuing.

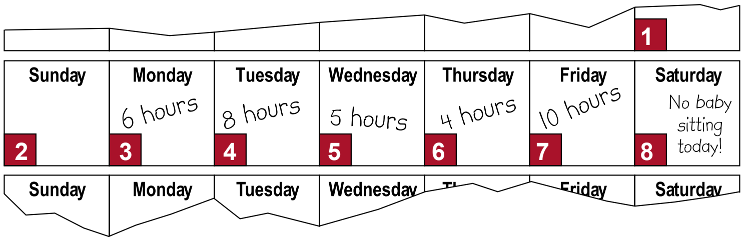
Write an **equation** *to represent each* **situation**. *Then* **solve** *the equation* **for *x***. *Show* **essential steps**.

9. The sum of $2x$, $3x$, and $5x$ is 50.

10. The difference of $3x$ and $8x$ is -15.

11. The sum of $6x$, $-2x$, and $10x$, decreased by 15 is 13.

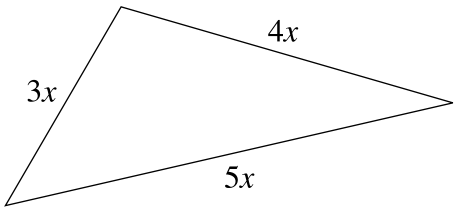
12. Your neighbor hired you to babysit for $\$x$ per hour. Here is a record of last week:



Your total salary for the week was $198.00. How much do you earn per hour?

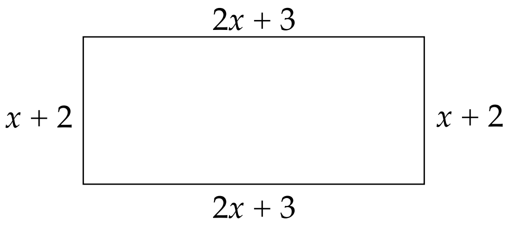
13. The **perimeter (*P*)** of the **triangle** is 48 inches. What is x?

Remember: The *perimeter* of a figure is the distance around a figure, or the *sum* of the **lengths (*l*)** of the **sides**.



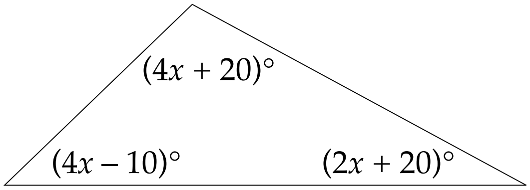
14. Use the answer from problem 13 to find the *length (l)* of each side of the triangle. Do the sides add up to 48 inches?

15. The perimeter of the **rectangle** is 58 inches. What is x?



16. Use the answer from problem 15 to find the length and **width (*w*)** of the rectangle. Do the sides add up to 58 inches?

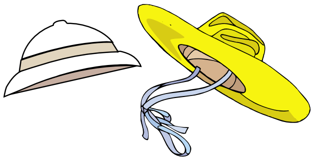
17. In any triangle, the sum of the **measures (m) of the angles (∠)** is always 180 **degrees (°)**. What is x?



18. Using the answer from problem 17, find the measure of each **angle (∠)**. Do the angles add up to 180 degrees?

Circle the letter of the correct answer.

19. You and your friend go to a popular theme park in central Florida. Admission for two comes to a total of $70. Both of you immediately buy 2 hats to wear during the day. Later, as you are about to leave you decide to buy 4 more hats for your younger brothers and sisters who didn't get to come. The total bill for the day is $115.00. Which equation could you use to find the cost of a single hat?



a. $6x = 115$
b. $70 + 2x + 4x = 115$
c. $70 - 6x = 115$
d. $70 + 2x = 115 + 6x$

Complete the following.

20. A common mistake in algebra is to say that

    $3x + 4x = 7x^2$,

    instead of

    $3x + 4x = 7x$.

    Let $x = 2$ and substitute into both expressions below.

Remember: When you are doing $7x^2$ the rules for the order of operation require that you square *before* you multiply!

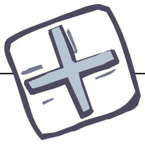
$$\boxed{\mathbf{x = 2}}$$

$3x + 4x = 7x$

$3x + 4x = 7x^2$

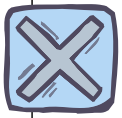
Are you convinced that $3x + 4x$ is not equal to ($\neq$) $7x^2$?

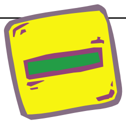
Putting It All Together

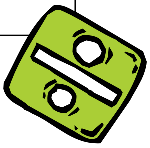


Guidelines for Solving Equations

1. Use the distributive property to clear parentheses.
2. Combine like terms. We want to isolate the variable.
3. Undo addition or subtraction using **inverse operations**.
4. Undo multiplication or division using *inverse operations*.
5. Check by substituting the solution in the original equation.







SAM = **S**implify (steps 1 and 2) then
Add (or subtract)
Multiply (or divide)

Example 1

Solve:

$$\begin{aligned}
6y + 4(y + 2) &= 88 \\
6y + 4y + 8 &= 88 && \longleftarrow \text{use distributive property} \\
10y + 8 - 8 &= 88 - 8 && \longleftarrow \text{combine like terms and undo addition} \\
& && \ \text{by subtracting 8 from each side} \\
\frac{10y}{10} &= \frac{80}{10} && \longleftarrow \text{undo multiplication by dividing by 10} \\
y &= 8
\end{aligned}$$

Check solution in the original equation:

$$\begin{aligned}
6y + 4(y + 2) &= 88 \\
6(8) + 4(8 + 2) &= 88 \\
48 + 4(10) &= 88 \\
48 + 40 &= 88 \\
88 &= 88 && \textit{It checks!}
\end{aligned}$$

Example 2

Solve:

$$\begin{aligned}
-\tfrac{1}{2}(x+8) &= 10 \\
-\tfrac{1}{2}x-4 &= 10 && \longleftarrow \text{ use distributive property} \\
-\tfrac{1}{2}x-4+4 &= 10+4 && \longleftarrow \text{ undo subtraction by adding 4 to both sides} \\
-\tfrac{1}{2}x &= 14 \\
(-2)-\tfrac{1}{2}x &= 14(-2) && \longleftarrow \text{ isolate the variable by multiplying} \\
x &= -28 && \longleftarrow \text{ each side by the reciprocal of } -\tfrac{1}{2}
\end{aligned}$$

Check solution in the original equation:

$$\begin{aligned}
-\tfrac{1}{2}(x+8) &= 10 \\
-\tfrac{1}{2}(-28+8) &= 10 \\
-\tfrac{1}{2}(-20) &= 10 \\
10 &= 10 && \textit{It checks!}
\end{aligned}$$

Example 3

Solve:

$$\begin{aligned}
26 &= \tfrac{2}{3}(9x-6) \\
26 &= \tfrac{2}{3}(9x)-\tfrac{2}{3}(6) && \longleftarrow \text{ use distributive property} \\
26 &= 6x-4 \\
26+4 &= 6x-4+4 && \longleftarrow \text{ undo subtraction by adding 4 to each side} \\
\tfrac{30}{6} &= \tfrac{6x}{6} && \longleftarrow \text{ undo multiplication by dividing each side by 6} \\
5 &= x
\end{aligned}$$

Check solution in the original equation:

$$\begin{aligned}
26 &= \tfrac{2}{3}(9x-6) \\
26 &= \tfrac{2}{3}(9 \bullet 5-6) \\
26 &= \tfrac{2}{3}(39) \\
26 &= 26 && \textit{It checks!}
\end{aligned}$$

Example 4

Solve:

$$\begin{aligned}
x - (2x + 3) &= 4 \\
x - 1(2x + 3) &= 4 && \longleftarrow \text{use the multiplicative property of -1} \\
x - 2x - 3 &= 4 && \longleftarrow \text{use the multiplicative identity of 1} \\
& && \text{ and use the distributive property} \\
-1x - 3 &= 4 && \longleftarrow \text{combine like terms} \\
-1x - 3 + 3 &= 4 + 3 && \longleftarrow \text{undo subtraction} \\
\frac{-1x}{-1} &= \frac{7}{-1} && \longleftarrow \text{undo multiplication} \\
x &= -7
\end{aligned}$$

Examine the solution steps above. See the use of the *multiplicative property of -1* in front of the parentheses on line two.

line 1: $x - (2x + 3) = 4$
line 2: $x \mathbf{- 1}(2x + 3) = 4$

Also notice the use of *multiplicative identity* on line three.

line 3: $\mathbf{1}x - 2x - 3 = 4$

The simple variable x was multiplied by 1 ($1 \bullet x$) to equal $1x$. The $1x$ helped to clarify the number of variables when combining like terms on line four.

Check solution in the original equation:

$$\begin{aligned}
x - (2x + 3) &= 4 \\
-7 - (2 \bullet -7 + 3) &= 4 \\
-7 - (-11) &= 4 \\
4 &= 4 && \textit{It checks!}
\end{aligned}$$

Practice

Solve *and* **check**. *Show* **essential steps**.

1. $10(2n + 3) = 130$

2. $4(y - 3) = -20$

3. $\frac{x}{-2} + 4 = -10$

4. $6x + 6(x - 4) = 24$

5. $6 = \frac{2}{3}(3n - 6)$

6. $10p - 4(p - 7) = 42$

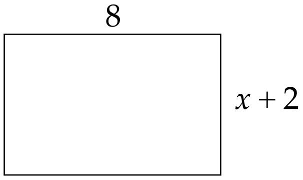
7. $28 = \frac{1}{2}(x - 8)$

8. $x - (3x - 7) = 11$

Solve *and* **Check. Show essential steps.**

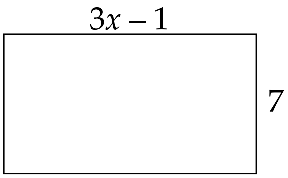
9. Write an equation for the area of the rectangle. Then solve for x.

Remember: To find the **area (A)** of a rectangle, multiply the length (l) times the width (w). $A = (lw)$



The area is 64 **square units**.

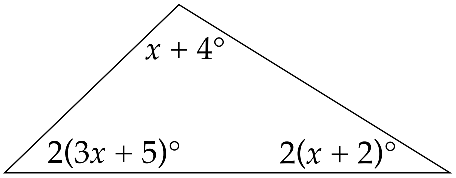
10. Write an equation for the area of the rectangle. Then solve for x.



The area is 56 square units.

11. Write an equation for the measure of degrees (°) in the triangle. Then solve for x.

Remember: For any triangle, the sum of the measures of the angles is 180 degrees.



Practice

Use the list below to write the correct term for each definition on the line provided.

area (A)
length (l)
perimeter (P)
rectangle
sum
triangle
width (w)

________________________ 1. a one-dimensional measure that is the measurable property of line segments

________________________ 2. a parallelogram with four right angles

________________________ 3. a one-dimensional measure of something side to side

________________________ 4. the result of adding numbers together

________________________ 5. the distance around a figure

________________________ 6. the measure, in square units, of the inside region of a closed two-dimensional figure; the number of square units needed to cover a surface

________________________ 7. a polygon with three sides

Practice

Use the list below to write the correct term for each definition on the line provided.

angle (∠)
degree (°)
inverse operation
measure (m) of an angle (∠)
side
square (of a number)
square units

________________________ 1. two rays extending from a common endpoint called the vertex

________________________ 2. the number of degrees (°) of an angle

________________________ 3. units for measuring area; the measure of the amount of an area that covers a surface

________________________ 4. common unit used in measuring angles

________________________ 5. the edge of a polygon, the face of a polyhedron, or one of the rays that make up an angle

________________________ 6. an action that cancels a previously applied action

________________________ 7. the result when a number is multiplied by itself or used as a factor twice

Lesson Three Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.

- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.

- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Algebra Body of Knowledge

Standard 3: Linear Equations and Inequalities

- MA.912.A.3.1
  Solve linear equations in one variable that include simplifying algebraic expressions.

- MA.912.A.3.2
  Identify and apply the distributive, associative, and commutative properties of real numbers and the properties of equality.

- MA.912.A.3.4
  Solve and graph simple and compound inequalities in one variable and be able to justify each step in a solution.

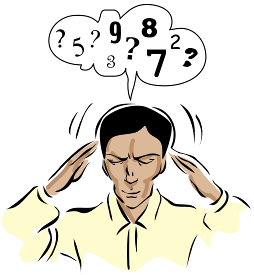
Standard 10: Mathematical Reasoning and Problem Solving

- MA.912.A.10.1
  Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing- and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.

Solving Equations with Variables on Both Sides

I am thinking of a number. If you multiply my number by 3 and then subtract 2, you get the same answer that you do when you add 4 to my number. What is my number?

To solve this riddle, begin by translating these words into an algebraic sentence. Let x represent my number.



If you multiply my number by 3 and then subtract 2
$3x - 2$

you get the same answer
$=$

that you do when you add 4 to my number
$4 + x$

Putting it all together, we get the equation $3x - 2 = 4 + x$. Note that this equation is different from equations in previous units. There is a *variable* on both sides. To solve such an equation, we do what we've done in the past: make sure both sides are simplified, and that there are no parentheses.

Strategy: Collect the variables on one side. Collect the numbers on the other side.

Now let's go back to the equation which goes with our riddle.

Solve:

$$\begin{aligned}
3x - 2 &= 4 + x & &\longleftarrow \text{both sides are simplified}\\
3x - 2 &= 4 + 1x & &\longleftarrow \text{multiplicative identity of 1}\\
3x - 2 - 1x &= 4 + 1x - 1x & &\longleftarrow \text{collect variables on the left}\\
2x - 2 &= 4 & &\longleftarrow \text{combine like terms; simplify}\\
2x - 2 + 2 &= 4 + 2 & &\longleftarrow \text{collect numbers on the right}\\
\frac{2x}{2} &= \frac{6}{2} & &\longleftarrow \text{divide both sides by 2}\\
x &= 3
\end{aligned}$$

Check solution in the original equation and the original riddle:

$$\begin{aligned}
3x - 2 &= 4 + x\\
3 \bullet 3 - 2 &= 4 + 3\\
9 - 2 &= 7\\
7 &= 7 & &\textit{It checks!}
\end{aligned}$$

Study the equation below.

Solve:

$$\begin{aligned}
2(3x + 4) &= 5(x - 2)\\
6x + 8 &= 5x - 10 & &\longleftarrow \text{distributive property}\\
6x + 8 - 5x &= 5x - 10 - 5x & &\longleftarrow \text{variables on the left}\\
x + 8 &= -10 & &\longleftarrow \text{simplify}\\
x + 8 - 8 &= -10 - 8 & &\longleftarrow \text{numbers on the right}\\
x &= -18
\end{aligned}$$

Check solution in the original equation:

$$\begin{aligned}
2(3x + 4) &= 5(x - 2)\\
2(3 \bullet -18 + 4) &= 5(-18 - 2)\\
2(-50) &= 5(-20)\\
-100 &= -100 & &\textit{It checks!}
\end{aligned}$$

Let's work this next example in two different ways.

1. Collect the *variables* on the *left* and the *numbers* on the *right*.

Solve:

$$\begin{aligned}
6y &= 4(5y-7) \\
6y &= 20y-28 && \longleftarrow \text{distributive property} \\
6y-20y &= 20y-28-20y && \longleftarrow \text{variables on the left} \\
\frac{-14y}{-14} &= \frac{-28}{-14} && \longleftarrow \text{divide both sides by -14} \\
y &= 2
\end{aligned}$$

Check solution in the original equation:

$$\begin{aligned}
6y &= 4(5y-7) \\
6 \bullet 2 &= 4(5 \bullet 2 - 7) \\
12 &= 4 \bullet 3 \\
12 &= 12 && \textit{It checks!}
\end{aligned}$$

2. Collect the *variables* on the *right* and *numbers* on the *left*.

Solve:

$$\begin{aligned}
6y &= 4(5y-7) \\
6y &= 20y-28 && \longleftarrow \text{distributive property} \\
6y-6y &= 20y-28-6y && \longleftarrow \text{variables on the right} \\
0 &= 14y-28 && \longleftarrow \text{simplify} \\
0+28 &= 14y-28+28 && \longleftarrow \text{numbers on the left} \\
\frac{28}{14} &= \frac{14y}{14} && \longleftarrow \text{divide both sides by 14} \\
2 &= y
\end{aligned}$$

We get the *same* answer, so the choice of which side you put the variable on is up to you!

Practice

Solve each **equation** *below. Then* **find** *your* **solution** *at the bottom of the page. Write the* **letter** *next to the number of that equation on the line provided above the solution. Then you will have the answer to this question*:

Which great explorer's last words were,

"I have not told half of what I saw!"

r 1. $2x - 4 = 3x + 6$

l 2. $2(-12 - 6x) = -6x$

a 3. $x - 3 = 2(-11 + x)$

c 4. $-7(1 - 4m) = 13(2m - 3)$

p 5. $-2x + 6 = -x$

m 6. $7x = 3(5x - 8)$

o 7. $2(12 - 8x) = 1x - 11x$

8.

___	___	___	___	___	___	___	___	___
3	19	-10	-16	4	6	4	-4	4

✓ **Check yourself:** Use the answer above to check your solutions to problems 1-7. Did your solutions spell out the great explorer's name? If not, correct your work before continuing.

Solve *and* **check**. *Show* **essential steps**.

9. Six more than 5 times a number is the same as 9 less than twice the number. Find the number.

10. Twelve less than a number is the same as 6 decreased by 8 times the number. Find the number.

11. The product of 5 and a number, plus 17, is equal to twice the sum of the same number and -5. Find the number.

Complete *to* **solve** *the following.*

12. $-12 + 7(x + 3) = 4(2x - 1) + 3$

Remember: Always multiply before you add.

$-12 +$ ______ $+$ ______ $=$ ______ $-$ ______ $+ 3$

$7x +$ ______ $=$ $8x -$ ______

Now finish the problem.

13. $-56 + 10(x - 1) = 4(x + 3)$

14. $5(2x + 4) + 3(-2x - 3) = 2x + 3(x + 4)$

______ $+$ ______ $+$ ______ $-$ ______ $= 2x +$ ______ $+$ ______

distributive property distributive property

______ $x + 11$ $=$ ______ $x +$ ______

add like terms add like terms

Now finish the problem.

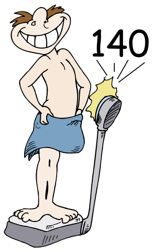
15. $-16x + 10(3x - 2) = -3(2x + 20)$

$-16x +$ ______ – ______ = ______ – ______
distributive property   distributive property

Add like terms and finish the problem.

16. $6(-2x - 4) + 2(3x + 12) = 37 + 5(x - 3)$

Problems That Lead to Equations



Joshua presently weighs 100 pounds, but is steadily growing at a rate of 2 pounds per week. When will he weigh 140 pounds?

The answer is 20 weeks. Let's use this simple problem to help us think algebraically.

Step 1: Read the problem and label the variable. <u>Underline</u> all clues.

Joshua presently weighs <u>100 pounds</u>, but is steadily growing at a rate of <u>2 pounds per week</u>. <u>When</u> will he weigh <u>140 pounds</u>?

Let x represent the number of weeks.

Step 2: Plan.

Let $2x$ represent the weight Joshua will gain.

Step 3: Write the equation.

$$\begin{array}{ccccc} \text{present weight} & + \text{ gain} & = & \text{desired weight} \\ 100 & + \;\; 2x & = & 140 \end{array}$$

Step 4: Solve the equation.

$$\begin{aligned} 100 + 2x &= 140 \\ 100 + 2x - 100 &= 140 - 100 \quad \longleftarrow \text{subtract 100 from both sides} \\ 2x &= 40 \\ \frac{2x}{2} &= \frac{40}{2} \qquad\qquad \longleftarrow \text{divide both sides by 2} \\ x &= 20 \end{aligned}$$

Step 5: Check your solution. Does your answer make sense?

$$\overset{\text{now}}{100} + \overset{\text{gain}}{2(20)} = 140$$

We will use this 5-step approach on the following problems. You will find that many times a picture or chart will also help you arrive at an answer. Remember, we are learning to think algebraically, and to do that the procedure is as important as the final answer!

5-Step Plan for Thinking Algebraically

Step 1: **Read** the problem and **label** the variable. **Underline** all clues.
Decide what *x* represents.

Step 2: **Plan.**

Step 3: **Write** an equation.

Step 4: **Solve** the equation.

Step 5: **Check** your solution. Does your answer make sense?

Practice

Use the **5-step plan** *to solve and check the following. Show* **essential steps***.*

1. Leon's television breaks down. Unfortunately he has only *$100.00* in savings for emergencies. The repairman charges *$35.00 for coming to Leon's house* and then another *$20.00 per hour* for fixing the television. *How many hours* can Leon pay for the repairman to work?

   a. Step 1: Read the problem and label the variable. Underline all clues. (*Note*: The clues have been italicized for you.)

      Let x represent ____ .

   b. Step 2: Plan. Let $20x$ represent ___________________________

      ____ .

   c. Step 3: Write an equation. _______________________________

   d. Step 4: Solve the equation.

   e. Step 5: Check your solution. Does it make sense? __________

2. Samantha charges $16.00 to deliver sand to your house, plus $3.50 per **cubic** foot for the sand that you buy. How much sand can you buy for $121.00?

   a. Step 1: Read the problem and label the variable. Underline all clues.

      Let x represent ______________________________________ .

   b. Step 2: Plan. Let ______ x represent ______________________

      ____ .

   c. Step 3: Write an equation. ______________________________

   d. Step 4: Solve the equation.

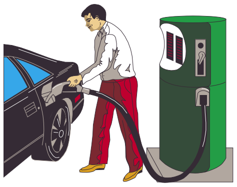
   e. Step 5: Check your solution. Does it make sense? __________

   f. If the installation of a child's sand box requires 29 cubic feet of sand, will you be able to complete this project for $121.00?

      Explain. ____

      ____

3. Suppose that the gas tank of a car holds 20 gallons, and that the car uses $\frac{1}{10}$ of a gallon per mile. How far has the car gone when 5 gallons remain?



a. Step 1: Read the problem and label the variable. Underline all clues.

Let x represent ____ .

b. Step 2: Plan. Let $\frac{1}{10}x$ represent ___________________________

____ .

c. Step 3: Explain why the appropriate equation is $20 - \frac{1}{10}x = 5$.

____

____

d. Step 4: Solve the equation.

e. Step 5: Check your solution. Does it make sense? __________

4. Jared weighs 250 pounds and is steadily losing 3 pounds per week. How long will it take him to weigh 150 pounds?

   a. Step 1: Read the problem and label the variable. Underline all clues.

      Let x represent ____ .

   b. Step 2: Let $3x$ represent ________________________________ .

   c. Step 3: Write an equation. ______________________________

      ____

   d. Step 4: Solve the equation.

   e. Step 5: Check your solution. Does it make sense? __________

5. Batman has $100.00 and spends $3.00 per day. Robin has $20.00 but is adding to it at the rate of $5.00 per day. When will they have the same amount of money?

   a. Step 1: Read the problem and label the variable. Underline all clues.

      Let x represent ____ .

   b. Step 2: Plan. 100 – 3x is what Batman will have after x days.

      How much will Robin have after x days? _________________

      ____

   c. Step 3: Write an equation stating that Batman's money is the same amount as Robin's money after x days.

      ____

   d. Step 4: Solve the equation.

   e. What does your solution mean? _________________________

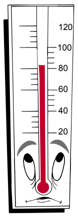
      ____

      ____

      ____

   f. Step 5: Check your solution. Does it make sense? ____________

6. Suppose you live in Tallahassee, Florida, where the temperature is 84 degrees and going down 3 degrees per hour. A friend lives in Sydney, Australia, where the temperature is 69 degrees and going up at a rate of 2 degrees per hour. How long would you and your friend have to wait before the temperatures in both places are equal?



a. Step 1: Read the problem and label the variable. Underline all clues.

Let x represent ____ .

b. Step 2: Plan. Let ________________________ represent

Tallahassee's temperature and ________________________

represent Sydney's temperature.

c. Step 3: Write an equation. Let the Tallahassee temperature *equal*

Sydney's temperature. __________________________________

d. Step 4: Solve the equation.

e. Step 5: Check your solution. Does it make sense? __________

Practice

Use the **5-step plan** *to* **solve** *and* **check** *the following. Show* **essential steps**.

Sometimes a *chart* helps *organize* the information in a problem.

1. I am thinking of *3 numbers*. The *second number is 4 more than the first number*. The *third number is twice the first number*. The *sum* of all 3 numbers *is 28*. Find the numbers.

   Algebraic Thinking:

- Step 1: Read the problem and label the variable. Underline all clues. (*Note*: The clues have been italicized for you.)

  What does x represent? Since the second and third numbers are described in terms of the first number, let x represent the *first* number.

- Step 2: Plan. See the **table** below.

Description		**Value**
first number	x	=
second number	$x + 4$	=
third number	$2x$	=
sum	$4x + 4$	= 28

- Step 3: Write an equation.

  $4x + 4 = 28$

- Step 4: Solve the equation.

  a. The equation $4x + 4 = 28$ will give you the *value* of only the *first* number. Substitute your answer back into the *expressions* in the *table* on the previous page to find the *second* and *third* numbers.

  b. Solve the equation to find the value of the *first* number.

  $4x + 4 = 28$

  c. Substitute the first number's value in the expression from the *table* on the previous page to get the value of the *second* number.

  $x + 4$

  d. Substitute the first number's value in the expression from the *table* on the previous page to get the value of the *third* number.

  $2x$

- Step 5: Check your solution. Does it make sense?

  e. Check solution in original equation.

  f. Do your numbers add up to 28? ______

  Write an equation and solve to prove that the sum of the 3 numbers equal 28.

Sometimes a *picture* helps *organize* the information in a problem.

2. A triangle has a perimeter (P) of 30 inches. The longest side is 8 inches longer than the shortest side. The third side is 1 inch shorter than the longest side. Find the sides.

Remember: The *perimeter* is the sum of all the lengths of all sides.

- Step 1: Read the problem and label the variable. Underline all clues.

Hint: Let the shortest side be x inches long.

- Step 2: Plan.

a. Draw a triangle. Label the shortest side x. Label the other two sides in terms of x.

b. Let ______________________________ represent adding up the sides of the triangle.

- Step 3: Write an equation.

c. Use the fact that the perimeter is 30 inches to write an equation.

____

- Step 4: Solve the equation.

  d. Find x by solving the equation.

- Step 5: Check your solution. Does it make sense?

  e. Check solution in original equation.

  f. Use the value of x to find the other 2 sides.

  g. Do the sides add up to 30? ______

  Write an equation and solve to prove that the sum of the 3 sides of the triangle equals 30.

3. A rectangle has a perimeter of 38 inches and a width of x inches. The length of the rectangle is *4 more than twice the width*. Label all 4 sides.

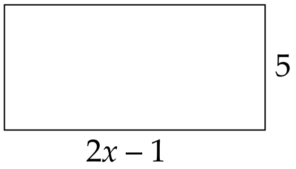
   Draw and label a rectangle and use the 5-step plan to find the dimensions of the rectangle.

4. The measures of the angles in any triangle add up to 180 degrees. Let the smallest angle be x degrees. The second angle is *twice the smallest*. The third angle is *30 degrees more than the second angle*. Find the measures of all the angles.

   Draw and label a triangle. Use the 5-step plan to find all angles.

5. Write an equation for the area of the rectangle. ________________

Remember: To find the *area* of a rectangle we multiply the length times the width. $A = l \bullet w$

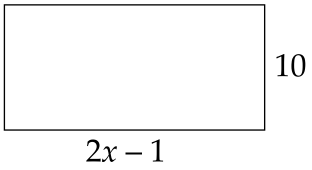


The area is 35 square inches.

Solve the equation for x, then substitute it in $2x - 1$ to find the length.

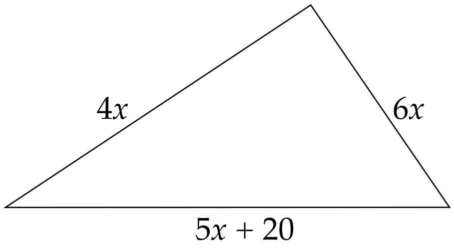
Is the product of the length and width 35 square inches? ________

6. Consider the rectangle and the triangle below. What is the value of x if the area of the rectangle equals the perimeter of the triangle?



Let ______________________________________ equal

the area of the rectangle.

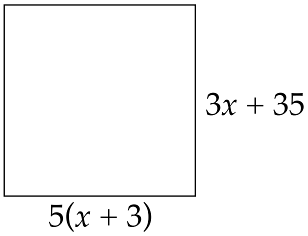


Let ______________________________________ equal

the perimeter of the triangle.

Now set up an equation. Let the area of the rectangle equal the perimeter of the triangle. Solve for x.

7. A **square** is a four-sided figure with *all sides the same length*. Find the value of x so that the figure is a square.



Circle the letter of the correct answer.

8. Mrs. Jones brings \$142.50 to pay for her family's expenses to see Florida A&M University play football. She has to pay \$10.00 to park. An adult ticket costs \$45.00. She has 4 children who qualify for student tickets. She has \$27.50 left at the end of the day. Which equation can you use to find the cost of a student ticket?

   a. $4x + 45 = \$142.50$

   b. $\$27.50 + 4x + 45 = \$142.50$

   c. $\$142.50 - 10 - 45 - 4x = \$27.50$

   d. $\$142.50 - 10 - 45 + 4x = \$27.50$

Answer the following. Show **essential steps**.

Consecutive **even integers** are numbers like 6, 8, and 10 or 14, 16, and 18. Note that you add 2 to the smallest to get the second number and 4 to the smallest to get the third number. Use this information to solve the following problem.

9. The sum of three consecutive even integers is 198. Find the numbers.

Description		**Value**	
first number	x	=	
second number	$x + 2$	=	
third number	$x + 4$	=	
sum	______________	=	198

Set up an equation and solve for x. Substitute your answer back into the table above to find all answers. Do the numbers add up to 198?

Practice

Use the list below to complete the following statements.

consecutive	**square**
even	**table**
integers	

1. A data display that organizes information about a topic into categories is called a(n) ______________________ or chart.
2. A rectangle with four sides the same length is called a ______________________ .
3. Consecutive even ______________________ are numbers like 6, 8, and 10 or 14, 16, and 18.
4. When numbers are in order they are ______________________ .
5. Any integer divisible by 2 is a(n) ______________________ integer.

Lesson Four Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.

- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.

- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Algebra Body of Knowledge

Standard 3: Linear Equations and Inequalities

- MA.912.A.3.1
  Solve linear equations in one variable that include simplifying algebraic expressions.

- MA.912.A.3.2
  Identify and apply the distributive, associative, and commutative properties of real numbers and the properties of equality.

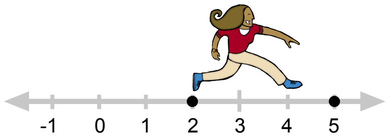
- MA.912.A.3.4
  Solve and graph simple and compound inequalities in one variable and be able to justify each step in a solution.

Standard 10: Mathematical Reasoning and Problem Solving

- MA.912.A.10.1
  Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing- and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.

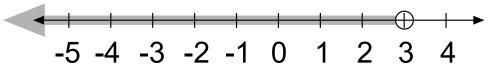
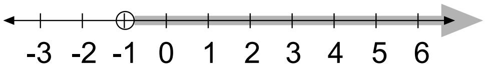
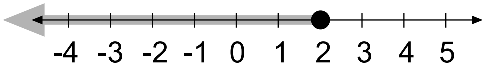
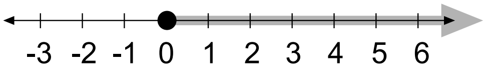
Graphing Inequalities on a Number Line

In this unit we will graph **inequalities** on a **number line**. A **graph of a number** is the point on a *number line* paired with the number. Graphing solutions on a number line will help you visualize solutions.



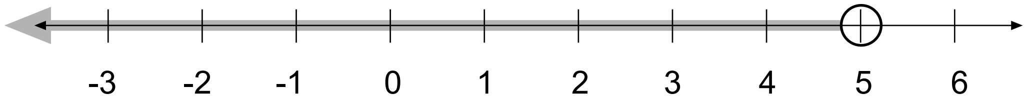
Here are some examples of *inequalities*, their verbal meanings, and their graphs.

Inequalities

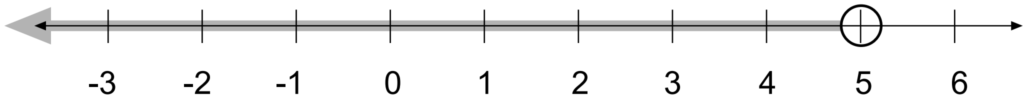
Inequality	Meaning	Graph
a. $x < 3$	All real numbers less than 3.	 The open circle means that 3 is *not* a solution. Shade to left.
b. $x > -1$	All real numbers greater than -1.	 The open circle means that -1 is *not* a solution. Shade to right.
c. $x \leq 2$	All real numbers less than or equal to 2.	 The solid circle means that 2 is a solution. Shade to left.
d. $x \geq 0$	All real numbers greater than or equal to 0.	 The solid circle means that 0 is a solution. Shade to right.

For each example, the inequality is written with the variable on the left. Inequalities can also be written with the variable on the right. However, graphing is easier if the variable is on the left.

Consider $x < 5$, which means the same as $5 > x$. Note that the graph of $x < 5$ is all real numbers less than 5.



The graph of $5 > x$ is all real number that 5 is greater than.



To write an inequality that is *equivalent* to (or the same as) $x < 5$, move the number and variable to the opposite side of the inequality, and then reverse the inequality.

$x < 5$

$5 > x$

$x < 5$ means the same as

$5 > x$

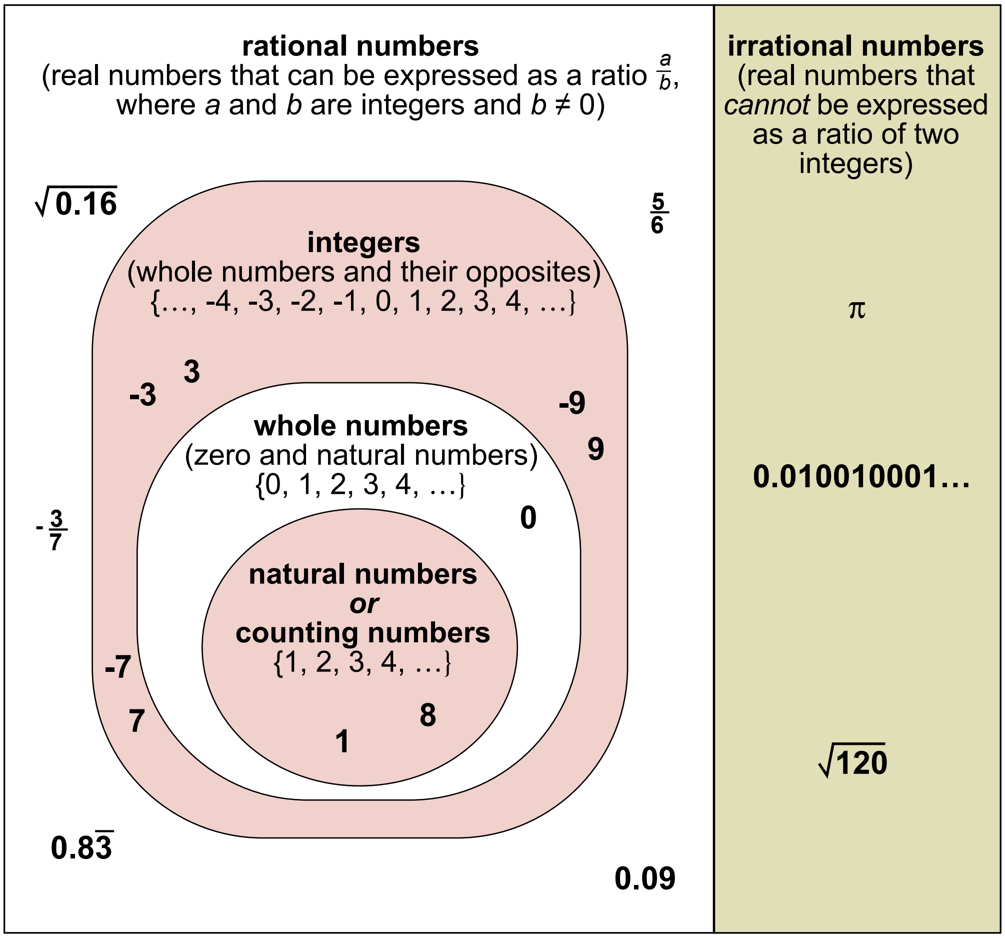
The inequality $y \geq -2$ is equivalent to $-2 \leq y$. Both inequalities can be written as the *set of all* **real numbers** that are *greater than or equal to* -2.

The inequality $0 \leq x$ is equivalent to $x \geq 0$. Each can be written as the *set of all real numbers* that are *greater than or equal to* zero.

Remember: *Real numbers* are all **rational numbers** and all **irrational numbers**.

The Venn diagram below is a graphic organizer that aids in visualizing what real numbers are.

The Set of Real Numbers



Rational numbers can be expressed as a **ratio** $\frac{a}{b}$, where a and b are *integers* and $b \neq 0$.

rational numbers	4	$-3\frac{3}{4}$	0.250	0	$0.\overline{3}$
expressed as ratio of two integers	$\frac{4}{1}$	$-\frac{15}{4}$	$\frac{1}{4}$	$\frac{0}{1}$	$\frac{1}{3}$

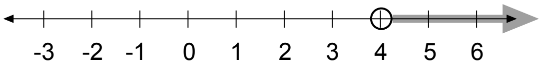
Note: All integers are rational numbers.

A *ratio* is the comparison of two quantities. For example, a ratio of 8 and 11 is 8:11 or $\frac{8}{11}$.

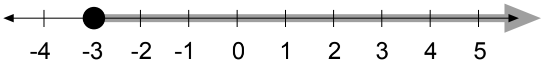
Practice

Match each **inequality** *with the correct* **graph**.

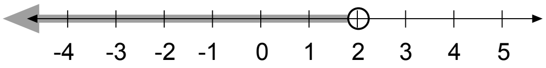
_______ 1. $x \geq -3$

a. 

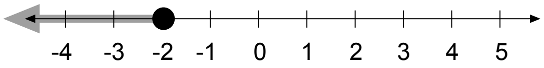
_______ 2. $x \leq 0$

b. 

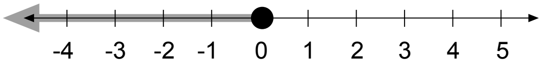
_______ 3. $x > 4$

c. 

_______ 4. $2 > x$

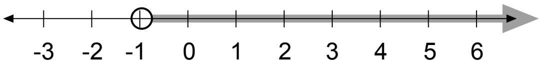
d. 

_______ 5. $x \leq -2$

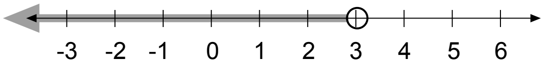
e. 

Write an **inequality** *for each* **graph**.

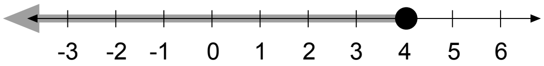
6. ______________________________



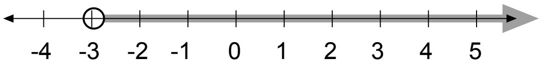
7. ______________________________



8. ______________________________

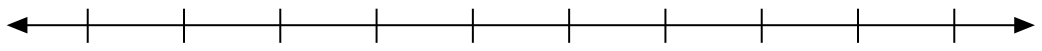


9. ______________________________

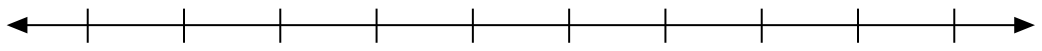


Graph each **inequality**.

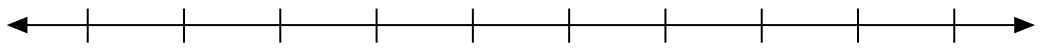
10. $x \geq -1$



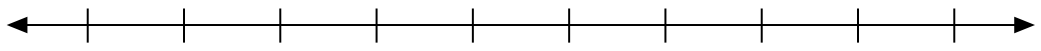
11. $x < 0$



12. $x > 5$



13. $x \leq -3$



Solving Inequalities

We have been solving *equations* since Unit 1. When we solve inequalities, the procedures are the same except for one important difference.

When we multiply or divide both sides of an inequality by the same *negative number*, we *reverse* the direction of the inequality symbol.

Example: Solve by *dividing* by a *negative number* and *reversing* the inequality sign.

$$-3x < 6$$

$$\frac{-3x}{-3} > \frac{6}{-3} \longleftarrow$$ divide each side by -3 and reverse the inequality symbol

$$x > -2$$

To check this solution, pick any number *greater than* -2 and substitute your choice into the original inequality. For instance, -1, 0, or 3, or 3,000 could be substituted into the original problem.

Check with different solutions of numbers *greater than* -2:

substitute -1

$$-3x < 6$$
$$-3(-1) < 6$$
$$3 < 6$$ *It checks!*

substitute 3

$$-3x < 6$$
$$-3(3) < 6$$
$$-9 < 6$$ *It checks!*

substitute 0

$$-3x < 6$$
$$-3(0) < 6$$
$$0 < 6$$ *It checks!*

substitute 3,000

$$-3x < 6$$
$$-3(3{,}000) < 6$$
$$-9{,}000 < 6$$ *It checks!*

Notice that -1, 0, 3, and 3,000 are all *greater than* -2 and each one *checks* as a solution.

Study the following examples.

Example: Solve by *multiplying* by a *negative number* and *reversing* the inequality sign.

$$\begin{aligned}
-\tfrac{1}{3}y &\geq 4 \\
(-3)\ -\tfrac{1}{3}y &\leq 4(-3) && \longleftarrow \text{ multiply each side by -3 and} \\
& && \text{ reverse the inequality symbol} \\
y &\leq -12
\end{aligned}$$

Example: Solve by first adding, then *dividing* by a *negative number*, and *reversing* the inequality sign.

$$\begin{aligned}
-3a - 4 &> 2 \\
-3a - 4 + 4 &> 2 + 4 && \longleftarrow \text{ add 4 to each side} \\
-3a &> 6 \\
\frac{-3a}{-3} &< \frac{6}{-3} && \longleftarrow \text{ divide each side by -3 and} \\
& && \text{ reverse the inequality symbol} \\
a &< -2
\end{aligned}$$

Example: Solve by first subtracting, then *multiplying* by a *negative number*, and *reversing* the inequality sign.

$$\begin{aligned}
\frac{y}{-2} + 5 &\leq 0 \\
\frac{y}{-2} + 5 - 5 &\leq 0 - 5 && \longleftarrow \text{ subtract 5 from each side} \\
\frac{y}{-2} &\leq -5 \\
\frac{(-2)y}{-2} &\geq (-5)(-2) && \longleftarrow \text{ multiply each side by -2 and} \\
& && \text{ reverse the inequality symbol} \\
y &\geq 10
\end{aligned}$$

Example: Solve by first subtracting, then *multiplying* by a *positive number* and **not** *reversing* the inequality sign.

$\frac{n}{2} + 5 \le 2$

$\frac{n}{2} + 5 - 5 \le 2 - 5$ ← subtract 5 from each side

$\frac{n}{2} \le -3$

$\frac{(2)n}{2} \le -3(2)$ ← multiply each side by 2, *but* do *not* reverse the inequality symbol because we multiplied by a positive number

$n \le -6$

When multiplying or dividing both sides of an inequality by the same *positive number*, do *not* reverse the inequality symbol—leave it alone.

Example: Solve by first adding, then *dividing* by a positive number, and **not** *reversing* the inequality sign.

$7x - 3 > -24$

$7x - 3 + 3 > -24 + 3$ ← add 3 to each side

$7x > -21$

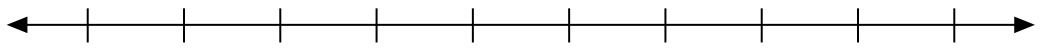
$\frac{7x}{7} > \frac{-21}{7}$ ← divide each side by 7 do *not* reverse the inequality symbol because we divided by a positive number

$x > -3$

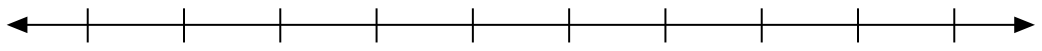
Practice

Solve each **inequality**. *Show* **essential steps**. *Then* **graph the solutions**.

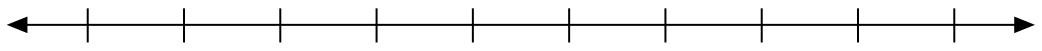
1. $x + 5 \geq 2$



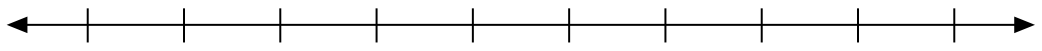
2. $y - 1 \leq 5$



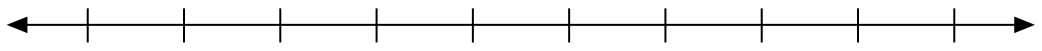
3. $4 < n - 1$



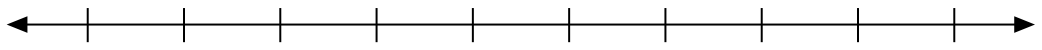
4. $2 \geq y - 4$



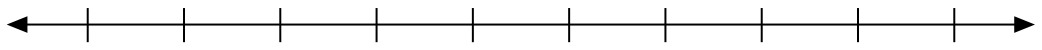
5. $5a - 2 \leq 3$



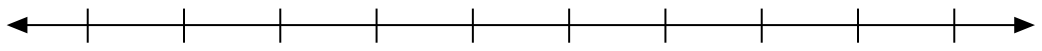
6. $-\frac{1}{4}y > 0$



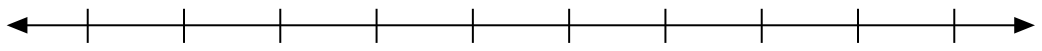
7. $-2a \geq -12$



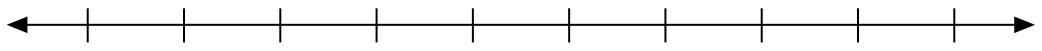
8. $\frac{a}{3} - 3 < 1$



9. $\frac{y}{2} - 6 < -5$



10. $\frac{a}{-3} + 9 < 8$



Practice

Solve *the following. Show* **essential steps**.

1. $2y + 1 \leq 4$

2. $-\frac{1a}{3} - 4 > 2$

3. $-11a + 3 < -30$

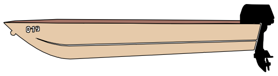
4. $\frac{1}{5}y + 9 \leq 8$

5. $-10 < 2b - 14$

6. $10y + 3 \leq 8$

Study the following.

Many problems in everyday life involve inequalities.



Example: A summer camp needs a boat with a motor. A local civic club will donate the money on the condition that the camp will spend *less than* \$1,500 for both. The camp decides to buy a boat for \$1,050. How much can be spent on the motor?

1. *Choose* a variable. *Let* x = cost of the motor,
   then let $x + 1{,}050$ = cost of motor and boat,
   and cost of motor + cost of boat < total money.
2. *Write as an inequality.* $x + 1{,}050 < 1{,}500$
3. *Solve.* $x + 1{,}050 - 1{,}050 < 1{,}500 - 1{,}050$
   $x < \$450$
4. *Interpretation of solution:* The camp can spend *any* amount *less than* \$450 for the motor. (**Note:** The motor *cannot* cost \$450.)

Use the **steps** *below for the* **word problems** *on the following pages.*

1. **Choose a variable**
2. **Write as an inequality**
3. **Solve**
4. **Interpret your solution**

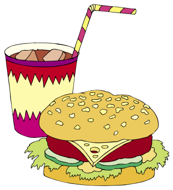
7. If $50 is added to 2 times the amount of money in a wallet, the result is less than $150. What is the greatest amount of money that could be in the wallet?

Interpretation of solution: ______________________________

____

____

8. Sandwiches cost $2.50 and a drink is $1.50. If you want to buy one drink, what is the greatest number of sandwiches you could also buy and spend less than $10.00?



Interpretation of solution: ______________________________

____

____

9. Annie babysits on Friday nights and Saturdays for $3.00 an hour. Find the fewest number of hours she can babysit and earn more than $20.00 a week.

Interpretation of solution: ________________________________

____

____

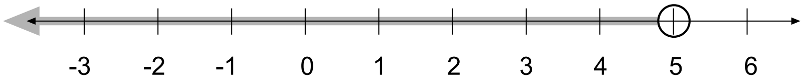
Practice

Write **True** *if the statement is correct. Write* **False** *if the statement is* not *correct.*

__________ 1. Graphing solutions on a number line will help you visualize solutions.

__________ 2. An inequality can only be written with the variable on the right.

__________ 3. The graph below of $x < 5$ shows all real numbers greater than 5.



__________ 4. *Real numbers* are all rational and irrational numbers.

__________ 5. A *ratio* is the comparison of two quantities.

__________ 6. To write an inequality that is equivalent to $x < 5$, move the number and variable to the opposite side of the inequality, and then reverse the inequality.

__________ 7. When we multiply or divide each side of an inequality by the same negative number, we reverse the direction of the inequality symbol.

__________ 8. There are no problems in everyday life that involve inequalities.

__________ 9. An *inequality* is a sentence that states one expression is greater than, greater than or equal to, less than, less than or equal to, or not equal to another expression.

Practice

Use the list below to write the correct term for each definition on the line provided.

decrease
difference
equation
increase
reciprocals
simplify an expression
solve
sum

________________________ 1. a number that is the result of subtraction

________________________ 2. to find all numbers that make an equation or inequality true

________________________ 3. to make less

________________________ 4. a mathematical sentence stating that the two expressions have the same value

________________________ 5. to make greater

________________________ 6. any two numbers with a product of 1; also called *multiplicative inverse*

________________________ 7. the result of adding numbers together

________________________ 8. to perform as many of the indicated operations as possible

Practice

Match each definition with the correct term. Write the letter on the line provided.

_______ 1. a sentence that states one expression is greater than, greater than or equal to, less than, less than or equal to, or not equal to another expression

_______ 2. the distance around a figure

_______ 3. the point on a number line paired with the number

_______ 4. a one-dimensional measure that is the measurable property of line segments

_______ 5. a data display that organizes information about a topic into categories

_______ 6. the number of degrees (°) of an angle

_______ 7. a polygon with three sides

_______ 8. two rays extending from a common endpoint called the vertex

_______ 9. any integer not divisible by 2; any integer with the digit 1, 3, 5, 7, or 9 in the units place; any integer in the set {… , -5, -3, -1, 1, 3, 5, …}

_______ 10. a parallelogram with four right angles

A. angle (∠)

B. graph (of a number)

C. inequality

D. length (*l*)

E. measure (m) of an angle (∠)

F. odd integer

G. perimeter (*P*)

H. rectangle

I. table (or chart)

J. triangle

Lesson Five Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.

Algebra Body of Knowledge

Standard 3: Linear Equations and Inequalities

- MA.912.A.3.3
  Solve literal equations for a specified variable.

Standard 10: Mathematical Reasoning and Problem Solving

- MA.912.A.10.1
  Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing- and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.

Formulas Using Variables

There may be times when you need to solve an equation, such as

$d = r \bullet t$ distance = rate • time

for one of its variables. When you know both rate and time, it is easy to calculate the distance using the **formula** above.

If you drive 60 mph for 5 hours, how far will you go? Use the following *formula*.

$d = r \bullet t$

Substitute 60 for the rate and 5 for the time to get the following.

$d = 60 \bullet 5$
$d = 300$ miles

But what if you know your destination is 385 miles away and that the speed limit is 55 mph? It would be helpful to have a formula that gives you the amount of time you will need to get there. Rather than trying to remember a new formula for each situation, you could transform the one you already have using the rules you know.

Use the same algebraic rules that we used before, and solve the following.

Remember: *t* is for time.

$d = r \bullet t$

We want to get *t* alone on one side of the equation.

$$\begin{aligned} d &= r \bullet t \\ \frac{d}{r} &= \frac{\cancel{r}^{1} \bullet t}{\cancel{r}_{1}} \\ \frac{d}{r} &= t \end{aligned}$$

⟵ divide both sides by r

Now see below how dividing distance (385) by rate (55), you get time, 7 hours.

$$\begin{aligned} d &= r \bullet t \\ 385 &= 55 \bullet t \\ \frac{\cancel{385}^{7}}{\cancel{55}_{1}} &= \frac{\cancel{55}^{1} \bullet t}{\cancel{55}_{1}} \\ 7 &= t \\ &\text{7 hours} \end{aligned}$$

Let’s try the examples below.

Example 1

Solve

$$\begin{aligned} A &= \tfrac{1}{2}bh \text{ for } b. \\ A &= \tfrac{1}{2}bh \\ 2A &= bh & \longleftarrow \text{multiply both sides by 2} \\ 2\tfrac{A}{h} &= b & \longleftarrow \text{divide both sides by } h \end{aligned}$$

Now let’s try a more challenging example.

Example 2

Solve

$$\begin{aligned} A &= \tfrac{1}{2}(b_1 + b_2)h \text{ for } h \\ A &= \tfrac{1}{2}(b_1 + b_2)h \\ 2A &= (b_1 + b_2)h & \longleftarrow \text{multiply both sides by 2} \\ \frac{2A}{(b_1 + b_2)r} &= h & \longleftarrow \text{divide both sides by } (b_1 + b_2) \end{aligned}$$

Your turn.

Practice

Solve *each* **formula** *or* **equation** *for the* **variable** *given.*

1. $ax + by = c$ Solve for x.

2. $(n - 2)180 = x$ Solve for n.

3. $2a + b = c$ Solve for b.

4. $a(1 + b) = c$ Solve for a.

5. $2a + 2b = 4c$ Solve for b.

6. $4(x + 5) = y$ Solve for x.

7. $t = a + (n - 1)$ Solve for a.

8. $c = \frac{5}{9}(F - 32)$ Solve for F.

Unit Review

Solve these equations. ***Show*** **essential steps.**

1. $4y + 2 = 30$

2. $-5x - 6 = 34$

3. $\frac{x}{3} + 7 = -3$

4. $\frac{x}{-4} - 2 = 10$

5. $\frac{1x}{6} + 2 = 8$

6. $\frac{-3x}{4} - 8 = -2$

7. $5 - x = 12$

8. $12 = -7 - x$

9. $8 - \frac{2x}{3} = 12$

10. What is the reciprocal of $-\frac{3}{4}$? ____________

Number 11 is a **gridded-response item**.
Write answer along the top of the grid and correctly mark it below.

11. What is the reciprocal of 8?

Mark your answer on the grid to the right.

	⊘	⊘	⊘	
⊙	⊙	⊙	⊙	⊙
⓪	⓪	⓪	⓪	⓪
①	①	①	①	①
②	②	②	②	②
③	③	③	③	③
④	④	④	④	④
⑤	⑤	⑤	⑤	⑤
⑥	⑥	⑥	⑥	⑥
⑦	⑦	⑦	⑦	⑦
⑧	⑧	⑧	⑧	⑧
⑨	⑨	⑨	⑨	⑨

Simplify *the following.*

12. $-5(x + 2) + 16$

14. $5x - 7x$

13. $15 + 2(x + 8)$

15. $-8x - 14 + 10x - 20$

Solve these equations. *Show* **essential steps**.

16. $7x + 3 - 8x + 12 = -6$

17. $7x + 3(x + 2) = 36$

18. $-\frac{1}{2}(x + 10) = -15$

19. $5x - 8 = 4x + 10$

20. $-8(1 - 2x) = 5(2x - 6)$

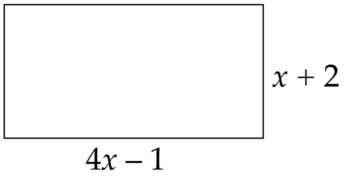
Write an **equation** *and* **solve for** *x*.

21. The *sum* of $2x$ and 7 equals 19. What is the number?

22. $\frac{1}{2}$ of x *decreased* by 7 is -10. What is the number?

23. The *difference* between 14 and $2x$ is -10. What is the number?

24. The *perimeter* (P) is the distance around a figure, or sum of the lengths of the sides of a figure. The perimeter of the rectangle below is 52. Write an equation and solve for x.



Answer the following. Show **essential steps**.

Consecutive **odd integers** are numbers like 3, 5, and 7 or 15, 17, and 19. Note that you add 2 to the smallest to get the second number and 4 to the smallest to get to the third number. Use this information to solve the following problem.

25. The sum of three consecutive *odd* integers is 159.

Description		**Value**
first number	x	=
second number	$x + 2$	=
third number	$x + 4$	=
sum	__________	= 159

Set up an equation and solve for x. Substitute your answer back into the table to find all answers. Do the numbers add up to 159?

Match each **inequality** *with its* **graph**.

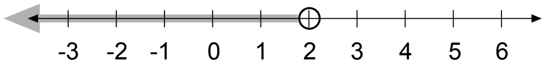
_______ 26. $x \geq 2$

_______ 27. $x < 2$

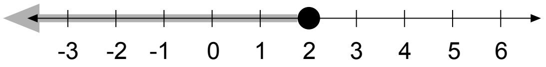
_______ 28. $2 \geq x$

_______ 29. $x < -2$

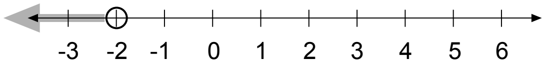
a.



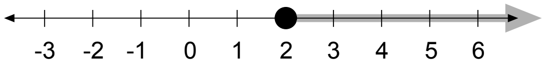
b.



c.

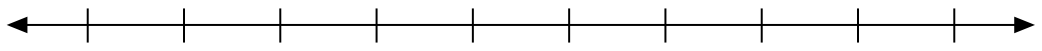


d.

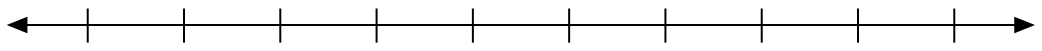


Solve and graph.

30. $-5x + 6 > -34$



31. $\frac{x}{-2} + 6 \leq 0$



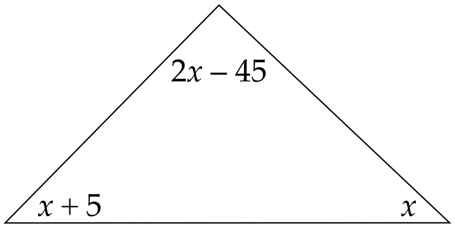
Solve *the* **equation** *for the* **variable** *given.*

32. $x(1 + b) = y$ Solve for x.

Bonus Problems

Answer the following.

33. The sum of the measures of the angles in any triangle is 180 degrees. Find x, and then find the *measure* of each angle for the triangle below.



34. Solve and graph this inequality.

$-20 < -2x - 14$

Unit 3: Working with Polynomials

This unit emphasizes the skills necessary to add, subtract, multiply, and divide rational expressions, simplify them efficiently, and use strategies necessary for operations involving polynomials.

Unit Focus

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.

- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.

- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Algebra Body of Knowledge

Standard 1: Real and Complex Number Systems

- MA.912.A.1.8
  Use the zero product property of real numbers in a variety of contexts to identify solutions to equations.

Standard 3: Linear Equations and Inequalities

- MA.912.A.3.2
  Identify and apply the distributive, associative, and commutative properties of real numbers and the properties of equality.

Standard 4: Polynomials

- MA.912.A.4.1
  Simplify monomials and monomial expressions using the laws of integral exponents.

- MA.912.A.4.2
  Add, subtract, and multiply polynomials.

- MA.912.A.4.3
  Factor polynomial expressions.

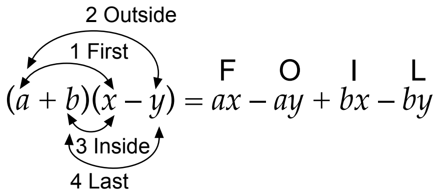
- MA.912.A.4.4
  Divide polynomials by monomials and polynomials with various techniques, including synthetic division.

Vocabulary

Use the vocabulary words and definitions below as a reference for this unit.

additive inverses	a number and its opposite whose sum is zero (0); also called *opposites* *Example*: In the equation 3 + (-3) = 0, the additive inverses are 3 and -3.
base (of an exponent) (algebraic)	the number used as a factor in exponential form *Example*: 2^3 is the exponential form of 2 x 2 x 2. The numeral two (2) is called the base, and the numeral three (3) is called the exponent.
binomial	the sum of two monomials; a polynomial with exactly *two* terms *Examples*: $4x^2 + x$ $2a - 3b$ $8qrs + qr^2$
canceling	dividing a numerator and a denominator by a common factor to write a fraction in lowest terms or before multiplying fractions *Example*: $\frac{15}{24} = \frac{{}^{1}\cancel{3} \bullet 5}{2 \bullet 2 \bullet 2 \bullet \cancel{3}_{1}} = \frac{5}{8}$
coefficient	the number that multiplies the variable(s) in an algebraic expression *Example*: In $4xy$, the coefficient of xy is 4. If no number is specified, the coefficient is 1.
common factor	a number that is a factor of two or more numbers *Example*: 2 is a common factor of 6 and 12.

commutative property	the order in which two numbers are added or multiplied does *not* change their sum or product, respectively *Examples*: 2 + 3 = 3 + 2 *or* 4 x 7 = 7 x 4
composite number	a whole number that has more than two factors *Example*: 16 has five factors—1, 2, 4, 8, and 16.
counting numbers (natural numbers)	the numbers in the set {1, 2, 3, 4, 5, …}
denominator	the bottom number of a fraction, indicating the number of equal parts a whole was divided into *Example*: In the fraction $\frac{2}{3}$ the denominator is 3, meaning the whole was divided into 3 equal parts.
distributive property	the product of a number and the sum or difference of two numbers is equal to the sum or difference of the two products *Examples*: $x(a + b) = ax + bx$ $5(10 + 8) = 5 \bullet 10 + 5 \bullet 8$
exponent (exponential form)	the number of times the base occurs as a factor *Example*: 2^3 is the exponential form of 2 x 2 x 2. The numeral two (2) is called the *base*, and the numeral three (3) is called the *exponent*.

expression	a mathematical phrase or part of a number sentence that combines numbers, operation signs, and sometimes variables *Examples*: $4r^2$; $3x + 2y$; $\sqrt{25}$ An expression does *not* contain equal (=) or inequality ($<$, $>$, $\leq$, $\geq$, or $\neq$) signs.
factor	a number or expression that divides evenly into another number; one of the numbers multiplied to get a product *Examples*: 1, 2, 4, 5, 10, and 20 are factors of 20 and $(x + 1)$ is one of the factors of $(x^2 - 1)$.
factored form	a number or expression expressed as the product of prime numbers and variables, where no variable has an exponent greater than 1
FOIL method	a pattern used to multiply two binomials; multiply the first, outside, inside, and last terms: **F** **F**irst terms **O** **O**utside terms **I** **I**nside terms **L** **L**ast terms. *Example*: 
fraction	any part of a whole *Example*: One-half written in fractional form is $\frac{1}{2}$.

greatest common factor (GCF)	the largest of the common factors of two or more numbers *Example*: For 6 and 8, 2 is the greatest common factor.
grouping symbols	parentheses ( ), braces { }, brackets [ ], and fraction bars indicating grouping of terms in an expression
integers	the numbers in the set {… , -4, -3, -2, -1, 0, 1, 2, 3, 4, …}
like terms	polynomials with exactly the same variable combinations; terms that have the same variables and the same corresponding exponents *Example*: In $5x^2 + 3x^2 + 6$, the like terms with the same variable combinations are $5x^2$ and $3x^2$.
monomial	a number, variable, or the product of a number and one or more variables; a polynomial with only *one* term *Examples*: 8 $\quad x$ $\quad 4c$ $\quad 2y^2$ $\quad -3$ $\quad \frac{xyz^2}{9}$
natural numbers (counting numbers)	the numbers in the set {1, 2, 3, 4, 5, …}
numerator	the top number of a fraction, indicating the number of equal parts being considered *Example*: In the fraction $\frac{2}{3}$, the numerator is 2.

opposites	two numbers whose sum is zero; also called *additive inverses* *Examples*: $\underset{\uparrow}{-5} + \underset{\uparrow}{5} = 0$ *or* $\underset{\uparrow}{\frac{2}{3}} + \underset{\uparrow}{\left(-\frac{2}{3}\right)} = 0$ opposites opposites
order of operations	the order of performing computations in parentheses first, then exponents or powers, followed by multiplication and/or division (as read from left to right), then addition and/or subtraction (as read from left to right); also called *algebraic order of operations* *Example*: $5 + (12 - 2) \div 2 - 3 \times 2 =$ $5 + 10 \div 2 - 3 \times 2 =$ $5 + 5 - 6 =$ $10 - 6 =$ 4
polynomial	a monomial or sum of monomials; any rational expression with no variable in the denominator *Examples*: $x^3 + 4x^2 - x + 8$ $5mp^2$ $-7x^2y^2 + 2x^2 + 3$
power (of a number)	an exponent; the number that tells how many times a number is used as a factor *Example*: In 2^3, 3 is the power.
prime factorization	writing a number as the product of prime numbers *Example*: $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$
prime number	any whole number with only two whole number factors, 1 and itself *Examples*: 2, 3, 5, 7, 11, etc.

product	the result of multiplying numbers together *Example*: In 6 x 8 = 48, the product is 48.
quotient	the result of dividing two numbers *Example*: In 42 ÷ 7 = 6, the quotient is 6.
rational expression	a fraction whose numerator and/or denominator are polynomials *Examples:* $\frac{x}{8}$ $\frac{5}{x+2}$ $\frac{4x^2+1}{x^2+1}$
simplest form (of an expression)	an expression that contains no grouping symbols (except for a fraction bar) and all like terms have been combined *Examples*: $6 + y + 3z + 4z = 6 + y + 7z$ $\frac{6xy^2}{5} + \frac{7xy^2}{5} = \frac{13xy^2}{5}$
standard form (of a quadratic equation)	$ax^2 + bx + c = 0$, where a, b, and c are integers (not multiples of each other) and $a > 0$
sum	the result of adding numbers together *Example*: In 6 + 8 = 14, the sum is 14.
term	a number, variable, product, or quotient in an expression *Examples*: In the expression $4x^2 + 3x + x$, the terms are $4x^2$, $3x$, and x.
trinomial	the sum of three monomials; a polynomial with exactly *three* terms *Examples*: $x + y + 2$ $m^2 + 6m + 3$ $b^2 - 2bc - c^2$ $8j^2 - 2n + rp^3$

variable	any symbol, usually a letter, which could represent a number
whole numbers	the numbers in the set {0, 1, 2, 3, 4, …}
zero property of multiplication or **zero product property**	for all numbers a and b, if $ab = 0$, then $a = 0$ and/or $b = 0$

Unit 3: Working with Polynomials

Introduction

We will see that numbers and expressions can be written in a variety of different ways by simplifying and performing operations on polynomials. Reformatting a number does *not* change the value of the number. Simplified expressions often lead us to see important information that unsimplified versions do not.

Lesson One Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Algebra Body of Knowledge

Standard 3: Linear Equations and Inequalities

- MA.912.A.3.2
  Identify and apply the distributive, associative, and commutative properties of real numbers and the properties of equality.

Polynomials

Any **expression** in which the operations are addition, subtraction, multiplication, and division, and all **powers** of the **variables** are **natural numbers** (also known as **counting numbers**). These types of expressions are called **rational expressions**. *Rational expressions* are **fractions** whose **numerator** and/or **denominator** are **polynomials**. Examples of rational expressions are as follows:

$$\frac{x+y}{3x} \qquad x - \frac{1}{x} \qquad \frac{2x+3y}{x-y}$$

Any rational expression with no *variable* in the *denominator* is called a *polynomial*. Examples of polynomials are as follows:

x^2 $\quad$ 7 $\quad$ $3y^2 - 2y + 1$ $\quad$ $x^2y + 2x - y$

A **term** is a number, variable, **product**, or **quotient** in an expression.

- If a polynomial has only one *term*, we call it a **monomial**, because "mono" means *one*.

  Examples of *monomials*:
  3 $\quad$ a^3b $\quad$ $3xy$

- If a polynomial has exactly two terms, we call it a **binomial**, because "bi" means *two*.

  Examples of *binomials*:
  $x + y$ $\quad$ $2x + 3y$ $\quad$ $3a^2 - 4b$ $\quad$ $-3y + 7$

- If a polynomial has three terms, we call it a **trinomial**, because "tri" means *three*.

  Examples of *trinomials*:

  $4x + 2y - 3z$ $\qquad x^2 + 3x + 2$ $\qquad 5ab + 2a - 3b$

Notice above that a plus or minus sign separates the terms in all polynomials. Be careful to notice where those signs occur in the expression.

Note: A polynomial is named *after* it is in its **simplest form**. For example, $3(x + 2y^3)$ must first be *simplified*. Therefore, $3(x + 2y^3) = 3x + 6y^3$, which is a binomial.

Practice

Use the list below to **identify each polynomial**. *Write the word on the line provided.*

binomial	monomial	trinomial

____________________ 1. $3b^2 - b$

____________________ 2. $4x^5$

____________________ 3. $5t^2 - 3t^5$

____________________ 4. $5x^3 - 4x^2 + 3x$

____________________ 5. $3r^2st^2$

____________________ 6. $x - y + 3$

Practice

Use the list below to **identify each polynomial**. *Write the word on the line provided.*

binomial	**monomial**	**trinomial**

________________________ 1. $3x^3 - 2x^2 + 1$

________________________ 2. $4xy^2z$

________________________ 3. $a - b + 2$

________________________ 4. $2a^2 - a$

________________________ 5. $6b^2$

________________________ 6. $3x^2 - 5y^2$

Practice

Match each definition with the correct term. Write the letter on the line provided.

_______ 1. a monomial or sum of monomials

_______ 2. a polynomial with only *one* term

_______ 3. an exponent; the number that tells how many times a number is used as a factor

_______ 4. any symbol, usually a letter, which could represent a number

_______ 5. a polynomial with exactly *three* terms

_______ 6. a mathematical phrase or part of a number sentence that combines numbers, operation signs, and sometimes variables

_______ 7. a polynomial with exactly *two* terms

_______ 8. the numbers in the set {1, 2, 3, 4, 5, …}

A. binomial

B. expression

C. monomial

D. natural numbers (counting numbers)

E. polynomial

F. power (of a number)

G. trinomial

H. variable

Lesson Two Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.

- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.

- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Algebra Body of Knowledge

Standard 3: Linear Equations and Inequalities

- MA.912.A.3.2
  Identify and apply the distributive, associative, and commutative properties of real numbers and the properties of equality.

Addition and Subtraction of Polynomials

Polynomials with *exactly* the same variable combinations can be added or subtracted. For example, $7xy$ and $3xy$ have the same variable combination. We call these **like terms**.

$$7xy + 3xy = 10xy \qquad \text{and} \qquad 7xy - 3xy = 4xy$$

A polynomial is in *simplest form* if it contains no **grouping symbols** (except a *fraction* bar) and all *like terms* have been combined.

Polynomials can be arranged in any order. In **standard form**, polynomials are arranged from *left to right*, from *greatest to least* degree of *power*. For example:

$$x^7 - x^2 + 8x$$

Polynomials can be added or subtracted in vertical (↕) or horizontal (⟷) form.

Addition

vertical form

$$(3y^2 + 2y + 3) + (y^2 + 1)$$

Align like terms in columns and add.

$$\begin{array}{rrrrl}
 & 3y^2 + & 2y + & 3 & \swarrow \text{write degrees of powers left to right from greatest to least} \\
(+) & y^2 & + & 1 & \longleftarrow \text{align like terms} \\
\hline
 & 4y^2 + & 2y + & 4 & \longleftarrow \text{add like terms}
\end{array}$$

horizontal form

$$(3y^2 + 2y + 3) + (y^2 + 1)$$

Regroup and add like terms.

$$\begin{array}{ll}
(3y^2 + y^2) + (2y) + (3 + 1) = & \longleftarrow \text{group like terms} \\
\quad 4y^2 \quad + \;\; 2y \quad + \quad 4 & \longleftarrow \text{add like terms}
\end{array}$$

Subtraction

You subtract a polynomial by adding its **additive inverse** or **opposite**. To do this, multiply each term in the *subtracted* polynomial by -1 and add.

polynomial	**additive inverse**
$-8y + 4x$	$8y - 4x$
$3q^2 - 6r + 11$	$-3q^2 + 6r - 11$
$2a + 7b - 3$	$-2a - 7b + 3$

vertical form

$(3y^2 - 2y + 3) - (y^2 - 1)$

Align like terms in columns and subtract by adding the additive inverse.

$$\begin{array}{rr} & 3y^2 - 2y + 3 \\ (-) & \underline{y^2 \qquad\quad - 1} \end{array}$$

write degrees of powers left to right from greatest to least

align like terms

add additive inverse

add like terms

$$\begin{array}{rr} & 3y^2 - 2y + 3 \\ (+) & \underline{-y^2 \qquad\quad + 1} \\ & 2y^2 - 2y + 4 \end{array}$$

Remember: $-y^2 = -1y^2$

horizontal form

$(3y^2 - 2y + 3) - (y^2 - 1)$

Subtract by adding *additive inverse* and group like terms.

$[3y^2 + (-2y) + 3] + [(-y^2) + 1] = \longleftarrow$ add additive inverse of $2y$, which is $-2y$, and $y^2 - 1$, which is $-y^2 + 1$

$[3y^2 + (-y^2)] + (-2y) + (3 + 1) = \longleftarrow$ group like terms

$2y^2 \quad + \ -2y \ + \quad 4 \quad = \longleftarrow$ add like terms

vertical form

Subtract $2t^2 - 3t + 4$ from the **sum** of $t^2 + t - 6$ and $3t^2 + 2t - 1$.

$(t^2 + t - 6) + (3t^2 + 2t - 1) - (2t^2 - 3t + 4)$

$$\begin{array}{rcr}
t^2 + t - 6 & \longleftarrow \text{write degrees of powers left to right from greatest to least} \longrightarrow & t^2 + t - 6 \\
3t^2 + 2t - 1 & \longleftarrow \text{align like terms} \longrightarrow & 3t^2 + 2t - 1 \\
(-)\ \underline{2t^2 - 3t + 4} & \longleftarrow \text{add additive inverse} \longrightarrow & (+)\ \underline{-2t^2 + 3t - 4} \\
 & & 2t^2 + 6t - 11
\end{array}$$

horizontal form

Subtract $2t^2 - 3t + 4$ from the *sum* of $t^2 + t - 6$ and $3t^2 + 2t - 1$.

$$(t^2 + t - 6) + (3t^2 + 2t - 1) - (2t^2 - 3t + 4)$$

$$\boxed{t^2}\enclose{circle}{+\ t}\underline{-\ 6}\boxed{+\ 3t^2}\enclose{circle}{+\ 2t}\underline{-\ 1}\boxed{-\ 2t^2}\enclose{circle}{+\ 3t}\underline{-\ 4} = \longleftarrow \text{add additive inverse of } -(2t^2 - 3t + 4)\text{, which is } -2t^2 + 3t - 4$$

$$(t^2 + 3t^2 - 2t^2) + (t + 2t + 3t) + (-6 - 1 - 4) = \longleftarrow \text{group like terms}$$

$$2t^2 + 6t - 11 \longleftarrow \text{combine like terms}$$

Practice

Write each **expression** *in* **simplest form**. *Use either the* **horizontal** *or* **vertical** *form. Refer to examples on pages 206-208 as needed. Show* **essential steps**.

Remember: Write answers with the degree of powers arranged from left to right and from greatest to least.

Example: $(3y^2 - 2y + 3) - (y^2 - 1)$

horizontal form		**vertical form**
$(3y^2 - 2y + 3) - (y^2 - 1) =$ ←	add additive inverse and group like terms	→ $3y^2 - 2y + 3$
$(3y^2 + -y^2) + (-2y) + (3 + 1) =$ ←		→ $(+)\ \underline{-y^2 \qquad + 1}$
$2y^2 \quad + \ -2y \quad + \quad 4$ ←	add like terms	→ $2y^2 - 2y + 4$

1. $3ab^2 - 5a^2b + 5ab^2$

2. $(2x^2 - 3x + 7) - (3x^2 + 3x - 5)$

3. $(2x^3 - 3x^2 + 2x) + (4x - 2x^2 - 3x^3)$

4. $(4a^2 + 6a - 6) + (3a^2 - 2a + 4) - (5a^2 - 5a - 9)$

5. $(-3y^3 + 4y^2 + 6y) - (y^3 - 2y^2 + y + 6) + (4y^3 + 2y^2 - 4y - 1)$

6. $(a^3 - 3a^2b - 4ab^2 + 6b^3) - (a^3 + a^2b - 2ab^2 - 5b^3)$

7. $3a + [5a - (a + 3)]$

8. $[x^2 - (2x - 3)] - [2x^2 + (x - 2)]$

9. $5 - [3y + (y - 2) - 1]$

10. $y - \{y - [x - (2x - y)] + 2y\}$

Example: Subtract $2t^2 - 3t + 4$ from the sum of $t^2 + t - 6$ and $3t^2 + 2t - 1$.

horizontal form		**vertical form**
$(t^2 + t - 6) + (3t^2 + 2t - 1) - (2t^2 - 3t + 4) =$ ←	add additive inverse	$t^2 + t - 6$
$t^2 + t - 6 + 3t^2 + 2t - 1 - 2t^2 + 3t - 4 =$		$3t^2 + 2t - 1$
$2t^2 + 6t - 11$ ←	group like terms and add	$(+)\ \underline{-2t^2 + 3t - 4}$
		$2t^2 + 6t - 11$

11. Subtract $4x^2 - 3x + 3$ from the sum of $x^2 - 2x - 3$ and $x^2 - 4$.

12. Subtract $2t^2 - 3t + 5$ from the sum of $4t^2 - 3t + 4$ and $-t^2 + 5t + 7$.

Practice

Write each **expression** *in* **simplest form**. *Use either the* **horizontal** *or* **vertical** *form. Refer to examples on previous practice and pages 206-208 as needed. Show* **essential steps**.

1. $5xy^2 + 2x^2y - 6xy^2$

2. $(6a^2 - 4a - 3) - (5a^2 + 2a + 1)$

3. $(3y^3 - 4y^2 + 9y) + (5y^3 - 6y^2 + 6)$

4. $(8x^2 + 2x - 6) - (4x^2 - 3x + 9) + (5x^2 + 2x - 3)$

5. $(8a^3 - 2a^2 + 3a) - (9a^3 + 5a - 4) + (6a^2 - 8a + 5)$

6. $(x^3 - 4x^2y - 6xy^2 + 2y^3) - (x^3 + 6x^2y - 9xy^2 + 6y^3)$

7. $5x + [3x - (x + 2)]$

8. $b^2 - [4b - (b + 6)]$

9. $7 – [4x + (x – 2) – 4]$

10. $x – \{2x – [x + (2x – y)] + 5y\}$

11. Subtract $3x^2 + 2x – 1$ from the sum of $8x^2 – 6x + 9$ and $x^2 – 8$.

12. Subtract $2a^2 – 6a + 4$ from the sum of $a^2 + 4$ and $4a^2 – 9a + 8$.

Lesson Three Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.

- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.

- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Algebra Body of Knowledge

Standard 1: Real and Complex Number Systems

- MA.912.A.1.8
  Use the zero product property of real numbers in a variety of contexts to identify solutions to equations.

Standard 3: Linear Equations and Inequalities

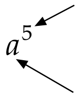
- MA.912.A.3.2
  Identify and apply the distributive, associative, and commutative properties of real numbers and the properties of equality.

Standard 4: Polynomials

- MA.912.A.4.1
  Simplify monomials and monomial expressions using the laws of integral exponents.

Multiplying Monomials

First Law of Exponents



exponent (exponential form) is the number of times the **base (of an exponent)** occurs as a **factor**

base (of an exponent) is the number that is used as a *factor* in *exponential form*

For example, a^5a^3 means a^5 times a^3 or $(aaaaa)(aaa)$. By *counting* the factors of a, which is 8, you can see that

$$a^5a^3 = a^8.$$

This is an example of the *first law of exponents*, which states that $a^xa^y = a^{x+y}$.

Below are other examples.

$x^2x^3 = x^5 \qquad xx^4x^5 = x^{10} \qquad b^4b^2b^3 = b^9$

When there are **coefficients** (the numbers you multiply the variables by), you must multiply those first and then use the first law of exponents. In the expression $8x^2y$, the *coefficient* is 8. In the expression $3xy^4$, the coefficient is 3. If no number is specified, the coefficient is 1.

First Law of Exponents—Product of Powers

You *multiply exponential forms* with the same base by *adding the exponents*.

$$4^3 \bullet 4^2 = 4^{3+2} \text{ or } 4^5$$

$$x^a \bullet x^b = x^{a+b}$$

If we multiply $2x^2y$ and $3xy^4$, this would be

$$(2x^2y)(3xy^4) =$$
$$\underbrace{2 \bullet 3}_{6} \bullet \underbrace{x^2 \bullet x}_{x^3} \bullet \underbrace{y \bullet y^4}_{y^5} =$$
$$6x^3y^5$$

- The coefficients are multiplied.

$$2 \bullet 3 = 6$$

- The exponents are added.

$$\overbrace{x^2 \bullet x}^{x^2 + x^1} = x^3 \text{ and } \overbrace{y \bullet y^4}^{y^1 + y^4} = y^5$$

If we multiply $7b$ and $-b^3$, this would be

$$(7b)(-b^3) = (7b)(-1b^3)$$
$$-7b^4$$

- The coefficients are multiplied.

$$7 \bullet -1 = -7$$

- The exponents are added. The term $-b^3$ is understood to be $-1b^3$.

$$b \bullet \ -b^3 = -b^4$$
$$1b \bullet -1b^3 = -1b^4 = -b^4$$

Remember: Use the rules for the **order of operations**. Complete multiplication as it occurs, from left to right, including all understood coefficients.

Example

$-x^3(x^4)(5x)(-2x^4) =$

$(-1)(1)(5)(-2) \bullet (x^3x^4x^1x^4) =$ ⟵ multiply the coefficients left to right

$10 \bullet x^{3+4+1+4} =$ ⟵ add the exponents

$10 \bullet x^{12} =$

$10x^{12}$

Practice

Write each **product** *as a* **polynomial** *in* **simplest form**.

Remember: Multiply the coefficients and add the exponents.

Example: $(7a^2)(5a^3b^4) =$
$35a^5b^4$

1. $(6t)(-3t^3)$

2. $(5x)(-x^4)$

3. $(-6r^2s)(4r^2s^3)$

4. $(-5a)(ab^3)(-3a^2bc)$

5. $(y^2z)(-3x^2z^2)(-y^4z)$

6. $-a^2(ab^2)(3a)(-2b^3)$

7. $(-t)^2(2t^2)(5t)^2$

Hint: Notice with $(-t)^2$ and $(5t)^2$, the exponent 2 is placed on the *outside* of the grouping symbols, the parentheses. Use the **distributive property** and raise every term in the parentheses to the exponent.

Example: $(-t)^2 = (-t)(-t) = t^2$
$(5t)^2 = (5t)(5t) = 25t^2$

8. $(3x^2)(-5x^3y^2)(0)(-4y)^2$

Hint: Notice the zero (0). The **zero property of multiplication**, also known as the **zero product property**, states that any number multiplied by 0 is 0.

Zero Property of Multiplication *or* Zero Product Property
For all numbers a and b, if $ab = 0$, then $a = 0$ and/or $b = 0$.

Practice

Write each **product** *as a* **polynomial** *in* **simplest form**.

1. $(8x)(-2x^2)$

2. $(5a)(-a^6)$

3. $(-4x^2y)(3x^3y^2)$

4. $(-6b)(ab^4)(-4a^2bc^2)$

5. $(x^3y^2)(-2x^2y)(-x^4y^2)$

6. $-s^3(s^2t^2)(4s)(-2t^4)$

7. $(-a)^2(4a^2)(3a)^2$

8. $(6x)^2(-2x^2y^3)(0)(-2x)^2$

Lesson Four Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Algebra Body of Knowledge

Standard 3: Linear Equations and Inequalities

- MA.912.A.3.2
  Identify and apply the distributive, associative, and commutative properties of real numbers and the properties of equality.

Standard 4: Polynomials

- MA.912.A.4.1
  Simplify monomials and monomial expressions using the laws of integral exponents.

- MA.912.A.4.4
  Divide polynomials by monomials and polynomials with various techniques, including synthetic division.

Dividing Monomials

Second Law of Exponents

When dividing monomials it is important to remember that

$$\frac{a^5}{a^3} = \frac{aaaaa}{aaa}.$$

It is also important to remember the following.

$$\frac{a}{a} = 1$$

$$a^0 = 1$$

Therefore, the three factors of *a* in the *denominator* **cancel** three of the five factors of *a* in the *numerator*. This leaves $a \bullet a$ or a^2 in the numerator and 1 in the denominator.

Remember: To *cancel* means to divide a numerator (the top part of the fraction) and a denominator (the bottom part of the fraction) by a **common factor**. This is done in order to write the fraction in lowest terms or before multiplying the fractions.

$$\frac{\overset{\text{numerator}}{\overset{\downarrow}{\cancel{a}\cancel{a}\cancel{a}aa}}}{\underset{\text{denominator}}{\underset{\uparrow}{\cancel{a}\cancel{a}\cancel{a}}}} = \frac{aa}{1} \text{ or } \frac{a^2}{1} = a^2$$

Another way to look at this is

$$\frac{a^5}{a^3} = a^{5-3} = a^2.$$

This is an example of the *second law of exponents*, which states that

$$\frac{a^x}{a^y} = a^{x-y}$$

as long as $a \neq 0$.

Second Law of Exponents

You *divide exponential forms* by *subtracting the exponents*.

$$9^7 \div 9^3 = 9^{7-3} = 9^4$$

Remember: The *fraction bar* represents division. So, $\frac{8^4}{8^2}$ means $8^4 \div 8^2$.

$$\frac{9^7}{9^3} = 9^{7-3} = 9^4$$

$$\frac{a^m}{a^n} = a^{m-n}$$

If the exponents are the same,

$$\frac{a^x}{a^x} = 1 \text{ and}$$

$$\frac{a^x}{a^x} = a^{x-x} = a^0 = 1.$$

Any number (except zero) raised to the zero power is equal to 1.

$$a^0 = 1$$

Example

$$\frac{x^4b^3}{xb^3} =$$

$$x^{4-1} \bullet \frac{b^3}{b^3} = \longleftarrow \frac{\cancel{b^3}}{\cancel{b^3}} = 1$$

$$x^3 \bullet 1 =$$

$$x^3$$

When there are coefficients with variables, simply reduce those as you do when working with fractions.

Example

$$\frac{12a^3b^5}{-4ab^3} = \longleftarrow \frac{\cancel{12}^{3}}{\cancel{-4}_{-1}} = \frac{3}{-1} = -3$$

$$-3a^{3-1}b^{5-3} =$$

$$-3a^2b^2$$

Practice

Write each **quotient** *as a* **polynomial** *in* **simplest form**. *Refer to examples on pages 223-225 as needed. Show* **essential steps**.

1. $\dfrac{6x^3y^4}{3xy}$

2. $\dfrac{14c^4d^3}{-7c^4d^2}$

3. $\dfrac{100m^5n}{-20m^3n}$

4. $\dfrac{-22a^2bc^5}{11abc}$

5. $\dfrac{12r^2st^3}{-3rst^3}$

6. $\dfrac{a^5b^6c^7}{a^4b^3c^2}$

7. $\dfrac{(t+4)^5}{(t+4)^2}$

Hint: Notice that the exponents 5 and 2 are on the *outside* of the grouping symbols, the parentheses. Since the bases ($t + 4$) are the same, just subtract the exponents. Do *not* raise each term in the parentheses to the exponent.

8. $\dfrac{9(x-3)^3}{-3(x-3)^2}$

Practice

Write each **quotient** *as a* **polynomial** *in* **simplest form**. *Refer to examples on pages 223-225 as needed. Show* **essential steps**.

1. $\frac{8a^2b^4}{4ab^2}$

2. $\frac{16x^5y^4}{-8x^3y}$

3. $\frac{-36a^2b^5c^4}{3ab^2c}$

4. $\frac{48x^2yz^3}{12xyz}$

5. $\dfrac{20a^2bc^3}{10abc^2}$

6. $\dfrac{x^5y^7z^6}{x^2y^2z^6}$

7. $\dfrac{(x + 1)^5}{(x + 1)^3}$

8. $\dfrac{10(x + 7)^4}{-5(x + 7)^2}$

Lesson Five Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.

- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.

- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Algebra Body of Knowledge

Standard 3: Linear Equations and Inequalities

- MA.912.A.3.2
  Identify and apply the distributive, associative, and commutative properties of real numbers and the properties of equality.

Standard 4: Polynomials

- MA.912.A.4.1
  Simplify monomials and monomial expressions using the laws of integral exponents.

- MA.912.A.4.2
  Add, subtract, and multiply polynomials.

- MA.912.A.4.3
  Factor polynomial expressions.

Multiplying Polynomials

Using the Distributive Property to Multiply a Monomial and a Trinomial

Multiplication of a *monomial* and a *polynomial* is simply an extension of the *distributive property*. Make sure that *every term in the parentheses* is multiplied by the *term in front of the parentheses*.

$$\begin{aligned} &3x(2a + 3b - 4c) = \\ &\quad 6xa + 9xb - 12xc \end{aligned}$$

← multiply every term in the parentheses by the term 3*x* in front of the parentheses

Typically, mathematicians like to put things in order. They will rearrange the variables in the answer above so that the variables in each term are *alphabetical*. Therefore, the final answer would be as follows.

$$\begin{aligned} &6xa + 9xb - 12xc = \\ &6ax + 9bx - 12cx \end{aligned}$$

Using the FOIL Method to Multiply Two Binomials

When we multiply two polynomials, we extend the distributive property even further to make sure that every term in the *first* set of parentheses is multiplied by every term in the *next* set of parentheses.

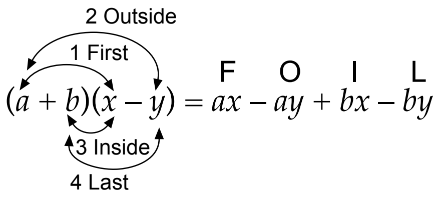
Look carefully at the product below.

$$(a + b)(x - y) = ax - ay + bx - by$$

Notice that both x and y were multiplied by a, and then by b. This is called the **FOIL method** because

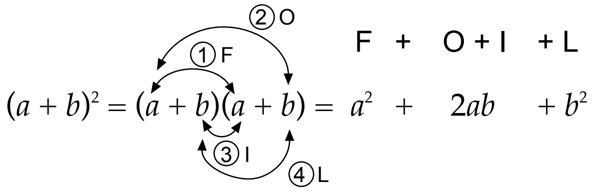
- the two **F**irst terms (a and x) are multiplied
- then the two **O**utside terms (a and $-y$) are multiplied
- then the two **I**nside terms (b and x) are multiplied and lastly
- the two **L**ast terms (b and $-y$) are multiplied together.

F **F**irst terms
O **O**utside terms
I **I**nside terms
L **L**ast terms



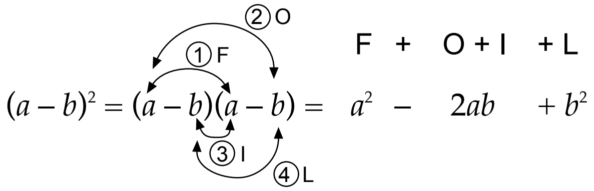
It is important to be *orderly* when you multiply to ensure that you don't leave out a step. Also, be very careful to watch the positive (+) and negative (–) signs as you work.

Special patterns often occur. Knowing these may help you.



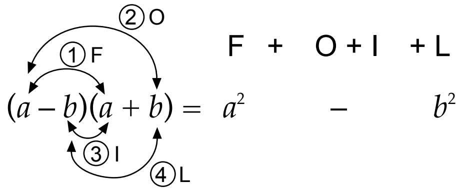
F + O + I + L

$$(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2$$



F + O + I + L

$$(a-b)^2 = (a-b)(a-b) = a^2 - 2ab + b^2$$



F + O + I + L

$$(a-b)(a+b) = a^2 - b^2$$

Alert!

$(a+b)^2 \neq a^2 + b^2$ ← To write this expression in simplest form, the power of 2 is not simply distributed over *a* + *b*. Instead…

$(a+b)^2 = (a+b)(a+b)$ ← $(a+b)^2$ is multiplied by itself, $(a+b)(a+b)$.

Using the Distributive Property to Multiply Any Two Polynomials

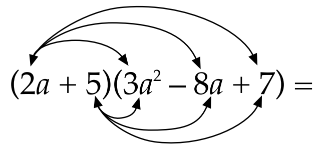
Let's look at using the distributive property to do the following.

- multiply a binomial and a trinomial in horizontal form
- multiply two trinomials in horizontal form
- multiply polynomials in vertical form

Example 1

Find the product of a binomial and a trinomial in **horizontal form**.

$(2a + 5)(3a^2 - 8a + 7) =$



$(2a + 5)(3a^2 - 8a + 7) =$

$2a(3a^2 - 8a + 7) + 5(3a^2 - 8a + 7) =$ ← distributive property

$(6a^3 - 16a^2 + 14a) + (15a^2 - 40a + 35) =$

combine

$6a^3 \boxed{-16a^2} (+14a) \boxed{+15a^2} (-40a) + 35 =$ ← combine like terms

combine

$6a^3 - a^2 - 26a + 35$

Example 2

Find the product of two trinomials in **horizontal form**.

$$(b^2 + 4b - 5)(3b^2 - 7b + 2) =$$

$$(b^2 + 4b - 5)(3b^2 - 7b + 2) =$$

distributive property

$$b^2(3b^2 - 7b + 2) + 4b(3b^2 - 7b + 2) - 5(3b^2 - 7b + 2) =$$

$$(3b^4 - 7b^3 + 2b^2) + (12b^3 - 28b^2 + 8b) - (15b^2 - 35b + 10) =$$

$$3b^4 - 7b^3 + 2b^2 \;+\; 12b^3 - 28b^2 + 8b \;-\; 15b^2 + 35b - 10 \;=$$

combine like terms

$$3b^4 + 5b^3 - 41b^2 + 43b - 10$$

Example 3

Find the product of polynomials in **vertical form**.

$$(c^3 - 8c^2 + 9)(3c + 4) =$$

Note: There is no c term in $c^3 - 8c^2 + 9$, so $0c$ is used as a placeholder.

$$\begin{array}{r}
c^3 - \;\; 8c^2 + \;\; 0c + \;\; 9 \\
(\mathrm{x})\underline{\qquad\qquad 3c + \;\; 4} \\
4c^3 - 32c^2 + \;\; 0c + 36 \\
\underline{3c^4 - 24c^3 + \;\; 0c^2 + \; 27c \qquad} \\
3c^4 - 20c^3 - 32c^2 + 27c + 36
\end{array}$$

Practice

Write each **expression** *as a* **polynomial** *in* **simplest form**. *Refer to examples on pages 231-235 as needed. Show* **essential steps**.

Example: $-3(2x + 4y - z) =$ ← distributive property

$-6x - 12y + 3z$

1. $2a(a + 3b)$

2. $-5x(3x - 2y + 6z)$

Example: $(x - 4)^2 =$

$(x - 4)(x - 4) =$ ← FOIL method

$x^2 - 4x - 4x + 16 =$

$x^2 - 8x + 16$

3. $(x + 4)^2$

4. $(x + 8)^2$

Example: $(y - 3)(y + 4) =$ ← FOIL method

$y^2 + 4y - 3x - 12 =$

$y^2 + y - 12$

5. $(a - 3)(a + 6)$

6. $(x + 2)(x - 2)$

7. $(2x + 5)(3x - 6)$

8. $(3t - 1)(3t + 5)$

9. $(3g - 4)(2g - 3)$

Example:

horizontal form

$$(x-4)(x^2+2x-3) =$$ ← distributive property

$$x^3 + 2x^2 - 3x - 4x^2 - 8x + 12 =$$
$$x^3 - 2x^2 - 11x + 12$$

vertical form

$$\begin{array}{rr}
 & x^2 + 2x - 3 \\
\text{(x)} & \underline{x - 4} \\
 & -4x^2 - 8x + 12 \\
\multicolumn{2}{l}{\underline{x^3 + 2x^2 - 3x\phantom{{}+ 12}}} \\
\multicolumn{2}{r}{x^3 - 2x^2 - 11x + 12}
\end{array}$$

Notice the order of the terms in the answer above. The values of the exponents are in *decreasing* order: x^3, x^2, x^1, x^0.

10. $(x + 2)(x^2 - 2x + 3)$

11. $(x^2 - 3x + 5)(x - 6)$

12. $(2a^2 - 3a + 1)(3a^2 + 2a + 1)$

Practice

Write each **expression** *as a* **polynomial** *in* **simplest form**. *Refer to examples on pages 231-235 as needed. Show* **essential steps**.

Use the **distributive property**.

1. $6s(s^2 - 3s + 2)$

2. $2y^3(3y^2 - 4y + 7)$

Use the **FOIL method**.

3. $(x - 3)^2$

4. $(x - 10)^2$

5. $(b + 5)(b + 4)$

6. $(c - 5)(c + 5)$

7. $(2z - 3)(4z + 2)$

Use the **distributive property**.

8. $(b + 5)(b^2 + 4b - 9)$

9. $(y^2 - 3y + 7)(y^2 + 4)$

10. $(a + 3)(a - 4)(a - 5)$

    Hint: Multiply the first two binomials. Then multiply that product by the third binomial. Use either the vertical or horizontal form to do this.

Lesson Six Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.

- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.

- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Algebra Body of Knowledge

Standard 3: Linear Equations and Inequalities

- MA.912.A.3.2
  Identify and apply the distributive, associative, and commutative properties of real numbers and the properties of equality.

Standard 4: Polynomials

- MA.912.A.4.1
  Simplify monomials and monomial expressions using the laws of integral exponents.

- MA.912.A.4.2
  Add, subtract, and multiply polynomials.
- MA.912.A.4.3
  Factor polynomial expressions.

Factoring Polynomials

If we look at the product *abc*, we know *a*, *b*, and *c* are *factors* of this product. In the same way, 2 and 3 are *factors* of 6. Other factors of 6 are 6 and 1.

Remember: A factor is a number or expression that divides evenly into another number.

factors of $abc = a, b$, and c

factors of 6 = 1, 2, 3, and 6

Some numbers, like 5, have no factors other than the number itself and the number 1. These numbers are called **prime numbers**. A *prime number* is any **whole number** {0, 1, 2, 3, 4, …} with only two factors, 1 and itself. The prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, and 19.

prime numbers < 20 = 2, 3, 5, 7, 11, 13, 17, and 19

Natural numbers greater than 1 that are *not* prime are called **composite numbers**. A *composite number* is a whole number with more than two factors. For example, 16 has five factors, 1, 2, 4, 8, and 16. Therefore, 4, 6, 8, 9, 10, 12, 14, 15, 16, and 18 are the composite numbers less than 20.

composite numbers < 20 = 4, 6, 8, 9, 10, 12, 14, 15, 16, and 18

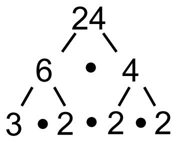
Every composite number can be written as a *product* of prime numbers. We can find this **prime factorization** by factoring the factors and repeating this process until all factors are primes.

For example, find the *prime factorization* of 24 and express it in completely **factored form**.

Factoring a Positive Number—
numbers greater than zero

Method One

Use a factor tree.



24 = 3 • 2 • 2 • 2

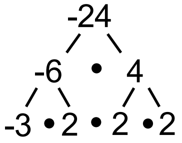
or

Method Two

24
6 • 4
(3 • 2)(2 • 2)
3 • 2 • 2 • 2

Factoring a Negative Number—
numbers less than zero

Method One



-24 = -3 • 2 • 2 • 2

or

Method Two

-24
-6 • 4 =
(-3 • 2)(2 • 2) =
-3 • 2 • 2 • 2

An Alternate Method for Factoring a Positive Number

Here is an alternate method for factoring a positive number called *upside-down dividing*. Divide by prime numbers starting with the number 2.

2)24
2)12
2) 6
3) 3
1

The prime factorization is down the left side.

2 • 2 • 2 • 3 or 2^3 • 3

To factor polynomials, you must look carefully at each term and decide if there is a factor that is *common* to each term. If there is, we basically "undistribute" or *factor out* that **greatest common factor (GCF)**. Look at the example below.

$6x^3 - 12x^2 + 3x =$ ⟵ Notice that each term can be divided by 3 and x. So, $3x$ is the *greatest factor* these terms have in *common*. Therefore, $3x$ is the *GCF* of $6x^3 - 12x^2 + 3x$.

$3x(2x^2 - 4x + 1)$ ⟵ *undistribute* the $3x$

All of the terms and symbols must be written to make sure that your new expression is *exactly* equal to the original one. You can check your work by distributing the $3x$ to everything within the parentheses to see if it matches the original expression.

$$3x(2x^2 - 4x + 1) = 6x^3 - 12x^2 + 3x$$

Remember: $(a + b) = (b + a)$

The **commutative property** of addition—numbers can be added in any order and the sum will be the same.

Alert!

$(a - b) \neq (b - a)$

The same is *not* true for $a - b$. The *commutative property* does *not* work with subtraction.

$a - b$ does *not* equal $b - a$

$(a - b) = -1(b - a)$

$a - b$ is understood as $a - {}^{+}b$, therefore,

$a - b$ equals $-1(b - a)$

Practice

Express each **integer {… , -3, -2, -1, 0, 1, 2, 3, …}** *in completely* **factored form**. *If the integer is a* **prime number**, *write* **prime**.

1. 8

2. 18

3. -16

4. 23

5. 56

Factor *the following. Show* **essential steps**.

Example: $18x^3y - 24x^2y^2 =$

$6x^2y(3x - 4y)$

Find the **GCF**, which is $6x^2y$, and *undistribute* it.

6. $3a - 9$

7. $2x^2y^2 + 3xy - 4xy^3$

8. $3m^4 + 6m^3 - 12m^2$

9. $ay^3b + a^2y^2 + ab$

Example: $x(b + 2) - 7(b + 2)$

$$x\boxed{(b + 2)} - 7\boxed{(b + 2)} =$$

$$(b + 2)(x - 7)$$

10. $a(a + 3) - 6(a + 3)$

11. $2x(x + 5) - 3(x + 5)$

12. $5(y - 7) + z(y - 7)$

Practice

Express each **integer {… , -3, -2, -1, 0, 1, 2, 3, …}** *in completely* **factored form***.*
If the integer is a **prime number***, write* **prime***.*

1. 12

2. 15

3. -25

4. 31

5. 72

Factor *the following. Show* **essential steps***.*

6. $4b^2 + 12b$

7. $y^4 - y^3 + y$

8. $15r^2s + 9rs^2 - 12rs$

9. $16x^2yz^3 + 8x^3y^2z^2 - 24x^4y^2z$

10. $y(y - 4) + 4(y - 4)$

11. $5x(3a + 1) - 4(3a + 1)$

12. $3a(2x - y) + 4a(2x - y)$

Practice

Match each definition with the correct term. Write the letter on the line provided.

_______ 1. the largest of the common factors of two or more numbers

_______ 2. any whole number with only two whole number factors, 1 and itself

_______ 3. the order in which two numbers are added or multiplied does *not* change their sum or product, respectively

_______ 4. a number or expression expressed as the product of prime numbers and variables, where no variable has an exponent greater than 1

_______ 5. writing a number as the product of prime numbers

_______ 6. a whole number that has more than two factors

A. commutative property

B. composite number

C. factored form

D. greatest common factor (GCF)

E. prime factorization

F. prime number

Lesson Seven Purpose

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  Identify and apply the distributive, associative, and commutative properties of real numbers and the properties of equality.

Standard 4: Polynomials

- MA.912.A.4.1
  Simplify monomials and monomial expressions using the laws of integral exponents.

- MA.912.A.4.2
  Add, subtract, and multiply polynomials.
- MA.912.A.4.3
  Factor polynomial expressions.
- MA.912.A.4.4
  Divide polynomials by monomials and polynomials with various techniques, including synthetic division.

Factoring Quadratic Polynomials

Polynomials that are written in the format $ax^2 + bx + c$ can be factored into two binomials. The following six-step method may help, especially if you have had difficulty with factoring in the past.

Example 1

Format	$ax^2 + bx + c$		
Step 1	$\underbrace{6x^2 + 17x + 5}_{\times}$	←	Write the problem. Factor out common factors, if there are any. Identify *a*, *b*, and *c*. *a* = 6, *b* = 17, and *c* = 5
Step 2	$ac = 6 \bullet 5$ $= 30$	←	Multiply *a* and *c*.
Step 3	$6x^2 + \underbrace{2x + 15x} + 5$	←	Rewrite the problem using factors of *ac*. The factors you choose must combine (add or subtract) to *equal the middle term*.
			Note: 2*x* + 15*x* = 17*x*, which is the *same* as the *original* middle term.
Step 4	$(6x^2 + 2x) + (15x + 5)$	←	Group the first two terms and the last two terms.
Step 5	$2x\underbrace{(3x + 1)} + 5\underbrace{(3x + 1)}$	←	Factor out the greatest common factor for each term. You will always be left with a matching pair of factors. Notice the factors of (3*x* + 1). If you do **not** have a matching pair, double-check your work at this point!
Step 6	$(3x + 1)(2x + 5)$	←	Write down the common factor (3*x* + 1). Then write the "leftovers" in parentheses. You have succeeded!

The next example shows how to handle minus signs. Watch carefully!

Example 2

Format	$ax^2 + bx + c$	
Step 1	$4x^2 - 5x + 1$	← Write the problem. Factor out common factors, if there are any. Identify *a*, *b*, and *c*. $a = 4$, $b = -5$, and $c = 1$
Step 2	$ac = 4 \bullet 1$ $= 4$	← Multiply *a* and *c*.
Step 3	$4x^2 - 4x - x + 1 =$ $4x^2 + -4x + -x + 1 =$	← Rewrite the problem using factors of *ac*. The factors you choose must combine (add or subtract) to *equal the middle term*.
Step 4	$(4x^2 + -4x) + (-x + 1) =$	← Group the first two terms and the last two terms. *If the second term in step 3 is followed by a minus sign, this* **requires** *a sign change to each term in the second group.*
Step 5	$4x(x - 1) + -1(x - 1) =$	← Factor out the greatest common factor for each term. You must always have a common factor, even if it is only a 1. You will always be left with a matching pair of factors. Notice the factors of $(x - 1)$. If you do **not** have a matching pair, double-check your work at this point!
Step 6	$(x - 1)(4x - 1)$	← Write down the common factor $(x - 1)$. Then write the "leftovers" in parentheses. You have succeeded!

Now, you try one!

Example 3

Format $ax^2 + bx + c$

Step 1 $4x^2 + 4x - 3$ ← Write the problem. Factor out common factors, if there are any. Identify *a*, *b*, and *c*.

a = ____ , *b* = ____ , and *c* = ____

Step 2 ac = _______ ← Multiply *a* and *c*.

= _______

Step 3 ________________ ← Rewrite the problem using factors of *ac*. The factors you choose must combine (add or subtract) to *equal the middle term*.

Step 4 ________________ ← Group the first two terms and the last two terms. *If the second term in step 3 is followed by a minus sign, this* **requires** *a sign change to each term in the second group.*

Step 5 ________________ ← Factor out the greatest common factor for each term. You must always have a common factor, even if it is only a 1. You will always be left with a matching pair of factors. Notice the factors of $(2x + 3)$. If you do **not** have a matching pair, double-check your work at this point!

Step 6 ________________ ← Write down the common factor. Then write the "leftovers" in parentheses.

Use FOIL to check your answer. If your answer is $(2x + 3)(2x - 1)$, you have succeeded!

Now you are ready to practice some problems on your own.

Practice

Factor *completely. Show* **essential steps**.

Format $ax^2 + bx + c$

Example: $8x^2 + 12x - 8 =$ ← $a = 2$, $b = 3$, and $c = -2$

$4(2x^2 + 3x - 2) =$ ← $ac = -4$

$4(2x^2 + 4x - x - 2) =$

$4[2x(x + 2) - 1(x + 2)] =$

$4[(x + 2)(2x - 1)]$

1. $6b^2 + 17b + 5$

2. $3x^2 - 8x + 5$

3. $3a^2 + 7a - 6$

4. $2y^2 + 7y + 5$

5. $8x^2 - 6x - 9$

6. $10a^2 + 11a - 6$

7. $3x^2 + 4x + 1$

8. $4a^2 - 5a + 1$

9. $2r^2 + 3r - 2$

Practice

Factor *completely. Show* **essential steps**.

Format $ax^2 + bx + c$

Example: $x^2 - 2x - 3 =$ ← a = 1, b = -2, and c = -3

$x^2 - 3x + x - 3 =$ ← ac = -3

$(x^2 - 3x) + (x - 3) =$

$x(x - 3) + 1(x - 3) =$

$(x - 3)(x + 1)$

1. $a^2 - a - 6$

2. $y^2 + 7y + 12$

3. $x^2 + 7x + 10$

4. $a^2 - 2a + 15$

5. $x^2 + 6x + 5$

Take opportunities to practice factoring problems like the ones in this practice, and use the factors of the middle term with trial and error tactics.

Practice

Factor *completely. Show* **essential steps**.

Format $ax^2 + bx + c$

Example: $x^2 - 4 =$ ← insert a middle term of 0*x*

$x^2 + 0x - 4 =$ ← *a* = 1, *b* = 0, and *c* = -4
ac = -4

$x^2 + 2x - 2x - 4 =$ ← rewrite *b* as +2*x* – 2*x*

$(x^2 + 2x) - (2x + 4) =$ ← group the first two and last two terms

Remember: If the second term is followed by a minus sign, this requires a sign change to each term in the second group.

$x(x + 2) - 2(x + 2) =$ ← take out common factors

$(x + 2)(x - 2)$ ← rewrite using common factors

1. $a^2 - 16$

2. $x^2 - 9$

3. $b^2 - 25$

4. $y^2 - 81$

5. $x^2 - 36y^2$

Notice that the final terms in the problems above were all *perfect squares* and the answers fit the pattern $a^2 - b^2 = (a + b)(a - b)$. Use this shortcut whenever possible. However, if you are unsure, you can always use the six-step method used in the previous practices.

Remember: A *perfect square* is a number whose *square root* is a whole number.
Example: 25 is a perfect square because 5 x 5 = 25.

Practice

Factor *completely. Show* **essential steps**.

Format $ax^2 + bx + c$

Example: $8x^2 + 12x - 8 =$ ← $a = 2$, $b = 3$, and $c = -2$

$4(2x^2 + 3x - 2) =$ ← $ac = -4$

$4(2x^2 + 4x - x - 2) =$

$4[2x(x + 2) - 1(x + 2)] =$

$4[(x + 2)(2x - 1)]$

1. $2x^2 + 3x - 20$

2. $15x^2 + 13x + 2$

3. $6x^2 - 7x - 10$

4. $x^2 + 6x + 8$

5. $x^2 + x - 12$

Practice

Factor *completely. Show* **essential steps**.

Format $ax^2 + bx + c$

Example: $x^2 - 2x - 3 =$ ← $a = 1$, $b = -2$, and $c = -3$
$x^2 - 3x + x - 3 =$ ← $ac = -3$
$(x^2 - 3x) + (x - 3) =$
$x(x - 3) + 1(x - 3) =$
$(x - 3)(x + 1)$

1. $x^2 - 3x - 4$

2. $x^2 - 3x + 2$

3. $x^2 - 8x + 15$

Example: $x^2 - 4 =$ ← insert a middle term of 0*x*

$x^2 + 0x - 4 =$ ← *a* = 1, *b* = 0, and *c* = -4
ac = -4

$x^2 + 2x - 2x - 4 =$ ← rewrite *b* as +2*x* – 2*x*

$(x^2 + 2x) - (2x + 4) =$ ← group the first two and last two terms

Remember: If the second term is followed by a minus sign, this requires a sign change to each term in the second group.

$x(x + 2) - 2(x + 2) =$ ← take out common factors

$(x + 2)(x - 2)$ ← rewrite using common factors

4. $a^2 - 4$

5. $x^2 - 64$

6. $r^2 - 9$

7. $y^2 - 100$

8. $a^2 - 25b^2$

Practice

Use the list below to write the correct term for each definition on the line provided.

binomial
coefficient
composite number
factor
like terms
monomial
polynomial
prime number
rational expression
simplest form (of an expression)
trinomial

________________________ 1. any whole number with only two whole number factors, 1 and itself

________________________ 2. the number that multiplies the variable(s)

________________________ 3. a polynomial with exactly *two* terms

________________________ 4. a polynomial with only *one* term

________________________ 5. polynomials with exactly the same variable combinations.

________________________ 6. a polynomial with exactly *three* terms

________________________ 7. any rational expression with no variable in the denominator

________________________ 8. an expression that contains no grouping symbols (except a fraction bar), and all like terms have been combined

________________________ 9. one of the numbers multiplied to get a product

________________________ 10. a fraction whose numerator and/or denominator are polynomials

________________________ 11. a whole number that has more than two factors

Unit Review

Use the list below to identify each **polynomial**. *Write the word on the line provided.*

binomial **monomial** **trinomial**

________________________ 1. $a + b + c$

________________________ 2. $8xy^3z^2$

________________________ 3. $4a^2 - b$

Write each **expression** *in* **simplest form**. *Show* **essential steps**.

4. $3a + [5a - (2a - b)]$

5. $(x^3 - 4x^2y + 5xy^2 + 4y^3) - (-2x^3 + x^2y + 6xy^2 - 5y^3)$

6. $8 - [2x - (3 + 5x) + 4]$

7. $(3x^2)(-6x)^2$

8. $(-a)^2(4a^2)(-2a)^3$

9. $(5x)^2(-2x^2y^2)(4x)$

10. $\frac{-16a^2b^5c^4}{4a^2bc^3}$

11. $\frac{a^5b^2c^4}{a^5bc^3}$

12. $\dfrac{(x-3)^4}{(x-3)^2}$

13. $(x+5)^2$

14. $(a+2)(a-6)$

15. $(3g-7)(2g+5)$

16. $(a+5)(a^2-4a+9)$

Factor *the following. Show* **essential steps***.*

17. -32

18. $6x - 18$

19. $4m^3 - 16m^2 + 12m$

20. $4(a - 2) - x(a - 2)$

21. $3x^2 - 8x + 5$

22. $15x^2 - 16x + 4$

23. $y^2 + 10y + 25$

24. $x^2 - 4x + 4$

25. $x^2 - 36$

Unit 4: Making Sense of Rational Expressions

This unit emphasizes performing mathematical operations on rational expressions and using these operations to solve equations and inequalities.

Unit Focus

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Algebra Body of Knowledge

Standard 3: Linear Equations and Inequalities

- MA.912.A.3.2
  Identify and apply the distributive, associative, and commutative properties of real numbers and the properties of equality.

- MA.912.A.3.4
  Solve and graph simple and compound inequalities in one variable and be able to justify each step in a solution.

Standard 4: Polynomials

- MA.912.A.4.1
  Simplify monomials and monomial expressions using the laws of integral exponents.

- MA.912.A.4.2
  Add, subtract, and multiply polynomials.

- MA.912.A.4.3
  Factor polynomial expressions.

- MA.912.A.4.4
  Divide polynomials by monomials and polynomials with various techniques, including synthetic division.

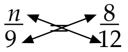
Vocabulary

Use the vocabulary words and definitions below as a reference for this unit.

canceling	dividing a numerator and a denominator by a common factor to write a fraction in lowest terms or before multiplying fractions *Example*: $\frac{15}{24} = \frac{{}^{1}\cancel{3} \bullet 5}{2 \bullet 2 \bullet 2 \bullet \cancel{3}_{1}} = \frac{5}{8}$
common denominator	a common multiple of two or more denominators *Example*: A common denominator for $\frac{1}{4}$ and $\frac{5}{6}$ is 12.
common factor	a number that is a factor of two or more numbers *Example*: 2 is a common factor of 6 and 12.
common multiple	a number that is a multiple of two or more numbers *Example*: 18 is a common multiple of 3, 6, and 9.

cross multiplication .... a method for solving and checking proportions; a method for finding a missing numerator or denominator in equivalent fractions or ratios by making the cross products equal
Example: Solve this proportion by doing the following.

$$\frac{n}{9} = \frac{8}{12}$$



$$12 \text{ x } n = 9 \text{ x } 8$$
$$12n = 72$$
$$n = \frac{72}{12}$$
$$n = 6$$

Solution:

$$\frac{\textcircled{6}}{9} = \frac{8}{12}$$

decimal number .... any number written with a decimal point in the number
Examples: A decimal number falls between two whole numbers, such as 1.5, which falls between 1 and 2. Decimal numbers smaller than 1 are sometimes called *decimal fractions*, such as five-tenths, or $\frac{5}{10}$, which is written 0.5.

denominator .... the bottom number of a fraction, indicating the number of equal parts a whole was divided into
Example: In the fraction $\frac{2}{3}$ the denominator is 3, meaning the whole was divided into 3 equal parts.

difference .... a number that is the result of subtraction
Example: In 16 – 9 = 7, the difference is 7.

distributive property	the product of a number and the sum or difference of two numbers is equal to the sum or difference of the two products *Examples*: $x(a + b) = ax + bx$ $5(10 + 8) = 5 \bullet 10 + 5 \bullet 8$
equation	a mathematical sentence stating that the two expressions have the same value *Example*: $2x = 10$
equivalent (forms of a number)	the same number expressed in different forms *Example*: $\frac{3}{4}$, 0.75, and 75%
expression	a mathematical phrase or part of a number sentence that combines numbers, operation signs, and sometimes variables *Examples*: $4r^2$; $3x + 2y$; $\sqrt{25}$ An expression does *not* contain equal (=) or inequality ($<$, $>$, $\leq$, $\geq$, or $\neq$) signs.
factor	a number or expression that divides evenly into another number; one of the numbers multiplied to get a product *Example*: 1, 2, 4, 5, 10, and 20 are factors of 20 and $(x + 1)$ is one of the factors of $(x^2 - 1)$.
factoring	expressing a polynomial expression as the product of monomials and polynomials *Example*: $x^2 - 5x + 4 = 0$ $(x - 4)(x - 1) = 0$

fraction	any part of a whole *Example*: One-half written in fractional form is $\frac{1}{2}$.
inequality	a sentence that states one expression is greater than (>), greater than or equal to (≥), less than (<), less than or equal to (≤), or not equal to (≠) another expression *Examples*: $a \neq 5$ *or* $x < 7$ *or* $2y + 3 \geq 11$
integers	the numbers in the set {… , -4, -3, -2, -1, 0, 1, 2, 3, 4, …}
inverse operation	an action that undoes a previously applied action *Example*: Subtraction is the inverse operation of addition.
irrational number	a real number that cannot be expressed as a ratio of two integers *Example*: $\sqrt{2}$
least common denominator (LCD)	the smallest common multiple of the denominators of two or more fractions *Example*: For $\frac{3}{4}$ and $\frac{1}{6}$, 12 is the least common denominator.
least common multiple (LCM)	the smallest of the common multiples of two or more numbers *Example*: For 4 and 6, 12 is the least common multiple.

like terms	terms that have the same variables and the same corresponding exponents *Example*: In $5x^2 + 3x^2 + 6$, the like terms are $5x^2$ and $3x^2$.
minimum	the smallest amount or number allowed or possible
multiplicative identity	the number one (1); the product of a number and the multiplicative identity is the number itself *Example*: $5 \times 1 = 5$
multiplicative property of -1	the product of any number and -1 is the opposite or additive inverse of the number *Example*: $-1(a) = -a$ and $a(-1) = -a$
negative numbers	numbers less than zero
numerator	the top number of a fraction, indicating the number of equal parts being considered *Example*: In the fraction $\frac{2}{3}$, the numerator is 2.
order of operations	the order of performing computations in parentheses first, then exponents or powers, followed by multiplication and/or division (as read from left to right), then addition and/or subtraction (as read from left to right); also called *algebraic order of operations* *Example*: $5 + (12 - 2) \div 2 - 3 \times 2 =$ $5 + 10 \div 2 - 3 \times 2 =$ $5 + 5 - 6 =$ $10 - 6 =$ 4

polynomial	a monomial or sum of monomials; any rational expression with no variable in the denominator *Examples*: $x^3 + 4x^2 - x + 8$ $5mp^2$ $-7x^2y^2 + 2x^2 + 3$
positive numbers	numbers greater than zero
product	the result of multiplying numbers together *Example*: In 6 x 8 = 48, the product is 48.
quotient	the result of dividing two numbers *Example*: In 42 ÷ 7 = 6, the quotient is 6.
ratio	the comparison of two quantities *Example* The ratio of a and b is $a{:}b$ or $\frac{a}{b}$, where $b \neq 0$.
rational expression	a fraction whose numerator and/or denominator are polynomials *Examples*: $\frac{x}{8}$ $\frac{5}{x+2}$ $\frac{4x^2+1}{x^2+1}$
rational number	a number that can be expressed as a ratio $\frac{a}{b}$, where a and b are integers and $b \neq 0$
real numbers	the set of all rational and irrational numbers
reciprocals	two numbers whose product is 1; also called *multiplicative inverses* *Examples:* 4 and $\frac{1}{4}$ are reciprocals because $\frac{4}{1}$ x $\frac{1}{4}$ = 1; $\frac{3}{4}$ and $\frac{4}{3}$ are reciprocals because $\frac{3}{4}$ x $\frac{4}{3}$ = 1; zero (0) has no multiplicative inverse

simplest form (of a fraction)	a fraction whose numerator and denominator have no common factor greater than 1 *Example*: The simplest form of $\frac{3}{6}$ is $\frac{1}{2}$.
simplify an expression	to perform as many of the indicated operations as possible
solution	any value for a variable that makes an equation or inequality a true statement *Example*: In $y = 8 + 9$ $y = 17$ 17 is the solution.
substitute	to replace a variable with a numeral *Example*: $8(a) + 3$ $8(5) + 3$
sum	the result of adding numbers together *Example*: In $6 + 8 = 14$, the sum is 14.
term	a number, variable, product, or quotient in an expression *Example*: In the expression $4x^2 + 3x + x$, the terms are $4x^2$, $3x$, and x.
variable	any symbol, usually a letter, which could represent a number

Unit 4: Making Sense of Rational Numbers

Introduction

Algebra students must be able to add, subtract, multiply, divide, and simplify rational expressions efficiently. These skills become more important as you progress in using mathematics. As an algebra student, you will have the opportunity to work with methods you will need for future mathematical success.

Lesson One Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Algebra Body of Knowledge

Standard 4: Polynomials

- MA.912.A.4.1
  Simplify monomials and monomial expressions using the laws of integral exponents.
- MA.912.A.4.2
  Add, subtract, and multiply polynomials.
- MA.912.A.4.3
  Factor polynomial expressions.
- MA.912.A.4.4
  Divide polynomials by monomials and polynomials with various techniques, including synthetic division.

Simplifying Rational Expressions

An **expression** is a mathematical phrase or part of a number sentence that combines numbers, operation signs, and sometimes **variables**. A **fraction**, or any part of a whole, is an *expression* that represents a **quotient**—the result of dividing two numbers. The same *fraction* may be expressed in many different ways.

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{5}{10}$$

If the **numerator** (top number) and the **denominator** (bottom number) are both **polynomials**, then we call the fraction a **rational expression**. A *rational expression* is a fraction whose *numerator* and/or *denominator* are *polynomials*. The fractions below are all rational expressions.

$$\frac{x}{x+y} \qquad \frac{a^2-2a+1}{a} \qquad \frac{1}{y^2+4} \qquad \frac{a}{b-3}$$

When the *variables* or any symbols which could represent numbers (usually letters) are replaced, the result is a numerator and a denominator that are **real numbers**. In this case, we say the entire *expression* is a *real number*. Real numbers are all **rational numbers** and **irrational numbers**. *Rational numbers* are numbers that can be expressed as a **ratio** $\frac{a}{b}$, where a and b are **integers** and $b \neq 0$. *Irrational numbers* are real numbers that *cannot* be expressed as a *ratio* of two *integers*. Of course, there is an exception: when a denominator is equal to 0, we say the fraction is *undefined*.

Note: In this unit, we will agree that *no* denominator equals 0.

Fractions have some interesting properties. Let's examine them.

- If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$. $\quad \frac{4}{8} = \frac{6}{12}$ therefore $4 \bullet 12 = 8 \bullet 6$

  $$\frac{a}{b} = \frac{c}{d}$$
  $$a \bullet d = b \bullet c$$
  $$ad = bc$$

  In other words, if two fractions are equal, then the **products** are equal when you **cross multiply**.

  $$\frac{4}{8} = \frac{6}{12}$$
  $$4 \bullet 12 = 8 \bullet 6$$
  $$48 = 48$$

- $\frac{a}{b} = \frac{ac}{bc}$ $\quad \frac{4}{7} = \frac{4 \bullet 3}{7 \bullet 3}$ therefore $\frac{4}{7} = \frac{12}{21}$

  Simply stated, if you *multiply* both the numerator and the denominator by the *same* number, the new fraction will be **equivalent** to the original fraction.

- $\frac{ac}{bc} = \frac{a}{b}$ $\quad \frac{9}{21} = \frac{9 \div 3}{21 \div 3}$ therefore $\frac{9}{21} = \frac{3}{7}$

  In other words, if you *divide* both the numerator and the denominator by the *same* number, the new fraction will be *equivalent* to the original fraction. The same rules are true for **simplifying** rational expressions by performing as many indicated operations as possible. Many times, however, it is necessary to **factor** and find numbers or expressions that divide the numerator or the denominator, or both, so that the **common factors** become easier to see. Look at the following example:

  $$\frac{3x + 3y}{3} = \frac{{}^{1}\cancel{3}(x + y)}{\cancel{3}_{1}} = x + y$$

  Notice that, by **factoring** a 3 out of the numerator, we can divide (or **cancel**) the 3s, leaving $x + y$ as the final result.

Before we move on, do the practice on the following pages.

Practice

Simplify *each* **expression**. *Refer to* **properties** *and* **examples** *on the previous pages as needed. Show* **essential steps**.

1. $\frac{4x-4}{x-1}$

2. $\frac{4m-2}{2m-1}$

3. $\frac{6x-3y}{3}$

Example:

$$\frac{4x-6}{6} = \frac{{}^{1}\cancel{2}(2x-3)}{{}_{3}\cancel{6}} = \frac{1(2x-3)}{3} = \frac{2x-3}{3}$$

4. $\dfrac{5a-10}{15}$

5. $\dfrac{2y-8}{4}$

6. $\dfrac{3m+6n}{3}$

7. $\dfrac{14r^3s^4 + 28rs^2 - 7rs}{7r^2s^2}$

Practice

Simplify *each* **expression**. *Refer to* **properties** *and* **examples** *on the previous pages as needed. Show* **essential steps**.

Example:

$$\frac{2x^2 - 8}{x + 2} = \frac{2(x^2 - 4)}{x + 2} = \frac{2(x - 2)\cancel{(x + 2)}^{1}}{\cancel{x + 2}_{1}} = 2(x - 2)$$

Note: In the above example, notice the following:

→ After we factored 2 from the numerator,

→ we were left with $x^2 - 4$,

→ which can be factored into $(x + 2)(x - 2)$.

→ Then the $(x + 2)$ is cancelled,

→ leaving $2(x - 2)$ as the final answer.

1. $\frac{3y^2 - 27}{y - 3}$

2. $\frac{a - b}{a^2 - b^2}$

3. $\dfrac{b-a}{a^2-b^2}$

4. $\dfrac{9x+3}{6x+2}$

5. $\dfrac{9x^2+3}{6x+3}$

Additional Factoring

Look carefully at numbers 2-5 in the previous practice. What do you notice about them?

Alert! You cannot cancel individual **terms** (numbers, variables, products, or quotients in an expression)—you can only cancel *factors* (numbers or expressions that exactly divide another number)!

$$\frac{2x+4}{4} \neq \frac{2x}{4} \qquad \frac{3x+6}{3} \neq \frac{x+6}{3} \qquad \frac{9x^2+3}{6x+3} \neq \frac{9x^2x}{6x}$$

Look at how simplifying these expressions was taken a step further. Notice that additional factoring was necessary.

Example

$$\frac{x^2+5x+6}{x+3} = \frac{{}^1\cancel{(x+3)}(x+2)}{{}_1\cancel{x+3}} = (x+2) = x+2$$

Look at the denominator above. It is one of the factors of the numerator. Often, you can use the problem for hints as you begin to factor.

Practice

Factor *each of these and then* **simplify**. *Look for* **hints** *within the problem. Refer to the previous page as necessary. Show* **essential steps**.

1. $\dfrac{a^2 - 3a + 2}{a - 2}$

2. $\dfrac{b^2 - 2b - 3}{b - 3}$

Sometimes, it is necessary to **factor both** *the numerator and denominator. Examine the example below, then* **simplify** *each of the following* **expressions**.

Example:

$$\frac{x^2 - 4}{x^2 + x - 6} = \frac{(x+2)\cancel{(x-2)}^{1}}{(x+3)\cancel{(x-2)}_{1}} = \frac{(x+2)}{(x+3)} = \frac{x+2}{x+3}$$

Note: The x's do **not** cancel.

3. $\dfrac{2r^2 + r - 6}{r^2 + r - 2}$

4. $\dfrac{x^2 + x - 2}{x^2 - 1}$

Practice

Simplify *each* **expression**. *Show* **essential steps**.

1. $\dfrac{5b-10}{b-2}$

2. $\dfrac{6a-9}{10a-15}$

3. $\dfrac{9x+3}{9}$

4. $\dfrac{6b+9}{12}$

5. $\dfrac{3a^2b + 6ab - 9b^2}{3b}$

6. $\dfrac{x^2 - 16}{x + 4}$

7. $\dfrac{2a - b}{b^2 - 4a^2}$

8. $\dfrac{6x^2 + 2}{9x^2 + 3}$

Practice

Factor *each of these* **expressions** *and then* **simplify**. *Show* **essential steps**.

1. $\dfrac{y^2 + 5y - 14}{y - 2}$

2. $\dfrac{a^2 - 5a + 4}{a - 4}$

3. $\dfrac{6m^2 - m - 1}{2m^2 + 9m - 5}$

4. $\dfrac{4x^2 - 9}{2x^2 + x - 6}$

Practice

Use the list below to write the correct term for each definition on the line provided.

denominator	**numerator**	**rational expression**
expression	**polynomial**	**real numbers**
fraction	**quotient**	**variable**

________________________ 1. a mathematical phrase or part of a number sentence that combines numbers, operation signs, and sometimes variables

________________________ 2. the top number of a fraction, indicating the number of equal parts being considered

________________________ 3. the bottom number of a fraction, indicating the number of equal parts a whole was divided into

________________________ 4. the set of all rational and irrational numbers

________________________ 5. any part of a whole

________________________ 6. a fraction whose numerator and/or denominator are polynomials

________________________ 7. any symbol, usually a letter, which could represent a number

________________________ 8. a monomial or sum of monomials; any rational expression with no variable in the denominator

________________________ 9. the result of dividing two numbers

Practice

Use the list below to complete the following statements.

canceling
cross multiplication
equivalent
factor
integers
product
simplify an expression
terms

1. If you *multiply* both the numerator and the denominator by the *same* number, the new fraction will be ______________________ because it is the same number expressed in a different form.

2. The numbers in the set {… , -4, -3, -2, -1, 0, 1, 2, 3, 4, …} are ______________________ .

3. If you divide a numerator and a denominator by a common factor to write a fraction in lowest terms, or before multiplying fractions, you are ______________________ .

4. To ______________________ , you need to perform as many of the indicated operations as possible.

5. Numbers, variables, products, or quotients in an expression are called ______________________ .

6. A ________________________ is a number or expression that divides evenly into another number.

7. When you multiply numbers together, the result is called the ________________________ .

8. To find a missing numerator or denominator in equivalent fractions or ratios, you can use a method called ________________________ and make the cross products equal.

Lesson Two Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.

- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.

- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Algebra Body of Knowledge

Standard 4: Polynomials

- MA.912.A.4.1
  Simplify monomials and monomial expressions using the laws of integral exponents.

- MA.912.A.4.2
  Add, subtract, and multiply polynomials.

- MA.912.A.4.3
  Factor polynomial expressions.

- MA.912.A.4.4
  Divide polynomials by monomials and polynomials with various techniques, including synthetic division.

Addition and Subtraction of Rational Expressions

In order to add and subtract rational expressions in fraction form, it is necessary for the fractions to have a **common denominator** (the same bottom number). We find those *common denominators* in the same way we did with simple fractions. The process requires careful attention.

- When we add $\frac{3}{7} + \frac{5}{8}$, we find a common denominator by multiplying 7 and 8.
- Then we change each fraction to an equivalent fraction whose denominator is 56.

$$\frac{3 \bullet 8}{7 \bullet 8} = \frac{24}{56} \qquad \text{and} \qquad \frac{5 \bullet 7}{8 \bullet 7} = \frac{35}{56}$$

- Next we add $\frac{24}{56} + \frac{35}{56} = \frac{59}{56}$.

Finding the Least Common Multiple (LCM)

By multiplying the denominators of the terms we intend to add or subtract, we can always find a common denominator. However, it is often to our advantage to find the **least common denominator (LCD)**, which is also the **least common multiple (LCM)**. The *LCD* or *LCM* is the smallest of the **common multiples** of two or more numbers. This makes simplifying the result easier. Look at the example on the following page.

Let's look at finding the LCM of 36, 27, and 15.

1. Factor each of the denominators and examine the results.

   $36 = 2 \bullet 2 \bullet 3 \bullet 3$ ← The new denominator must contain at least **two 2s** and **two 3s**.

   $27 = 3 \bullet 3 \bullet 3$ ← The new denominator must contain at least **three 3s**.

   $45 = 3 \bullet 3 \bullet 5$ ← The new denominator must contain at least **two 3s** and **one 5**.

2. Find the **minimum** combination of factors that is described by the combination of all the statements above—two 2s, three 3s, and one 5.

$$\text{LCM} = \underbrace{2 \bullet 2}_{\text{two 2s}} \bullet \underbrace{3 \bullet 3 \bullet 3}_{\text{three 3s}} \bullet \underbrace{5}_{\text{one 5}} = 540$$

3. Convert the *terms* to equivalent fractions using the new common denominator and then proceed to add or subtract.

$$\frac{5}{36} = \frac{75}{540}; \frac{8}{27} = \frac{160}{540}; \frac{4}{15} = \frac{144}{540} \longrightarrow \frac{75}{540} + \frac{160}{540} - \frac{144}{540} = \frac{91}{540}$$

Now, let's look at an algebraic example.

$$\frac{y}{y^2 - 9} - \frac{1}{y^2 - 4y - 21} =$$

1. Factor each denominator and examine the results.

$$y^2 - 9 = (y + 3)(y - 3)$$

← The new denominator must contain (y + 3) and (y – 3).

$$y^2 - 4y - 21 = (y - 7)(y + 3)$$

← The new denominator must contain (y – 7) and (y + 3).

2. Find the *minimum* combination of factors.

$$\text{LCM} = (y + 3)(y - 3)(y - 7)$$

3. Convert each fraction to an equivalent fraction using the new common denominator and proceed to subtract.

$$\frac{y(y-7)}{(y+3)(y-3)(y-7)} - \frac{1(y-3)}{(y+3)(y-3)(y-7)} =$$

$$\frac{y^2 - 7y - y + 3}{(y+3)(y-3)(y-7)} =$$

$$\frac{y^2 - 8y + 3}{(y+3)(y-3)(y-7)}$$

notice how the minus sign between the fractions *distributes* to make -y + 3 in the numerator (**distributive property**)

Hint: Always check to see if the numerator can be factored and then reduce, if possible. Do this to be sure the answer is in the lowest terms.

Practice

Write each **sum** *or* **difference** *as a single fraction in* **lowest terms**. *Show* **essential steps**.

1. $\frac{a}{7} + \frac{2a}{7} - \frac{5}{7}$

4. $\frac{x+1}{5} - \frac{x+1}{5}$

2. $\frac{x-2}{2y} + \frac{x}{2y}$

5. $\frac{5}{6} + \frac{y}{4}$

3. $\frac{x+1}{5} + \frac{x-1}{5}$

6. $\frac{2}{x+2} - \frac{3}{x+3}$

Practice

Write each **sum** *or* **difference** *as a single fraction in* **lowest terms**. *Show* **essential steps**.

Example: $\frac{5}{b^2 - 9} - \frac{1}{b - 3} = \frac{5}{(b + 3)(b - 3)} - \frac{1}{b - 3} =$

$$\frac{5}{(b + 3)(b - 3)} - \frac{1(b + 3)}{(b + 3)(b - 3)} =$$

$$\frac{5 - 1(b + 3)}{(b + 3)(b - 3)} =$$

$$\frac{5 - b - 3}{(b + 3)(b - 3)} =$$

$$\frac{2 - b}{(b + 3)(b - 3)} =$$

1. $\frac{1}{2z + 1} + \frac{3}{z - 2}$

2. $\frac{r}{r^2 - 16} + \frac{r + 1}{r^2 - 5r + 4}$

3. $\dfrac{8}{a^2 - 4} - \dfrac{2}{a^2 - 5a + 6}$

4. $m + \dfrac{1}{m - 1} - \dfrac{1}{(m - 1)^2}$

Practice

Write each **sum** *or* **difference** *as a single fraction in* **lowest terms**. *Show* **essential steps**.

1. $\frac{x}{3} - \frac{3y}{3} + \frac{4z}{3}$

2. $\frac{x-2}{2y} - \frac{x}{2y}$

3. $\frac{x+1}{5} - \frac{x-1}{5}$

4. $\frac{2}{2a-4b} - \frac{b-2}{2a-4b} + \frac{7b}{2a-4b}$

5. $\frac{x+3}{4} + \frac{5-x}{10}$

6. $\frac{5}{2m-6} - \frac{3}{m-3}$

Practice

Write each **sum** *or* **difference** *as a single fraction in* **lowest terms**. *Show* **essential steps**.

1. $\dfrac{2}{x^2 - x - 2} - \dfrac{2}{x^2 + 2x + 1}$

2. $\dfrac{1}{b^2 - 1} - \dfrac{1}{b^2 + 2b + 1}$

3. $\dfrac{3x}{x^2 + 3x - 10} - \dfrac{2x}{x^2 + x - 6}$

4. $\dfrac{1}{x^2 - 7x + 12} + \dfrac{2}{x^2 - 5x + 6} - \dfrac{3}{x^2 - 6x + 8}$

Lesson Three Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.

- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.

- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Algebra Body of Knowledge

Standard 4: Polynomials

- MA.912.A.4.1
  Simplify monomials and monomial expressions using the laws of integral exponents.

- MA.912.A.4.2
  Add, subtract, and multiply polynomials.

- MA.912.A.4.3
  Factor polynomial expressions.

Multiplication and Division of Rational Expressions

To multiply fractions, you learned to multiply the numerators together, then multiply the denominators together, and then reduce, if possible.

$$\frac{3}{8} \text{ x } \frac{5}{7} = \frac{15}{56}$$

We use this same process with rational expressions.

$$\frac{4}{5x} \text{ x } \frac{11x}{13} = \frac{44x}{65x} = \frac{44}{65}$$

Sometimes it is simpler to reduce or cancel *common factors* before multiplying.

$$\frac{4}{5x} \text{ x } \frac{11x}{13} = \frac{4}{5\cancel{x}} \text{ x } \frac{11\cancel{x}}{13} = \frac{44}{65}$$

When we need to divide fractions, we invert (flip over) the fraction to the right of the division symbol and then multiply.

invert

$$\frac{2x^2}{3y} \div \frac{4x}{5y^3} = \frac{2x^2}{3y} \bullet \frac{5y^3}{4x} = \frac{10x^2y^3}{12xy} = \frac{5xy^2}{6}$$

Pay careful attention to *negative* signs in the factors.

Decide before you multiply whether the answer will be positive *or* negative.

- If the number of negative factors is *even*, the result will be *positive*.
- If the number of negative factors is *odd*, the answer will be *negative*.

Remember: In this unit, we agreed that *no* denominator equals 0.

Practice

Write each **product** *as a single fraction in* **simplest terms**. *Show* **essential steps**.

1. $\frac{6x^3}{3} \bullet \frac{4b}{2x}$

2. $\frac{14a^3b}{3b} \bullet \frac{-6}{7ab}$

3. $\frac{-12ab^2}{5bc} \bullet \frac{10b^2c}{6ab}$

Practice

Write each **product** *as a single fraction in* **simplest terms**. *Show* **essential steps**.

Example:

$$\frac{4a^2-1}{a^2-4} \bullet \frac{a+2}{4a+2} =$$

$$\frac{\cancel{(2a+1)}(2a-1)}{\cancel{(a+2)}(a-2)} \bullet \frac{\cancel{a+2}}{2\cancel{(2a+1)}} =$$

$$\frac{(2a-1)}{2(a-2)} =$$

$$\frac{2a-1}{2(a-2)}$$

1. $\dfrac{5x+25}{4x} \bullet \dfrac{2x}{3x+15}$

2. $\dfrac{y^2-y-2}{y^2+4y+3} \bullet \dfrac{y^2-4y-5}{y^2-3y-10}$

   Hint: If you have trouble factoring, review the examples and explanation of processes on pages 304-306.

3. $\dfrac{2a^2 - a - 6}{3a^2 - 4a + 1} \bullet \dfrac{3a^2 + 7a + 2}{2a^2 + 7a + 6}$

4. $\dfrac{3x^2 - 3x}{5} \bullet \dfrac{x^2 - 9x - 10}{6x - 60} \bullet \dfrac{4}{1 - x^2}$

Practice

Write each **quotient** *as a single fraction in* **simplest terms***. Show* **essential steps**.

Remember: Invert and then multiply!

1. $\frac{9ab}{x} \div \frac{3a}{2x^2}$

2. $\frac{x^2 - x - 6}{x^2 - 2x - 15} \div \frac{x^2 - 4}{x^2 - 6x + 5}$

3. $\frac{10a^2 - 13a - 3}{2a^2 - a - 3} \div \frac{5a^2 - 9a + -2}{3a^2 + 2a - 1}$

4. $\frac{9r^2 + 3r - 2}{12r^2 + 5r - 2} \div \frac{9r^2 - 6r + 1}{8r^2 + 10r - 3}$

Practice

Write each **product** *as a single fraction in* **simplest terms**. *Show* **essential steps**.

1. $\frac{4a^3}{3} \bullet \frac{6b}{2a}$

2. $\frac{-18ab^2}{5bc} \bullet \frac{15b^3c}{6ab}$

3. $\frac{24a^3b}{3b} \bullet \frac{-9}{6ab}$

4. $\dfrac{9b^2-25}{2b-2} \bullet \dfrac{b^2-1}{6b-10}$

5. $\dfrac{x^2-x-20}{x^2+7x+12} \bullet \dfrac{x+3}{x-5}$

6. $\dfrac{7x+14}{14x-28} \bullet \dfrac{4-2x}{x+2} \bullet \dfrac{x+3}{x+1}$

Hint: $a - b = \text{-}(b - a)$

Practice

Write each **quotient** *as a single fraction in* **simplest terms**. *Show* **essential steps**.

1. $\dfrac{28x^2y^3}{10a^2} \div \dfrac{21x^3y}{5a}$

2. $\dfrac{4x-8}{3} \div \dfrac{-(6x-12)}{9}$

3. $\dfrac{6a^3b}{4x} \div \dfrac{3a}{2x^3}$

4. $\dfrac{r^2+2r-15}{r^2+3r-10} \div \dfrac{r^2-9}{r^2-9r+14}$

5. $\dfrac{y^2+y-2}{y^2+2y-3} \div \dfrac{y^2+7y+10}{y^2-2y-15}$

Lesson Four Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Algebra Body of Knowledge

Standard 3: Linear Equations and Inequalities

- MA.912.A.3.2
  Identify and apply the distributive, associative, and commutative properties of real numbers and the properties of equality.

Standard 4: Polynomials

- MA.912.A.4.1
  Simplify monomials and monomial expressions using the laws of integral exponents.

- MA.912.A.4.2
  Add, subtract, and multiply polynomials.
- MA.912.A.4.3
  Factor polynomial expressions.
- MA.912.A.4.4
  Divide polynomials by monomials and polynomials with various techniques, including synthetic division.

Solving Equations

Recall that an **equation** is a mathematical sentence stating the two expressions have the same value. The equality symbol or equal sign (=) shows that two quantities are equal. An *equation* equates one expression to another.

$3x - 7 = 8$ is an example of an equation.

You may be able to solve this problem mentally, without using paper and pencil.

$3x - 7 = 8$

The problem reads—3 times what number minus 7 equals 8?

Think: $3 \bullet 4 = 12$
$12 - 7 = 5$ ← too small

Think: $3 \bullet 5 = 15$
$15 - 7 = 8$ ← That's it!

$$\begin{aligned} 3x - 7 &= 8 \\ 3(5) - 7 &= 8 \end{aligned}$$

Practice

Solve each of the following **mentally**, *writing only the answer.*

1. $4y + 6 = 22$

   $y =$

2. $2a - 4 = 10$

   $a =$

3. $5x - 15 = -20$

   $x =$

4. $-7b + 6 = -22$

   $b =$

✓ **Check yourself:** Add all your answers for problems 1-4. Did you get a *sum* of 14? If not, correct your work before continuing.

Step-by-Step Process for Solving Equations

A problem like $\frac{x+12}{5} = -2(x-10)$ is a bit more challenging. You could use a guess and check process, but that would take more time, especially when answers involve **decimals** or fractions.

So, as problems become more difficult, you can see that it is important to have a process in mind and to write down the steps as you go.

Unfortunately, there is *no* exact process for solving equations. Every rule has an exception. That is why creative thinking, reasoning, and practice are necessary and keeping a written record of the steps you have used is extremely helpful.

Example 1

Let's look at a step-by-step process for solving the problem above.

$\frac{x+12}{5} = -2(x-10)$ ← Step 1: Copy the problem **carefully**!

$\frac{x+12}{5} = -2x + 20$ ← Step 2: Simplify each side of the equation as needed by *distributing* the 2.

$\left(\frac{x+12}{\cancel{5}}\right) \bullet \cancel{5} = (-2x+20) \bullet 5$ ← Step 3: Multiply both sides of the equation by 5 to "undo" the division by 5, which eliminates the fraction.

$x + 12 = -10x + 100$ ← Step 4: Simplify by distributing the 5.

$\boxed{+10x} + 1x + 12 = -10x \boxed{+10x} + 100$ ← Step 5: Add 10x to both sides.

$11x + 12 = 100$

$11x + 12 \boxed{-12} = 100 \boxed{-12}$ ← Step 6: Subtract 12 from both sides.

$11x = 88$

$11x \boxed{\div 11} = 88 \boxed{\div 11}$ ← Step 7: Divide both sides by 11.

$x = 8$

$\frac{x+12}{5} = -2(x-10)$ ← Step 8: Check by replacing the variable in the original problem.

$\frac{8+12}{5} = -2(8) + 20$

$4 = -16 + 20$

$4 = 4$ *It checks!*

Example 2

What if the original problem had been $5x + 12 = -2(x - 10)$? The process would have been different. Watch for differences.

$$5x + 12 = -2(x - 10)$$

← Step 1: Copy the problem **carefully**!

$$5x + 12 = -2x + 20$$

← Step 2: Simplify each side of the equation as needed by *distributing* the 2.

$$5x + 12 \boxed{- 12} = -2x + 20 \boxed{- 12}$$
$$5x = -2x + 8$$

← Step 3: Subtract 12 from both sides of the equation.

$$5x \boxed{+ 2x} = -2x \boxed{+ 2x} + 8$$
$$7x = 8$$

← Step 4: Add 2*x* to both sides of the equation.

$$7x \boxed{\div 7} = 8 \boxed{\div 7}$$
$$x = \frac{8}{7}$$

← Step 5: Divide both sides by 7.

$$5x + 12 = -2(x - 10)$$
$$5(\tfrac{8}{7}) + 12 = -2(\tfrac{8}{7}) + 20$$
$$\frac{40}{7} + 12 = \frac{-16}{7} + 20$$
$$5\tfrac{5}{7} + 12 = -2\tfrac{2}{7} + 20$$
$$17\tfrac{5}{7} = 17\tfrac{5}{7}$$

← Step 6: Check by replacing the variable in the original problem.

← *It checks!*

Did you notice that the steps were *not* always the same? The rules for solving equations change to fit the individual needs of each problem. You can see why it is a good idea to check your answers each time. You may need to do some steps in a different order than you originally thought.

Generally speaking the processes for solving equations are as follows.

- Simplify both sides of the equation as needed.
- “Undo” additions and subtractions.
- “Undo” multiplications and divisions.

You might notice that this seems to be the *opposite* of the **order of operations**. Typically, we “undo” in the *reverse* order from the original process.

Guidelines for Solving Equations

1. Use the **distributive property** to clear parentheses.
2. Combine **like terms**. We want to isolate the variable.
3. Undo addition or subtraction using **inverse operations**.
4. Undo multiplication or division using *inverse operations*.
5. Check by **substituting** the **solution** in the original equation.

SAM = **S**implify (steps 1 and 2) then
Add (or subtract)
Multiply (or divide)

Here are some additional examples.

Example 3

Solve:

$$\begin{aligned} 6y + 4(y + 2) &= 88 \\ 6y + 4y + 8 &= 88 && \longleftarrow \text{ use } \textit{distributive property} \\ 10y + 8 - 8 &= 88 - 8 && \longleftarrow \text{ combine } \textit{like terms} \text{ and undo addition} \\ & && \quad\;\; \text{by subtracting 8 from each side} \\ \tfrac{10y}{10} &= \tfrac{80}{10} && \longleftarrow \text{ undo multiplication by dividing} \\ y &= 8 && \quad\;\; \text{by 10} \end{aligned}$$

Check *solution* in the original equation:

$$\begin{aligned} 6y + 4(y + 2) &= 88 \\ 6(8) + 4(8 + 2) &= 88 \\ 48 + 4(10) &= 88 \\ 48 + 40 &= 88 \\ 88 &= 88 && \longleftarrow \textit{ It checks!} \end{aligned}$$

Example 4

Solve:

$$\begin{aligned} -\tfrac{1}{2}\,(x + 8) &= 10 \\ -\tfrac{1}{2}\,x - 4 &= 10 && \longleftarrow \text{ use distributive property} \\ -\tfrac{1}{2}x - 4 + 4 &= 10 + 4 && \longleftarrow \text{ undo subtraction by adding 4 to} \\ & && \quad\;\; \text{both sides} \\ -\tfrac{1}{2}x &= 14 \\ (-2)-\tfrac{1}{2}x &= 14(-2) && \longleftarrow \text{ isolate the variable by multiplying} \\ x &= -28 && \quad\;\; \text{each side by the } \textbf{reciprocal} \text{ of } \mathbf{-}\tfrac{1}{2} \end{aligned}$$

Check solution in the original equation:

$$\begin{aligned} -\tfrac{1}{2}(x + 8) &= 10 \\ -\tfrac{1}{2}(-28 + 8) &= 10 \\ -\tfrac{1}{2}(-20) &= 10 \\ 10 &= 10 && \longleftarrow \textit{ It checks!} \end{aligned}$$

Example 5

Solve:

$$\begin{aligned}
26 &= \tfrac{2}{3}(9x-6) \\
26 &= \tfrac{2}{3}(9x) - \tfrac{2}{3}(6) && \leftarrow \text{use distributive property} \\
26 &= 6x-4 \\
26+4 &= 6x-4+4 && \leftarrow \text{undo subtraction by adding 4 to each side} \\
\tfrac{30}{6} &= \tfrac{6x}{6} && \leftarrow \text{undo multiplication by dividing each side by 6} \\
5 &= x
\end{aligned}$$

Check solution in the original equation:

$$\begin{aligned}
26 &= \tfrac{2}{3}(9x-6) \\
26 &= \tfrac{2}{3}(9 \bullet 5 - 6) \\
26 &= \tfrac{2}{3}(39) \\
26 &= 26 && \leftarrow \textit{It checks!}
\end{aligned}$$

Example 6

Solve:

$x - (2x + 3) = 4$
$x - 1(2x + 3) = 4$ ← use the **multiplicative property of -1**
$x - 2x - 3 = 4$ ← use the **multiplicative identity of 1**
and use the distributive property
$-1x - 3 = 4$ ← combine like terms
$-1x - 3 + 3 = 4 + 3$ ← undo subtraction
$\frac{-1x}{-1} = \frac{7}{-1}$ ← undo multiplication
$x = -7$

Examine the solution steps above. See the use of the *multiplicative property of -1* in front of the parentheses on line two.

line 1: $x - (2x + 3) = 4$
line 2: $x \mathbf{-1}(2x + 3) = 4$

Also notice the use of *multiplicative identity* on line three.

line 3: $\mathbf{1}x - 2x - 3 = 4$

The simple variable x was multiplied by 1 ($1 \bullet x$) to equal $1x$. The $1x$ helped to clarify the number of variables when combining like terms on line four.

Check solution in the original equation:

$x - (2x + 3) = 4$
$-7 - (2 \bullet -7 + 3) = 4$
$-7 - (-11) = 4$
$4 = 4$ ← *It checks!*

Practice

Solve *and* **check** *each* **equation**. *Use the examples on pages 325-330 for reference. Show* **essential steps**.

Hint: Find a step that looks similar to the problem you need help with and follow from that point.

Remember: To check your work, replace the variable in the original problem with the answer you found.

1. $3x - 7 = 17$

2. $4x + 20 = x - 4$

3. $\frac{x}{6} = 1.5$

4. $\frac{2x}{5} = 3.2$

5. $5(x - 4) = 20$

6. $5(4x - 7) = 0$

7. $8x - 2x = 42$

8. $5x - 3 = 2x + 18$

9. $-2x + 4 = -4x - 10$

Practice

Solve *and* **check** *each* **equation**. *Use the examples on pages 325-330 for reference. Show* **essential steps**.

1. $2(3x - 4) + 6 = 10$

2. $3(x - 7) - x = -9$

3. $\frac{2}{3}x = 1$

   Hint: $\frac{2}{3}x = \frac{2x}{3}$. Rewrite 1 as $\frac{1}{1}$ and cross multiply.

4. $\frac{-1}{2}x - \frac{3}{4} = 4$

5. $-3x = \frac{-33}{8}$

6. $\frac{-2}{x} = 8$

7. $-3x - \frac{3}{2} = \frac{11}{2}$

Practice

Solve *and* **check** *each* **equation**.

1. $-87 = 9 - 8x$

2. $4k + 3 = 3k + 1$

3. $5a + 9 = 64$

4. $\frac{b}{3} + 5 = -2$

5. $4x = \text{-}(9 - x)$

6. $\frac{5}{x} = \text{-}10$

7. $3x - 1 = \text{-}x + 19$

Practice

Solve *and* **check** *each* **equation**. **Reduce** *fractions to* **simplest form**.

1. $5x - 3 = 2x + 18$

2. $6x - (4x - 12) = 3x + 5$

3. $\frac{x}{6} = \frac{-24}{5}$

4. $4(x - 2) = -3(x + 5)$

5. $5(\frac{1}{3}x - 2) = 4$

6. $\frac{4}{x} + \frac{3}{2} = \frac{5}{8}$

7. $\frac{2}{9}x = \frac{1}{5}$

8. $\frac{-1}{2} + \frac{8x}{5} = \frac{-7}{8}$

Lesson Five Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.
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Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Algebra Body of Knowledge

Standard 3: Linear Equations and Inequalities

- MA.912.A.3.2
  Identify and apply the distributive, associative, and commutative properties of real numbers and the properties of equality.
- MA.912.A.3.4
  Solve and graph simple and compound inequalities in one variable and be able to justify each step in a solution.

Standard 4: Polynomials

- MA.912.A.4.1
  Simplify monomials and monomial expressions using the laws of integral exponents.
- MA.912.A.4.2
  Add, subtract, and multiply polynomials.
- MA.912.A.4.3
  Factor polynomial expressions.
- MA.912.A.4.4
  Divide polynomials by monomials and polynomials with various techniques, including synthetic division.

Solving Inequalities

Inequalities are mathematical sentences that state two expressions are not equal. Instead of using the equal symbol (=), we use the following with *inequalities*.

• greater than	$>$
• less than	$<$
• greater than or equal to	$\geq$
• less than or equal to	$\leq$
• not equal to	$\neq$

Remember: The "*is greater than*" (>) or "*is less than*" (<) symbols always *point to the lesser number*.

For example:

$$5 \;\; > \dashrightarrow 3$$
$$3 \dashleftarrow < \;\; 5$$

We have been solving *equations* in this unit. When we solve inequalities, the procedures are the same except for one important difference.

> **When we multiply or divide both sides of an inequality by the same *negative number*, we *reverse* the direction of the inequality symbol.**

Example

Solve by *dividing* by a **negative number** and *reversing* the inequality sign.

$$-3x < 6$$

$$\frac{-3x}{-3} > \frac{6}{-3} \quad \leftarrow \text{ divide each side by -3 and reverse the inequality symbol}$$

$$x > -2$$

To check this solution, pick any number *greater than* -2 and substitute your choice into the original inequality. For instance, -1, 0, or 3, or 3,000 could be substituted into the original problem.

Check with different solutions of numbers *greater than* -2:

substitute -1

$$-3x < 6$$
$$-3(-1) < 6$$
$$3 < 6 \quad \leftarrow \textit{ It checks!}$$

substitute 0

$$-3x < 6$$
$$-3(0) < 6$$
$$0 < 6 \quad \leftarrow \textit{ It checks!}$$

substitute 3

$$-3x < 6$$
$$-3(3) < 6$$
$$-9 < 6 \quad \leftarrow \textit{ It checks!}$$

substitute 3,000

$$-3x < 6$$
$$-3(3{,}000) < 6$$
$$-9{,}000 < 6 \quad \leftarrow \textit{ It checks!}$$

Notice that -1, 0, 3, and 3,000 are all *greater than* -2 and each one *checks* as a solution.

Study the following examples.

Example 1

Solve by *multiplying* by a *negative number* and *reversing* the inequality sign.

$-\frac{1}{3}y \geq 4$

$(-3)-\frac{1}{3}y \leq 4(-3)$ ← multiply each side by -3 and reverse the inequality symbol

$y \leq -12$

Example 2

Solve by first adding, then *dividing* by a *negative number*, and *reversing* the inequality sign.

$-3a - 4 > 2$

$-3a - 4 + 4 > 2 + 4$ ← add 4 to each side

$-3a > 6$

$\frac{-3a}{-3} < \frac{6}{-3}$ ← divide each side by -3 and reverse the inequality symbol

$a < -2$

Example 3

Solve by first subtracting, then *multiplying* by a *negative number*, and *reversing* the inequality sign.

$\frac{y}{-2} + 5 \leq 0$

$\frac{y}{-2} + 5 - 5 \leq 0 - 5$ ← subtract 5 from each side

$\frac{y}{-2} \leq -5$

$\frac{(-2)y}{-2} \geq (-5)(-2)$ ← multiply each side by -2 and reverse the inequality symbol

$y \geq 10$

Example 4

Solve by first subtracting, then *multiplying* by a **positive number**. Do **not** *reverse* the inequality sign.

$$\begin{aligned} \tfrac{n}{2} + 5 &\leq 2 \\ \tfrac{n}{2} + 5 - 5 &\leq 2 - 5 \leftarrow \text{subtract 5 from each side} \\ \tfrac{n}{2} &\leq \text{-}3 \\ \tfrac{(2)n}{2} &\leq \text{-}3(2) \leftarrow \text{multiply each side by 2, } \textit{but} \\ n &\leq \text{-}6 \qquad \text{do } \textit{not} \text{ reverse the inequality symbol because} \\ & \qquad\qquad \text{we multiplied by a } \textit{positive number} \end{aligned}$$

When multiplying or dividing both sides of an inequality by the same positive number, do not reverse the inequality symbol—leave it alone.

Example 5

Solve by first adding, then *dividing* by a positive number. Do **not** *reverse* the inequality sign.

$$\begin{aligned} 7x - 3 &> \text{-}24 \\ 7x - 3 + 3 &> \text{-}24 + 3 \leftarrow \text{add 3 to each side} \\ 7x &> \text{-}21 \qquad \leftarrow \text{divide each side by 7, but} \\ \tfrac{7x}{7} &> \tfrac{-21}{7} \qquad \text{do } \textit{not} \text{ reverse the inequality symbol because} \\ x &> \text{-}3 \qquad \text{we divided by a positive number} \end{aligned}$$

Practice

Solve each **inequality** *on the following page. Use the examples below and pages 340-343 for reference. Show* **essential steps***.*

Remember: Reverse the inequality symbol every time we multiply or divide both sides of the inequality by a negative number. See the example below.

Example: $7 - 3x > 13$

$7 \boxed{-7} - 3x > 13 \boxed{-7}$ ← subtract 7 from both sides

$-3x > 6$

$\frac{\cancel{-3}x}{\cancel{-3}} > \frac{\cancel{6}^{2}}{\cancel{-3}}$ ← divide both sides by -3 and reverse the inequality symbol

$x < -2$

Notice in the example above that we first subtracted 7 from both sides of the sentence. Then we solved for x, we divided both sides by -3, and the > symbol became a < symbol.

Check your answer by choosing a number that fits your answer. Replace the variable in the original sentence with the chosen number. Check to see if it makes a *true* statement.

In the example above, choose a number that makes $x < -2$ a *true* statement. For example, let's try -3.

Now put -3 in place of the variable in the original problem and see what happens.

$7 - 3x > 13$ ← original sentence

$7 - 3(-3) > 13$ ← replace x with -3

$7 - (-9) > 13$ ↙

$7 + 9 > 13$

$16 > 13$ ← This is a true statement, so the answer ($x < -2$) is correct.

See **directions** *and* **examples** *on previous page.*

1. $6x - 7 > 17$

2. $13x + 20 < x - 4$

3. $\frac{x}{5} \geq 1.5$

4. $\frac{2x}{5} > 4.8$

5. $5(x - 4) < 20$

6. $3(4x - 7) \geq 15$

7. $3(x - 7) - x > -9$

8. $\frac{-1}{2}x - \frac{3}{4} \leq 6$

9. $2x - 9 < -21$

10. $\frac{-12}{x} < 8$

11. $4(x - 7) - x > -7$

12. $\frac{2}{3}x > 10$

Practice

Solve each **inequality**. *Show* **essential steps**.

1. $5x - 3 \leq 12$

2. $2a + 7 \geq 5a - 5$

3. $\frac{2x}{5} > 2.4$

4. $5(x - 5) < 20$

5. $-2(x + 6) > 14$

6. $8x - 12x > 48$

7. $5x - 3 \geq 2x + 18$

8. $-2x + 4 < -4x - 12$

9. $2(3x - 4) + 6 \leq 16$

10. $-5x - \frac{3}{2} \geq \frac{11}{2}$

11. $-3x > \frac{-33}{7}$

12. $\frac{2}{3}x > 11$

Practice

Write **True** *if the statement is correct. Write* **False** *if the statement is* not *correct.*

__________ 1. An equation is a mathematical sentence that uses an equal sign to show that two quantities are equal.

__________ 2. A product is the result of dividing two numbers.

__________ 3. A quotient is the result of multiplying two numbers.

__________ 4. An expression is a collection of numbers, symbols, and/or operation signs that stand for a number.

__________ 5. To simplify an expression, perform as many indicated operations as possible.

__________ 6. A common multiple is a number that is a multiple of two or more numbers.

__________ 7. The smallest of the common multiples of two or more numbers is called the least common multiple (LCM).

__________ 8. A number that is the result of subtraction is called the sum.

__________ 9. A number that is the result of adding numbers together is called the difference.

__________ 10. When solving an inequality, every time you add or subtract both sides of the inequality by a negative number, you will have to reverse the inequality symbol.

Unit Review

Simplify each **expression**.

1. $\dfrac{5x - 10}{x - 2}$

2. $\dfrac{6x - 9y}{3}$

3. $\dfrac{12a^2b^5 + 18a^3b^4 - 24a^4b^3}{-6a^2b^3}$

4. $\dfrac{12x - 6}{10x - 5}$

5. $\dfrac{x^2 - 4}{x^2 + x - 6}$

6. $\dfrac{x^2 + 3x - 10}{x + 5}$

Write each **sum** *or* **difference** *as a single fraction in* **lowest terms**.

7. $\dfrac{3a}{8} + \dfrac{a}{8} - \dfrac{6}{8}$

8. $\dfrac{x + 3}{6} - \dfrac{x - 3}{6}$

9. $\frac{x-2}{4} + \frac{x+2}{4}$

10. $\frac{2}{3x+1} + \frac{5}{x-3}$

11. $\frac{3}{a^2-9} - \frac{6}{a^2+a-6}$

Write each **product** *or* **quotient** *as a single fraction in* **simplest terms**.

12. $\frac{21x^2y^3}{3xy} \bullet \frac{-9}{7xy^2}$

13. $\frac{a}{a+4} \bullet \frac{3a+12}{6}$

14. $\frac{-12}{x^2-x} \div \frac{4x-2}{x^2-1}$

15. $\frac{x^2-x-20}{x^2+7x+12} \bullet \frac{x^2+9x+18}{x^2-7x+10}$

Solve each **equation**.

16. $3(4x - 2) = 30$

17. $7x - 2(x + 3) = 19$

18. $5 - \frac{x}{2} = 12$

19. $28 + 6x = 23 + 8x$

Solve each **inequality**.

20. $5x + 4 \geq 20$

21. $16 - 4x < 20$

22. $5(x + 2) > 4x + 7$

Unit 5: How Radical Are You?

This unit focuses on simplifying radical expressions and performing operations involving radicals.

Unit Focus

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Algebra Body of Knowledge

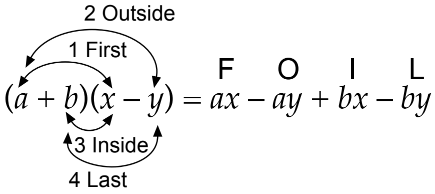
Standard 6: Radical Expressions and Equations

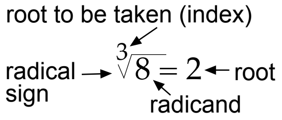
- MA.912.A.6.1
  Simplify radical expressions.
- MA.912.A.6.2
  Add, subtract, multiply and divide radical expressions (square roots and higher).

Vocabulary

Use the vocabulary words and definitions below as a reference for this unit.

coefficient	the number that multiplies the variable(s) in an algebraic expression *Example*: In $4xy$, the coefficient of xy is 4. If no number is specified, the coefficient is 1.
conjugate	if $x = a + b$, then $a - b$ is the conjugate of x *Example*: The expressions $(a + \sqrt{b})$ and $(a - \sqrt{b})$ are conjugates of each other.
decimal number	any number written with a decimal point in the number *Examples*: A decimal number falls between two whole numbers, such as 1.5, which falls between 1 and 2. Decimal numbers smaller than 1 are sometimes called *decimal fractions*, such as five-tenths, or $\frac{5}{10}$, which is written 0.5.
denominator	the bottom number of a fraction, indicating the number of equal parts a whole was divided into *Example*: In the fraction $\frac{2}{3}$ the denominator is 3, meaning the whole was divided into 3 equal parts.
digit	any one of the 10 symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9
distributive property	the product of a number and the sum or difference of two numbers is equal to the sum or difference of the two products *Examples*: $x(a + b) = ax + bx$ $5(10 + 8) = 5 \bullet 10 + 5 \bullet 8$

expression	a mathematical phrase or part of a number sentence that combines numbers, operation signs, and sometimes variables *Examples*: $4r^2$; $3x + 2y$; $\sqrt{25}$ An expression does *not* contain equal (=) or inequality ($<$, $>$, $\leq$, $\geq$, or $\neq$) signs.
factor	a number or expression that divides evenly into another number; one of the numbers multiplied to get a product *Examples*: 1, 2, 4, 5, 10, and 20 are factors of 20 and $(x + 1)$ is one of the factors of $(x^2 - 1)$.
FOIL method	a pattern used to multiply two binomials; multiply the first, outside, inside, and last terms: **F** **F**irst terms **O** **O**utside terms **I** **I**nside terms **L** **L**ast terms. *Example*: 
fraction	any part of a whole *Example*: One-half written in fractional form is $\frac{1}{2}$.
irrational number	a real number that cannot be expressed as a ratio of two integers *Example*: $\sqrt{2}$

like terms	terms that have the same variables and the same corresponding exponents *Example*: In $5x^2 + 3x^2 + 6$, the like terms are $5x^2$ and $3x^2$.
numerator	the top number of a fraction, indicating the number of equal parts being considered *Example*: In the fraction $\frac{2}{3}$, the numerator is 2.
perfect square	a number whose square root is a whole number *Example*: 25 is a perfect square because $5 \times 5 = 25$.
product	the result of multiplying numbers together *Example*: In $6 \times 8 = 48$, the product is 48.
radical	an expression that has a root (square root, cube root, etc.) *Example*: $\sqrt{25}$ is a radical Any root can be specified by an index number, b, in the form $\sqrt[b]{a}$ (e.g., $\sqrt[3]{8}$). A radical without an index number is understood to be a square root.  **radical**
radical expression	a numerical expression containing a radical sign *Examples*: $\sqrt{25}$ $2\sqrt{25}$
radical sign ($\sqrt{\ }$)	the symbol ($\sqrt{\ }$) used before a number to show that the number is a *radicand*

rationalizing the denominator	a method used to remove or eliminate radicals from the denominator of a fraction
rational number	a number that can be expressed as a ratio $\frac{a}{b}$, where a and b are integers and $b \neq 0$
simplest radical form	an expression under the radical sign that contains no perfect squares greater than 1, contains no fractions, and is not in the denominator of a fraction *Example*: $\sqrt{27} = \sqrt{9 \bullet 3} = \sqrt{9} \bullet \sqrt{3} = 3\sqrt{3}$
simplify an expression	to perform as many of the indicated operations as possible
square root	a positive real number that can be multiplied by itself to produce a given number *Example*: The square root of 144 is 12 or $\sqrt{144} = 12$.
term	a number, variable, product, or quotient in an expression *Example*: In the expression $4x^2 + 3x + x$, the terms are $4x^2$, $3x$, and x.
variable	any symbol, usually a letter, which could represent a number
whole numbers	the numbers in the set {0, 1, 2, 3, 4, …}

Unit 5: How Radical Are You?

Introduction

We will see that radical expressions can be rewritten to conform to the mathematical definitions of simplest terms. We will then be able to perform the operations of addition, subtraction, multiplication and division on these reformatted expressions. We will also explore the effects of multiplying a radical expression by its conjugate.

Lesson One Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.

- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.

- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Algebra Body of Knowledge

Standard 6: Radical Expressions and Equations

- MA.912.A.6.1
  Simplify radical expressions.

Simplifying Radical Expressions

A **radical expression** is any mathematical **expression** that contains a **square root** symbol. Look at the following examples:

$$\sqrt{5} \qquad \frac{\sqrt{6}}{3} \qquad \frac{3}{\sqrt{6}} \qquad \frac{7}{5+\sqrt{2}} \qquad \sqrt{36}$$

Certain numbers can be reformatted to make them easier to work with. To do so, mathematicians have rules that make working with numbers uniform. If we all play by the same rules, we should all have the same outcome.

With this in mind, here are the two basic rules for working with *square roots*.

1. Never leave a **perfect square factor** under a **radical sign** ($\sqrt{\ }$).
   Why? Because if you do, the radical expression is *not* simplified.

2. Never leave a *radical sign* in a **denominator**.
   Why? Because if you do, the radical expression is *not* simplified.

Important! Do **not** use your calculator with the *square roots*. It will change the numbers to **decimal** approximations. We are looking for exact answers.

Let's explore each of the rules…one at a time.

Rule One

First, let's review the idea of *perfect squares*. Perfect squares happen whenever you multiply a number times itself. In the following examples,

$3 \text{ x } 3 = 9 \qquad 7 \text{ x } 7 = 49 \qquad 9 \text{ x } 9 = 81$

9, 49, and 81 are all perfect squares.

It will be helpful to learn the chart below. You will be asked to use these numbers many times in this unit and in real-world applications. The chart shows the perfect squares underneath the radical sign, then gives the square root of each perfect square.

Perfect Squares:
Square Root = Whole Number

$\sqrt{1} = 1$
$\sqrt{4} = 2$
$\sqrt{9} = 3$
$\sqrt{16} = 4$
$\sqrt{25} = 5$
$\sqrt{36} = 6$
$\sqrt{49} = 7$
$\sqrt{64} = 8$
$\sqrt{81} = 9$
$\sqrt{100} = 10$
$\sqrt{121} = 11$
$\sqrt{144} = 12$
$\sqrt{169} = 13$
$\sqrt{196} = 14$
$\sqrt{225} = 15$
$\sqrt{256} = 16$
$\sqrt{289} = 17$
$\sqrt{324} = 18$
$\sqrt{361} = 19$
$\sqrt{400} = 20$

Any time you see a perfect square under a square root symbol, simplify it by writing it as the square root.

Sometimes, perfect squares are hidden in an *expression* and we have to search for them. At first glance, $\sqrt{45}$ looks as if it is in **simplest radical form**. However, when we realize that 45 has a *factor* that is a perfect square, we can rewrite it as

$$\sqrt{45} = \sqrt{9} \bullet \sqrt{5}.$$

From the information in the chart, we know that 9 is a perfect square and that

$\sqrt{9} = 3$. Therefore

$\sqrt{45} = 3 \bullet \sqrt{5}$ or $3\sqrt{5}$.

Let's look at some examples.

$$\begin{aligned}\sqrt{18} &= \sqrt{9} \bullet \sqrt{2} \\ &= 3 \bullet \sqrt{2} \\ &= 3\sqrt{2}\end{aligned}$$

$$\begin{aligned}\sqrt{20} &= \sqrt{4} \bullet \sqrt{5} \\ &= 2 \bullet \sqrt{5} \\ &= 2\sqrt{5}\end{aligned}$$

Now you try some in the following practices. Study the chart of perfect squares on page 367 before you start the practices.

Practice

Simplify *each* **radical expression**.

Remember: Never leave a perfect square factor under a radical sign.

1. $\sqrt{50}$

2. $\sqrt{27}$

3. $\sqrt{125}$

4. $\sqrt{64}$

5. $\sqrt{13}$

6. $\sqrt{32}$

7. $\sqrt{12}$

8. $\sqrt{45}$

9. $\sqrt{300}$

10. $\sqrt{8}$

Practice

Simplify ***each*** **radical expression**.

1. $\sqrt{48}$
2. $2\sqrt{40}$
3. $\sqrt{60}$
4. $\sqrt{242}$
5. $\sqrt{28}$
6. $-\sqrt{200}$
7. $-\sqrt{250}$
8. $\sqrt{108}$
9. $\sqrt{405}$
10. $5\sqrt{90}$

Rule Two

Now it's time to work on that second rule: never leave a square root in the *denominator*. Because if a square root is left in the denominator of a radical expression, the radical expression is *not* simplified.

If a **fraction** has a denominator that is a perfect square root, just rewrite the *fraction* using that square root. Let's look at examples.

$$\frac{2}{\sqrt{36}} = \frac{2}{6} = \frac{1}{3} \qquad\qquad \frac{4}{\sqrt{81}} = \frac{4}{9}$$

Many times, however, that denominator will *not* be a perfect square root. In those cases, we have to reformat the denominator so that it is a perfect square root. This is called **rationalizing the denominator** or the bottom number of the fraction. To do this, we make it into a **rational number** by using a method to eliminate **radicals** from the denominator of a fraction. Remember, we aren't concerned about what may happen to the format of the **numerator**, just the denominator.

To reformat an **irrational** denominator (one with a square root in it), we find a number to multiply it by that will produce a perfect square root.

Follow the explanation of this example carefully.

$\frac{2}{\sqrt{7}}$	⟵	Yikes! This denominator is irrational! I need to *rationalize* it.
$\frac{2}{\sqrt{7}} \bullet \frac{\sqrt{7}}{\sqrt{7}}$	⟵	Look what happens if I multiply the denominator by itself. (Since, $\frac{\sqrt{7}}{\sqrt{7}}$ = 1, I have *not* changed the value of the original fraction.)
$\frac{2}{\sqrt{7}} \bullet \frac{\sqrt{7}}{\sqrt{7}} = \frac{2\sqrt{7}}{\sqrt{49}}$	⟵	Because I remember the perfect square roots from the chart on page 240, I see that $\sqrt{49}$ is a perfect square root…and therefore rational!
$\frac{2}{\sqrt{7}} = \frac{2\sqrt{7}}{\sqrt{49}} = \frac{2\sqrt{7}}{7}$	⟵	This may *not* look like a simpler expression than I started with, but *it does conform to the second rule*.

Follow along with this example.

$$\frac{6}{\sqrt{3}} = \frac{6}{\sqrt{3}} \bullet \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{\sqrt{9}} = \frac{6\sqrt{3}}{3} = \frac{2\sqrt{3}}{1} = 2\sqrt{3}$$

In the above example, notice that we reduced the "real 6" and the "real 3," but not with the square root of 3. Do *not* mix a rational number with an irrational number, sometimes referred to as a *non-rational* number when you are reducing…they are *not* **like terms**!

It's time for you to practice.

Practice

Simplify *each* **radical expression**.

Remember: Never leave a square root in the denominator.

Example: $\frac{6}{\sqrt{5}} = \frac{6}{\sqrt{5}} \bullet \frac{\sqrt{5}}{\sqrt{5}} = \frac{6\sqrt{5}}{\sqrt{25}} = \frac{6\sqrt{5}}{5}$

Show all your steps.

1. $\frac{7}{\sqrt{2}}$

2. $\frac{5}{\sqrt{6}}$

3. $\frac{1}{\sqrt{3}}$

4. $\frac{3}{\sqrt{5}}$

5. $\frac{5}{\sqrt{18}}$

6. $\frac{4}{\sqrt{3}}$

7. $\frac{7}{\sqrt{10}}$

8. $\frac{3}{\sqrt{7}}$

9. $\frac{4}{\sqrt{11}}$

10. $\frac{\sqrt{2}}{\sqrt{15}}$

Practice

Simplify *each* **radical expression**.

Example: $\frac{10}{\sqrt{6}} = \frac{10}{\sqrt{6}} \bullet \frac{\sqrt{6}}{\sqrt{6}} = \frac{10\sqrt{6}}{\sqrt{36}} = \frac{10\sqrt{6}}{6} = \frac{5\sqrt{6}}{3}$

1. $\frac{9}{\sqrt{6}}$

2. $\frac{-2}{\sqrt{8}}$

3. $\frac{2}{\sqrt{7}}$

4. $\frac{5}{\sqrt{5}}$

5. $\sqrt{\frac{2}{3}}$

6. $\frac{3}{\sqrt{18}}$

7. $\sqrt{\frac{1}{3}}$

8. $\frac{-5}{\sqrt{20}}$

9. $\frac{3\sqrt{5}}{\sqrt{6}}$

10. $\frac{7\sqrt{3}}{\sqrt{5}}$

Practice

Match each definition with the correct term. Write the letter on the line provided.

_______ 1. a number whose square root is a whole number

_______ 2. an expression under the radical sign that contains no perfect squares greater than 1, contains no fractions, and is not in the denominator of a fraction

_______ 3. the symbol ($\sqrt{\ }$) used before a number to show that the number is a *radicand*

_______ 4. terms that have the same variables and the same corresponding exponents

_______ 5. a real number that cannot be expressed as a ratio of two integers

_______ 6. a number or expression that divides evenly into another number

_______ 7. a numerical expression containing a radical sign

_______ 8. a number that can be expressed as a ratio $\frac{a}{b}$, where a and b are integers and $b \neq 0$

_______ 9. a positive real number that can be multiplied by itself to produce a given number

A. factor

B. irrational number

C. like terms

D. perfect square

E. radical expression

F. radical sign

G. rational number

H. simplest radical form

I. square root

Lesson Two Purpose

Algebra Body of Knowledge

Standard 6: Radical Expressions and Equations

- MA.912.A.6.2
  Add, subtract, multiply and divide radical expressions (square roots and higher).

Add and Subtract Radical Expressions

We can add or subtract radical expressions only when those radical expressions match. For instance,

$$5\sqrt{2} + 6\sqrt{2} = 11\sqrt{2}.$$

Notice that we did not change the $\sqrt{2}$'s. We simply added the **coefficients** because they had matching radical parts.

Remember: *Coefficients* are any factor in a **term**. Usually, but not always, a coefficient is a number instead of a **variable** or a *radical*.

The same is true when we subtract radical expressions.

$$5\sqrt{7} - 3\sqrt{7} = 2\sqrt{7}$$

At first glance, it may sometimes appear that there are no matching numbers under the radical sign. But, if we **simplify** the expressions, we often find radical expressions that we can add or subtract.

Look at this example.

$3\sqrt{8} + 5\sqrt{2} - 4\sqrt{32}$

Notice that $\sqrt{8}$ and $\sqrt{32}$ each have perfect square factors and can be simplified. Follow the simplification process step by step and see what happens.

$3\sqrt{8} + 5\sqrt{2} - 4\sqrt{32} =$

$3\sqrt{4}\sqrt{2} + 5\sqrt{2} - 4\sqrt{16}\sqrt{2} =$ ← We *found* the perfect square factors of $\sqrt{8}$ and $\sqrt{32}$ and rewrote the problem.

$3 \bullet 2\sqrt{2} + 5\sqrt{2} - 4 \bullet 4\sqrt{2} =$ ← Next, we *simplified* the perfect square roots.

$6\sqrt{2} + 5\sqrt{2} - 16\sqrt{2} =$ ← We *multiplied* the new factors for each coefficient.

$-5\sqrt{2}$ ← Finally, we *add and subtract* matching radical expressions, in order, from *left to right*.

When Radical Expressions Don’t Match or Are Not in Radical Form

What happens when radical expressions don’t match, or there is a number that is not in radical form? Just follow the steps on the previous pages and leave your answer, with appropriate terms in descending order. Watch this!

$$\begin{aligned}
\sqrt{75} + \sqrt{27} - \sqrt{16} + \sqrt{80} &= \\
\sqrt{25}\sqrt{3} + \sqrt{9}\sqrt{3} - 4 + \sqrt{16}\sqrt{5} &= \\
5\sqrt{3} + 3\sqrt{3} - 4 + 4\sqrt{5} &= \\
8\sqrt{3} - 4 + 4\sqrt{5} &= \\
8\sqrt{3} + 4\sqrt{5} - 4 & \quad \longleftarrow \text{rewritten in descending order}
\end{aligned}$$

Practice

Simplify *each of the following. Refer to pages 376-378 as needed.*

1. $4\sqrt{7} + 10\sqrt{7}$

2. $-5\sqrt{2} + 7\sqrt{2} - 4\sqrt{2}$

3. $3\sqrt{7} + 5 - \sqrt{7}$

4. $2\sqrt{27} - 4\sqrt{12}$

5. $\sqrt{2} + \sqrt{18} - \sqrt{16}$

6. $\sqrt{3} + 5\sqrt{3} - \sqrt{27}$

7. $\sqrt{50} + \sqrt{18}$

8. $\sqrt{27} + \sqrt{12} - \sqrt{48}$

Practice

Simplify *each of the following. Refer to pages 376-378 as needed.*

1. $-3\sqrt{5} + 4\sqrt{2} - \sqrt{5} + \sqrt{8}$

2. $\sqrt{81} + \sqrt{24} - \sqrt{9} + \sqrt{54}$

3. $\sqrt{50} - \sqrt{45} + \sqrt{32} - \sqrt{80}$

4. $5\sqrt{7} + 2\sqrt{3} - 4\sqrt{7} - \sqrt{27}$

5. $\sqrt{200} - \sqrt{8} + 3\sqrt{72} - 6$

6. $12 - 3\sqrt{5} + 2\sqrt{144} - \sqrt{20}$

7. $\sqrt{18} + \sqrt{48} - \sqrt{32} - \sqrt{27}$

Lesson Three Purpose

Algebra Body of Knowledge

Standard 6: Radical Expressions and Equations

- MA.912.A.6.2
  Add, subtract, multiply and divide radical expressions (square roots and higher).

Multiply and Divide Radical Expressions

Radical expressions *don't* have to match when we multiply or divide them. The following examples show that we simply multiply or divide the **digits** under the radical signs and then simplify our results, if possible.

Example 1

$$\sqrt{5} \text{ x } \sqrt{6} = \sqrt{30}$$

Example 2

$$\sqrt{8} \text{ x } \sqrt{3} = \sqrt{24} = \sqrt{4}\sqrt{6} = 2\sqrt{6}$$

Example 3

$$\sqrt{18} \text{ x } \sqrt{2} = \sqrt{36} = 6$$

Example 4

$$\frac{\sqrt{12}}{\sqrt{3}} = \sqrt{4} = 2$$

Example 5

$$\frac{\sqrt{20}}{\sqrt{10}} = \sqrt{2}$$

Example 6

$$\frac{\sqrt{8}}{\sqrt{24}} = \frac{\sqrt{1}}{\sqrt{3}} \quad \text{(we must simplify this)} \quad \rightarrow \frac{\sqrt{1}}{\sqrt{3}} \bullet \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{9}} = \frac{\sqrt{3}}{3}$$

After studying the examples above, try the following practice.

Practice

Simplify *each of the following. Refer to the* **examples** *on the previous page as needed.*

1. $\sqrt{5} \bullet \sqrt{10}$

2. $\sqrt{2} \bullet \sqrt{50}$

3. $\sqrt{75} \bullet \sqrt{3}$

4. $\sqrt{6} \bullet \sqrt{10}$

5. $\frac{\sqrt{30}}{\sqrt{2}}$

6. $\frac{\sqrt{8}}{\sqrt{32}}$

7. $\frac{\sqrt{6}}{\sqrt{10}}$

8. $\frac{\sqrt{75}}{\sqrt{3}}$

9. $\frac{\sqrt{72}}{\sqrt{18}}$

10. $\frac{\sqrt{5}}{\sqrt{10}}$

Working with a Coefficient for the Radical

What happens when there is a coefficient for the *radical*? It is important to multiply or divide the radical numbers together separately from the coefficients. Then simplify each answer. Look at the following examples.

Example 1

multiply coefficients

$3 \cdot 5 = 15$

$$3\sqrt{7} \bullet 5\sqrt{2} = 15\sqrt{14}$$

multiply radicands

$\sqrt{7} \cdot \sqrt{2} = \sqrt{14}$

Example 2

$$6\sqrt{3} \bullet \sqrt{3} = 6\sqrt{9} = 6 \bullet 3 = 18$$

Remember: If there is *no* written coefficient, then it is understood to be a 1.

Example 3

$$\frac{2\sqrt{14}}{6\sqrt{7}} = \frac{1\sqrt{2}}{3} = \frac{\sqrt{2}}{3}$$

Example 4

$$\frac{12\sqrt{5}}{6\sqrt{10}} = \frac{2}{\sqrt{2}} \bullet \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2}}{\sqrt{4}} = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

Example 5

$$\frac{\sqrt{6} - \sqrt{12}}{\sqrt{3}} = \frac{\sqrt{6}}{\sqrt{3}} - \frac{\sqrt{12}}{\sqrt{3}} = \sqrt{2} - \sqrt{4} = \sqrt{2} - 2$$

Now it's time to practice on the following page.

Practice

Simplify *each of the following. Refer to the* **examples** *on the previous page as needed.*

1. $5\sqrt{3} \bullet 6\sqrt{5}$

2. $2\sqrt{5} \bullet 4\sqrt{2}$

3. $8\sqrt{2} \bullet 5\sqrt{3}$

4. $2\sqrt{7} \bullet \sqrt{7}$

5. $\frac{3\sqrt{10}}{6\sqrt{5}}$

6. $\frac{4\sqrt{6}}{2\sqrt{12}}$

7. $\frac{9\sqrt{5}}{3\sqrt{10}}$

8. $2\sqrt{7} \bullet 5\sqrt{7}$

9. $5\sqrt{6} \bullet 4\sqrt{2}$

Practice

Simplify *each of the following. Refer to the* **examples** *on page 386 as needed.*

1. $\frac{\sqrt{15} - \sqrt{20}}{\sqrt{5}}$

2. $\frac{\sqrt{8} - \sqrt{12}}{\sqrt{2}}$

3. $\frac{\sqrt{30} - \sqrt{50}}{\sqrt{10}}$

4. $\frac{3\sqrt{18}}{\sqrt{3}}$

5. $\frac{4\sqrt{6}}{6\sqrt{2}}$

6. $\frac{10\sqrt{8}}{12\sqrt{12}}$

7. $\frac{\sqrt{75}-\sqrt{50}}{\sqrt{25}}$

Lesson Four Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.

Algebra Body of Knowledge

Standard 6: Radical Expressions and Equations

- MA.912.A.6.2
  Add, subtract, multiply and divide radical expressions (square roots and higher).

Multiple Terms and Conjugates

Sometimes it is necessary to multiply or divide radical expressions with more than one *term*. To multiply radicals with multiple terms by a single term, we use the old reliable **distributive property**. See how the *distributive property* works for these examples.

Example 1

$$6(\sqrt{5}+\sqrt{3}) =$$
$$6\sqrt{5}+6\sqrt{3}$$

Example 2

$$
\begin{aligned}
&\sqrt{3}(2\sqrt{5} - 4\sqrt{3}) = \\
&\quad 2\sqrt{15} - 4\sqrt{9} = \\
&\quad 2\sqrt{15} - 4 \bullet 3 = \\
&\quad 2\sqrt{15} - 12
\end{aligned}
$$

Example 3

$$
\begin{aligned}
&6\sqrt{3}(2\sqrt{2} + 5\sqrt{6}) = \\
&12\sqrt{6} + 30\sqrt{18} = \\
&12\sqrt{6} + 30\sqrt{9}\sqrt{2} = \\
&12\sqrt{6} + 30 \bullet 3\sqrt{2} = \\
&12\sqrt{6} + 90\sqrt{2}
\end{aligned}
$$

Practice

Simplify *each of the following. Refer to the* **examples** *on the previous pages as needed.*

1. $2(\sqrt{6} + \sqrt{5})$

2. $\sqrt{2}(\sqrt{6} + \sqrt{5})$

3. $3\sqrt{2}(5\sqrt{3} - 4\sqrt{2})$

4. $6(3\sqrt{8} - 5\sqrt{2})$

5. $\sqrt{6}(3\sqrt{8} - 5\sqrt{2})$

6. $-2(\sqrt{5} + 7)$

7. $2\sqrt{3}(\sqrt{7} + \sqrt{10})$

8. $4(2\sqrt{3} - 5\sqrt{2})$

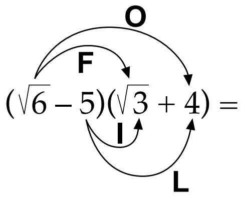
9. $4\sqrt{3}(2\sqrt{3} - 5\sqrt{2})$

10. $8\sqrt{6}(2\sqrt{6} + 5\sqrt{8})$

The FOIL Method

Another reliable method we can use when multiplying two radical expressions with multiple terms is the **FOIL method:** multiplying the **f**irst, **o**utside, **i**nside, and **l**ast terms. We use that same process in problems like these.

Example 1



$$(\sqrt{6}-5)(\sqrt{3}+4)=$$

$$\overset{\textbf{F}}{\sqrt{6}\bullet\sqrt{3}}+\overset{\textbf{O}}{\sqrt{6}\bullet 4}-\overset{\textbf{I}}{5\bullet\sqrt{3}}-\overset{\textbf{L}}{5\bullet 4}= \longleftarrow$$

Multiply the **first** terms, the **outside** terms, the **inside** terms, and then the **last** terms.

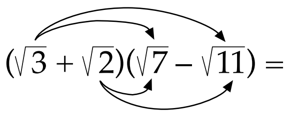
$$\sqrt{18}+4\sqrt{6}-5\sqrt{3}-20= \longleftarrow$$

Carefully write out the **products**.

$$3\sqrt{2}+4\sqrt{6}-5\sqrt{3}-20 \longleftarrow$$

Simplify each term and combine like terms (if needed).

Example 2



$$(\sqrt{3}+\sqrt{2})(\sqrt{7}-\sqrt{11})=$$

$$\sqrt{3}\sqrt{7}-\sqrt{3}\sqrt{11}+\sqrt{2}\sqrt{7}-\sqrt{2}\sqrt{11}=$$

$$\sqrt{21}-\sqrt{33}+\sqrt{14}-\sqrt{22} \longleftarrow$$

Notice that no term has a perfect square as a factor. Therefore, there is no further simplifying to be done.

Time to try the following practice.

Practice

Simplify *each of the following. Refer to the* **examples** *on page 395 as needed.*

1. $(\sqrt{6} - 2)(\sqrt{5} + 7)$

2. $(5 - \sqrt{3})(2 + \sqrt{7})$

3. $(4 + 5\sqrt{2})(2 - \sqrt{2})$

4. $(2\sqrt{5} - 3)(\sqrt{5} + 6)$

5. $(4 - 3\sqrt{10})(2 - \sqrt{10})$

6. $(2\sqrt{7} - 3)(5\sqrt{7} + 1)$

7. $(\sqrt{5} - 7)(3\sqrt{5} + 7)$

8. $(\sqrt{10} - \sqrt{6})(\sqrt{7} - \sqrt{13})$

9. $(3\sqrt{6} + 2\sqrt{3})(\sqrt{5} - 2)$

10. $(4\sqrt{3} - \sqrt{5})(3\sqrt{3} - \sqrt{5})$

11. $(3 + \sqrt{10})(3 - \sqrt{10})$

12. $(6\sqrt{5} + 4)(6\sqrt{5} - 4)$

Two-Term Radical Expressions

At the beginning of this unit, we learned that there are two rules we must remember when simplifying a radical expression. Rule one requires that we never leave a perfect square factor under a radical sign. Rule two insists that we never leave a radical in the denominator. With that in mind, let's see what to do with two-term radical expressions.

In a problem like $\frac{2+\sqrt{7}}{5-\sqrt{6}}$, we see that we must rationalize the denominator (reformat it without using a square root). At first glance, it may seem to you that multiplying that denominator by itself makes the square roots disappear. But when we try that, we realize that new square roots appear as a result of the FOILing.

$$(5 - \sqrt{6})(5 - \sqrt{6}) =$$

$$25 - 5\sqrt{6} - 5\sqrt{6} + \sqrt{6}\sqrt{6} =$$

$$25 - 10\sqrt{6} + 6$$

So there must be a better way to rationalize this denominator. Try multiplying $(5 - \sqrt{6}\,)$ by its **conjugates** $(5 + \sqrt{6}\,)$. These numbers are *conjugates* because they match, except for the signs between the terms. Notice that one has a "+" and the other has a "–".

$$(5 - \sqrt{6})(5 + \sqrt{6}) =$$

$$25 + 5\sqrt{6} - 5\sqrt{6} - \sqrt{6}\sqrt{6} =$$

$$25 - \sqrt{36} =$$

$$25 - 6 =$$

$$19$$

Remember, we only need to rationalize the denominator. It is acceptable to leave simplified square roots in the numerator. Now, let's take a look at the entire problem.

$\frac{2+\sqrt{7}}{5-\sqrt{6}} \bullet \frac{5+\sqrt{6}}{5+\sqrt{6}} =$	←	reformat the fraction by multiplying it by 1 $\frac{5+\sqrt{6}}{5+\sqrt{6}} = 1$
$\frac{(2)(5)+2\sqrt{6}+5\sqrt{7}+\sqrt{42}}{(5)(5)+5\sqrt{6}-5\sqrt{6}-\sqrt{6}\sqrt{6}} =$	←	FOIL the numerator and denominator
$\frac{10+2\sqrt{6}+5\sqrt{7}+\sqrt{42}}{25-\sqrt{36}} =$	←	simplify
$\frac{10+2\sqrt{6}+5\sqrt{7}+\sqrt{42}}{25-6} =$	←	simplify again
$\frac{10+2\sqrt{6}+5\sqrt{7}+\sqrt{42}}{19}$	←	and again, if necessary

Follow along with this one!

$$\frac{3+\sqrt{2}}{4+\sqrt{8}} \bullet \frac{4-\sqrt{8}}{4-\sqrt{8}} =$$

← reformat the fraction by multiplying it by 1

$$\frac{4-\sqrt{8}}{4-\sqrt{8}} = 1$$

$$\frac{(3)(4)-3\sqrt{8}+4\sqrt{2}-\sqrt{16}}{(4)(4)-4\sqrt{8}+4\sqrt{8}-\sqrt{8}\sqrt{8}} =$$

← FOIL the numerator and denominator

$$\frac{12-3\sqrt{4}\sqrt{2}+4\sqrt{2}-\sqrt{16}}{16-\sqrt{64}} =$$

← simplify

$$\frac{12-3\bullet 2\sqrt{2}+4\sqrt{2}-4}{16-8} =$$

← simplify again

$$\frac{12-6\sqrt{2}+4\sqrt{2}-4}{8} =$$

← and again

$$\frac{8-2\sqrt{2}}{8} = \frac{2(4-\sqrt{2})}{8} =$$

← and again

$$\frac{4-\sqrt{2}}{4}$$

← and again, if necessary

With more practice, you will be able to mentally combine some of those simplifying steps and finish sooner.

So let's practice on the following page.

Practice

Simplify *each of the following.*

1. $\frac{\sqrt{5}+2}{\sqrt{3}-1}$

2. $\frac{\sqrt{6}+5}{3\sqrt{6}-2}$

3. $\frac{5\sqrt{2}+7}{\sqrt{2}-3}$

4. $\frac{\sqrt{7}-\sqrt{5}}{\sqrt{5}+\sqrt{7}}$

5. $\frac{\sqrt{6} - \sqrt{3}}{\sqrt{6} + \sqrt{3}}$

6. $\frac{\sqrt{2} + \sqrt{3}}{2\sqrt{2} - 5}$

7. $\frac{\sqrt{5} + 7}{\sqrt{5} - 3}$

8. $\frac{\sqrt{6} + \sqrt{5}}{\sqrt{6} - 3\sqrt{5}}$

Practice

Simplify *each of the following.*

1. $\frac{4-\sqrt{7}}{3+\sqrt{7}}$

2. $\frac{4\sqrt{2}-\sqrt{3}}{\sqrt{2}+3\sqrt{3}}$

3. $\frac{6\sqrt{5}-2}{\sqrt{5}+\sqrt{2}}$

4. $\frac{6\sqrt{2} + 5}{1 + \sqrt{5}}$

5. $\frac{5 + 3\sqrt{2}}{1 - \sqrt{2}}$

6. $\frac{\sqrt{6} + 2}{2\sqrt{6} + 1}$

7. $\frac{\sqrt{5} + 2\sqrt{7}}{\sqrt{5} + \sqrt{7}}$

Practice

Match each **symbol or expression** *with the appropriate* **description**.

_______ 1. 7

_______ 2. $\sqrt{\ }$

_______ 3. $\frac{2}{\sqrt{7}} = \frac{2\sqrt{7}}{\sqrt{49}} = \frac{2\sqrt{7}}{7}$

_______ 4. $x - 4$

_______ 5. 5

_______ 6. $3x\sqrt{6}$

_______ 7. 121

A. coefficient in the expression $5\sqrt{x}$

B. conjugate of $x + 4$

C. perfect square of 11

D. radical expression

E. radical sign

F. rationalizing the denominator

G. square root of 49

Unit Review

Simplify *each of the following.*

1. $\sqrt{75}$

2. $-\sqrt{40}$

3. $5\sqrt{27}$

4. $\frac{3}{\sqrt{36}}$

5. $\frac{5}{\sqrt{8}}$

6. $\frac{1}{\sqrt{7}}$

7. $\sqrt{\frac{3}{8}}$

8. $\frac{5\sqrt{6}}{\sqrt{5}}$

9. $6\sqrt{3} - 8\sqrt{3}$

10. $4\sqrt{8} - 5\sqrt{2} + 3\sqrt{32}$

11. $\sqrt{75} - \sqrt{45} - \sqrt{80}$

12. $2\sqrt{50} - 3\sqrt{45} + \sqrt{32} + \sqrt{80}$

13. $\sqrt{5} + \sqrt{2} + \sqrt{8} + \sqrt{125}$

14. $\sqrt{2} \text{ x } \sqrt{10}$

15. $\dfrac{\sqrt{18}}{\sqrt{3}}$

16. $\sqrt{6} \bullet \sqrt{2}$

17. $\dfrac{2\sqrt{6}}{\sqrt{24}}$

18. $\sqrt{\dfrac{6}{30}}$

19. $\dfrac{\sqrt{60}}{3\sqrt{5}}$

20. $8\sqrt{3} \bullet 2\sqrt{8}$

21. $\dfrac{\sqrt{18} - \sqrt{12}}{\sqrt{6}}$

22. $(3 + 5\sqrt{6})(3 - 5\sqrt{6})$

23. $\frac{\sqrt{2} + \sqrt{6}}{\sqrt{2} - \sqrt{6}}$

24. $\frac{5 + 2\sqrt{3}}{2 + \sqrt{5}}$

25. $\frac{\sqrt{6} - 1}{2\sqrt{6} + 2}$

Unit 6: Extreme Fractions

This unit will illustrate the difference between shape and size as they relate to the concepts of congruency and similarity.

Unit Focus

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

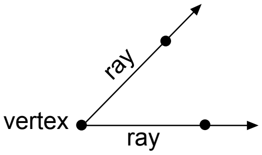
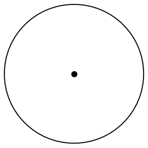
Algebra Body of Knowledge

Standard 5: Radical Expressions and Equations

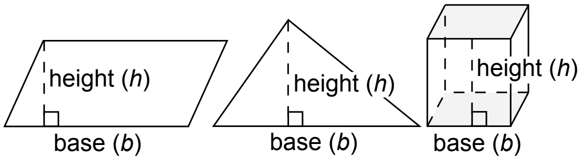
- MA.912.A.5.1
  Simplify algebraic ratios.
- MA.912.A.5.4
  Solve algebraic proportions.

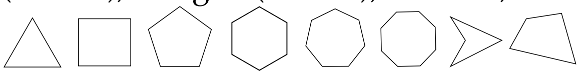
Vocabulary

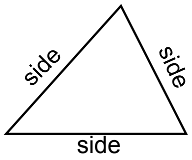
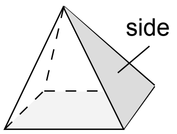
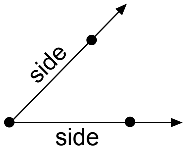
Use the vocabulary words and definitions below as a reference for this unit.

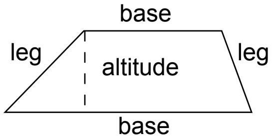
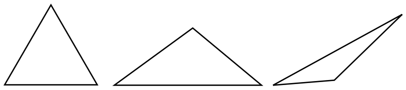
angle (∠)	two rays extending from a common endpoint called the vertex; measures of angles are described in degrees (°) 
circle	the set of all points in a plane that are all the same distance from a given point called the center 
congruent (≅)	having exactly the same shape and size
corresponding	in the same location in their respective figures
corresponding angles and sides	the matching angles and sides in similar figures
cross multiplication	a method for solving and checking proportions; a method for finding a missing numerator or denominator in equivalent fractions or ratios by making the cross products equal *Example*: Solve this proportion by doing the following. $\frac{n}{9} = \frac{8}{12}$ $\frac{n}{9} \times \frac{8}{12}$ $12 \text{ x } n = 9 \text{ x } 8$ $12n = 72$ $n = \frac{72}{12}$ $n = 6$ Solution: $\frac{\textcircled{6}}{9} = \frac{8}{12}$

degree (°)	common unit used in measuring angles
denominator	the bottom number of a fraction, indicating the number of equal parts a whole was divided into *Example*: In the fraction $\frac{2}{3}$ the denominator is 3, meaning the whole was divided into 3 equal parts.
distributive property	the product of a number and the sum or difference of two numbers is equal to the sum or difference of the two products *Examples*: $x(a + b) = ax + bx$ $5(10 + 8) = 5 \bullet 10 + 5 \bullet 8$
equation	a mathematical sentence stating that the two expressions have the same value *Example*: $2x = 10$
equiangular	a figure with all angles congruent
equilateral	a figure with all sides congruent
fraction	any part of a whole *Example*: One-half written in fractional form is $\frac{1}{2}$.
height (*h*)	a line segment extending from the vertex or *apex* (highest point) of a figure to its base and forming a right angle with the base or plane that contains the base



integers	the numbers in the set {… , -4, -3, -2, -1, 0, 1, 2, 3, 4, …}
length (*l*)	a one-dimensional measure that is the measurable property of line segments
numerator	the top number of a fraction, indicating the number of equal parts being considered *Example*: In the fraction $\frac{2}{3}$, the numerator is 2.
perimeter (*P*)	the distance around a figure
polygon	a closed-plane figure, having at least three sides that are line segments and are connected at their endpoints *Examples*: triangle (3 sides), quadrilateral (4 sides), pentagon (5 sides), hexagon (6 sides), heptagon (7 sides), octagon (8 sides); concave, convex 
proportion	a mathematical sentence stating that two ratios are equal *Example*: The ratio of 1 to 4 equals 25 to 100, that is $\frac{1}{4} = \frac{25}{100}$.
ratio	the comparison of two quantities *Example*: The ratio of a and b is a:b or $\frac{a}{b}$, where $b \neq 0$.

regular polygon	a polygon that is both *equilateral* (all sides congruent) and *equiangular* (all angles congruent)
rounded number	a number approximated to a specified place *Example*: A commonly used rule to round a number is as follows. • If the digit in the first place after the specified place is 5 or more, *round up* by adding 1 to the digit in the specified place ($\underline{4}\overset{\curvearrowleft}{6}1$ rounded to the nearest hundred is 500). • If the digit in the first place after the specified place is less than 5, *round down* by *not* changing the digit in the specified place ($\underline{4}\overset{\curvearrowleft}{4}1$ rounded to the nearest hundred is 400).
scale factor	the constant that is multiplied by the lengths of each side of a figure that produces an image that is the same shape as the original figure
side	the edge of a polygon, the face of a polyhedron, or one of the rays that make up an angle *Example*: A triangle has three sides.  side side side *edge of a polygon*  side *face of a polyhedron*  side side *ray of an angle*

similar figures (~)	figures that are the same shape, have corresponding congruent angles, and have corresponding sides that are proportional in length
solve	to find all numbers that make an equation or inequality true
trapezoid	a quadrilateral with just one pair of opposite sides parallel 
triangle	a polygon with three sides 
value (of a variable)	any of the numbers represented by the variable
variable	any symbol, usually a letter, which could represent a number

Unit 6: Extreme Fractions

Introduction

We should be able to see that changing the size of a geometric figure can occur without changing the shape of a figure. Working with ratios and proportions will help us understand the relationship between congruence and similarity.

Lesson One Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.

- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.

Algebra Body of Knowledge

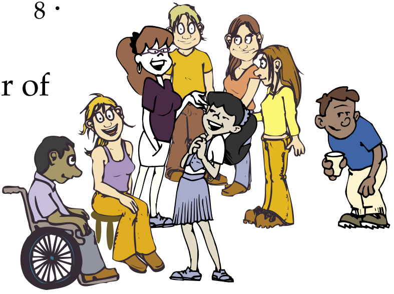
Standard 5: Radical Expressions and Equations

- MA.912.A.5.1
  Simplify algebraic ratios.

- MA.912.A.5.4
  Solve algebraic proportions.

Ratios and Proportions

Ratio is another word for a **fraction**. It is the comparison of two quantities: the **numerator** (top number of a *fraction*) and the **denominator** (bottom number of a fraction). For instance, if a classroom has 32 students and 20 of them are girls, we can say that the *ratio* of the number of girls to the number of students in the class is $\frac{20}{32} = \frac{5}{8}$ or 5:8. There are several other comparisons we can make using the information. We could compare the number of boys to the number of students, $\frac{12}{32} = \frac{3}{8}$.



What about the number of boys to the number of girls? $\frac{12}{20} = \frac{3}{5}$

Or, the number of girls to the number of boys? $\frac{20}{12} = \frac{5}{3}$

When two ratios are equal to each other, we have formed a **proportion**. A *proportion* is a mathematical sentence stating that two ratios are equal.

$$\frac{6}{9} = \frac{2}{3}$$

There are several properties of proportions that will be useful as we continue through this unit.

- We could *switch* the 6 with the 3 and still have a *true* proportion (Example 1).
- We could *switch* the 2 with the 9 and still have a *true* proportion (Example 2).
- We could even *flip* both fractions over and still have a *true* proportion (Example 3).

Example 1

$$\frac{\textcircled{6}}{9} = \frac{2}{\textcircled{3}}$$

$$\frac{\textcircled{3}}{9} = \frac{2}{\textcircled{6}}$$

Example 2

$$\frac{6}{\textcircled{9}} = \frac{\textcircled{2}}{3}$$

$$\frac{6}{\textcircled{2}} = \frac{\textcircled{9}}{3}$$

Example 3

$$\frac{6}{9} = \frac{2}{3}$$

$$\frac{9}{6} = \frac{3}{2}$$

Proportions are also very handy to use for problem solving. We use a process that involves **cross multiplying**, then **solve** the resulting **equation**. Look at the example below as we *solve* the *equation* and find the **value of the variable**.

$$\frac{3}{5} = \frac{x}{x+6}$$

$$\frac{3}{5} \rightleftarrows \frac{x}{x+6}$$

← cross multiply

$$\begin{aligned} 3(x+6) &= 5x \\ 3x + 18 &= 5x \end{aligned}$$

← distribute
(**distributive property**)

$$\begin{aligned} 3x - 3x + 18 &= 5x - 3x \\ 18 &= 2x \end{aligned}$$

← subtract 3*x* from each side

$$\begin{aligned} \frac{18}{2} &= \frac{2x}{2} \\ 9 &= x \end{aligned}$$

← divide each side by 2

Check your answer. Does $\frac{9}{9+6} = \frac{3}{5}$? Yes, $\frac{9}{15} = \frac{3}{5}$, so 9 is the correct *value* for x.

Try the following practice.

Practice

Find the **value of the variable** *in each of the following. Refer to previous pages as needed. Check your answers.* **Show all your work.**

1. $\frac{2}{x+1} = \frac{4}{x}$

2. $\frac{6}{z-2} = \frac{12}{4}$

3. $\frac{3}{2x-1} = \frac{7}{3x+1}$

4. $\frac{2}{x-9} = \frac{9}{x+12}$

5. $\frac{6}{x-1} = \frac{5}{x+2}$

6. $\frac{x-3}{18} = \frac{x+1}{30}$

7. $\frac{x-8}{x} = \frac{5}{7}$

8. $\frac{x+12}{2x+3} = \frac{5}{3}$

9. $\frac{2x}{x+3} = \frac{3}{2}$

Using Proportions Algebraically

We can use proportions in word problems as well. Here's an example.

In Coach Coffey's physical education class, the ratio of boys to girls is 3 to 4. If there are 12 boys in the class, how many girls are there?

When setting up proportions, you must have a plan and be consistent when you write the ratios. If you set up one ratio as $\frac{\text{boys}}{\text{girls}}$, the you must set up the other ratio in the same order, as $\frac{\text{boys}}{\text{girls}}$.

$\frac{3}{4} = \frac{12}{x}$ $\leftarrow$ notice that both fractions indicate $\frac{\text{boys}}{\text{girls}}$

$\frac{3}{4} \overset{\nwarrow\,\nearrow}{\underset{\swarrow\,\searrow}{=}} \frac{12}{x}$ $\leftarrow$ cross multiply

$3x = 4 \text{ x } 12$ $\leftarrow$ simplify

$3x = 48$ $\leftarrow$ divide each side by 3

$\frac{3x}{3} = \frac{48}{3}$

$x = 16$

Check your answer. Does $\frac{12}{16} = \frac{3}{4}$? Yes, so 16 is the correct answer.

Now it is your turn to practice on the following page.

Practice

Use **proportions** *to solve the following. Refer to the previous pages as needed. Check your answers.* **Show all your work.**

1. The ratio of two **integers** {… , -4, -3, -2, -1, 0, 1, 2, 3, 4, …} is 13:6. The smaller integer is 54. Find the *larger* integer.

   Answer: ____________

2. The ratio of two integers is 7:11. The larger integer is 187. Find the *smaller* integer.

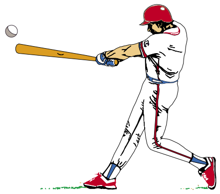
   Answer: ____________

3. A shopkeeper makes $85 profit when he sells $500 worth of clothing. At the same rate of profit, what will he make on a $650 sale?

   Answer: $ ____________

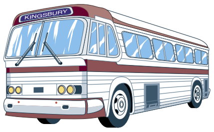
4. A baseball player made 43 hits in 150 times at bat. At the same rate, how many hits can he expect in 1,050 times at bat?

   Answer: ____________



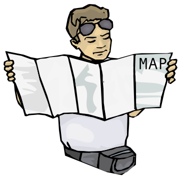
5. The cost of a 1,600-mile bus trip is $144. At the same rate per mile, what will be the cost of a 650-mile trip?

   Answer: $ ____________



6. On a map, 19 inches represent 250 miles. What **length** on the map will represent 600 miles?

   Answer: ____________ miles



Practice

Match each definition with the correct term. Write the letter on the line provided.

_______ 1. the comparison of two quantities

_______ 2. the numbers in the set {… , -4, -3, -2, -1, 0, 1, 2, 3, 4, …}

_______ 3. a mathematical sentence stating that the two expressions have the same value

_______ 4. to find all numbers that make an equation or inequality true

_______ 5. the bottom number of a fraction, indicating the number of equal parts a whole was divided into

_______ 6. the top number of a fraction, indicating the number of equal parts being considered

_______ 7. a mathematical sentence stating that two ratios are equal

_______ 8. $x(a + b) = ax + bx$
$5(10 + 8) = 5 \bullet 10 + 5 \bullet 8$

_______ 9. any part of a whole

_______ 10. a one-dimensional measure that is the measurable property of line segments

_______ 11. a method for solving and checking proportions; a method for finding a missing numerator or denominator in equivalent fractions or ratios by making the cross products equal

A. cross multiplication

B. denominator

C. distributive property

D. equation

E. fraction

F. integers

G. length (*l*)

H. numerator

I. proportion

J. ratio

K. solve

Lesson Two Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Algebra Body of Knowledge

Standard 5: Radical Expressions and Equations

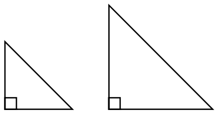
- MA.912.A.5.4
  Solve algebraic proportions.

Similarity and Congruence

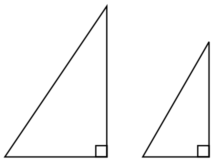
Geometric figures that are exactly the same shape, but not necessarily the same size, are called **similar figures (~)**. In *similar figures*, all the pairs of **corresponding angles** are the same measure, and all the pairs of **corresponding sides** are in the same ratio. This ratio, in its reduced form, is called the **scale factor**. When all pairs of *corresponding sides* are in the same ratio as the *scale factor*, we say that the **sides** are in proportion.

Some geometric figures are always similar.

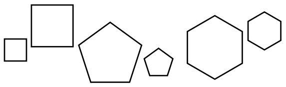
1. All **triangles** whose **angles' (∠)** measures of **degree (°)** are 45°, 45°, and 90° are similar to each other.



2. All *triangles* whose *angles' (∠)* measures of *degree* (°) are 30°, 60°, and 90° are similar to each other.



3. All **regular polygons** with the same number of *sides* are similar to each other.



Remember: A *regular polygon* is a **polygon** that is **equilateral** and **equiangular**. Therefore, all its sides are **congruent (≅)** and all angles are *congruent (≅)*.

Note: Circles seem to be similar, but since they have no angle measures, we don't include them in this group.

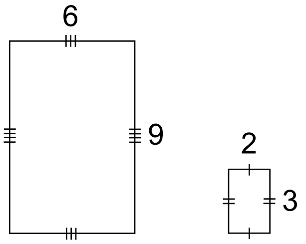
Practice

Look at each **pair of figures** *below. Determine if they are* **similar or not to each other**.

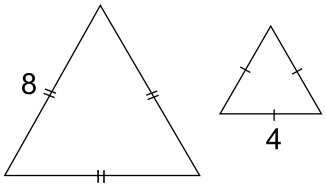
- *Write* **yes** *if they are similar.*
- *Write* **no** *if they are* not *similar.*
- *If they are similar, write the* **scale factor**.

The first one has been done for you.

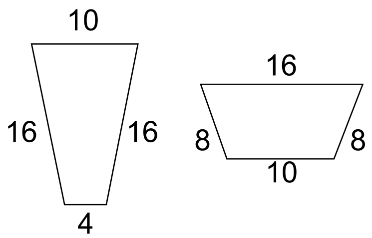
yes: 3:1 1.



______________________ 2.



______________________ 3.



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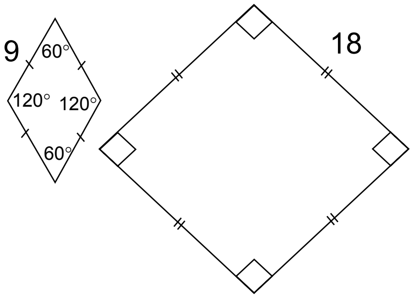

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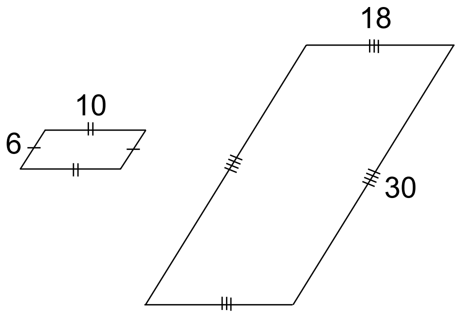

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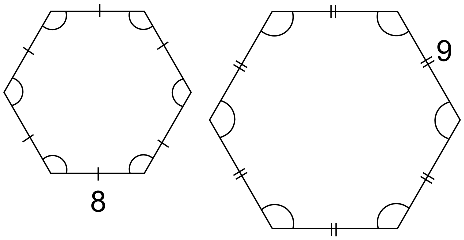

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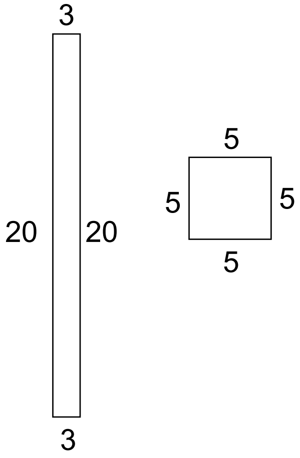

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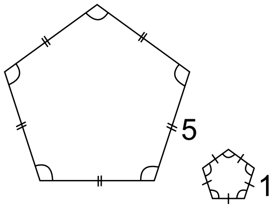

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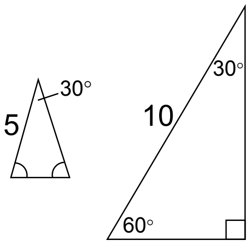
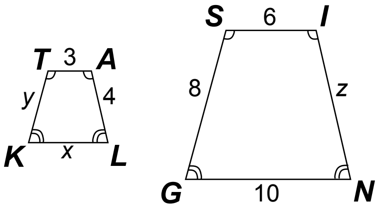


Using Proportions Geometrically

If we know two shapes are similar, and we know some of the lengths, we often can find some of the other measures of those shapes. Look at the two similar figures below. We have labeled the trapezoids *TALK* and *SING*.

Trapezoids *TALK* and *SING*



By locating the corresponding angles, we can say that

Trapezoid *TALK* ~ **Trapezoid** *SING*.

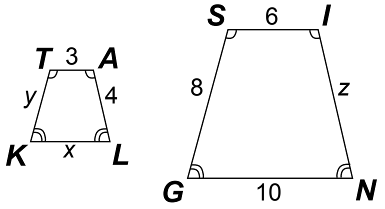
Note: ~ is the symbol for similar.

To find the values of x, y, and z, we must first find a pair of corresponding sides with lengths given.

- Side *TA* and side *SI* are a pair of corresponding sides.
- It is given that $TA = 3$ and $SI = 6$.
- So, we can set up a ratio $\frac{TA}{SI} = \frac{3}{6}$.
- When we reduce the ratio, we get the scale factor, which is $\frac{1}{2}$.
- This means that every length in *TALK* is one-half the **corresponding** length in *SING*.

Now we can use the scale factor to make proportions and find x, y, and z. Remember to be consistent as you set up the proportions. Since my scale factor was determined by a comparison of *TALK* to *SING*, I will continue in that order: ($\frac{TALK}{SING}$).

Trapezoids *TALK* and *SING*



$$\begin{array}{ccc}
\frac{1}{2} = \frac{x}{10} & \frac{1}{2} = \frac{y}{8} & \frac{1}{2} = \frac{4}{z} \\
2x = 10 & 2y = 8 & 8 = 1z \\
x = 5 & y = 4 & 8 = z
\end{array}$$

What is the **perimeter (*P*)**, or distance around the *polygon*, of *TALK*?

Did you get 16?

Can you guess the *perimeter* of *SING*?

If you guessed 32, you are correct.

Does it make sense that the perimeters should be in the same ratio as the scale factor?

Yes, because the perimeters of *TALK* and *SING* are corresponding lengths. In addition, all *corresponding* lengths in similar figures are in proportion!

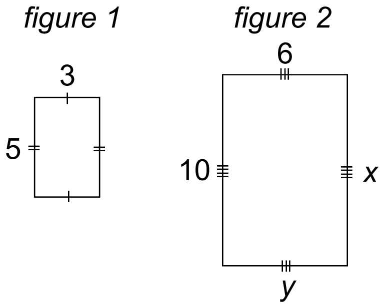
Practice

Find the following for each pair of **similar figures** *below.*

- *scale factor (SF)*
- $x =$
- $y =$
- P_1 = *perimeter of figure 1*
- P_2 = *perimeter of figure 2*

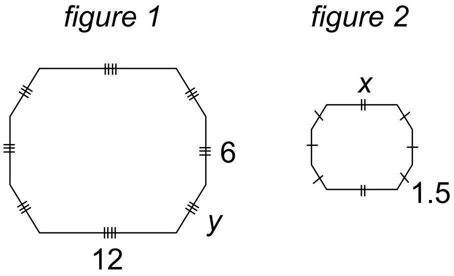
Refer to the previous pages as needed. **The first one has been done for you.**

1.



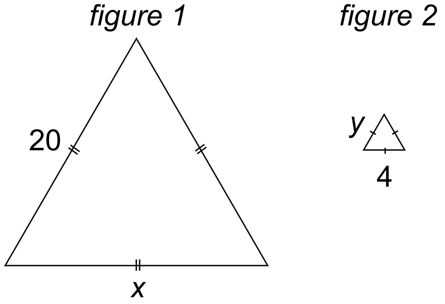
SF 1:2 ; x = 10 ; y = 6 P_1 16

2.



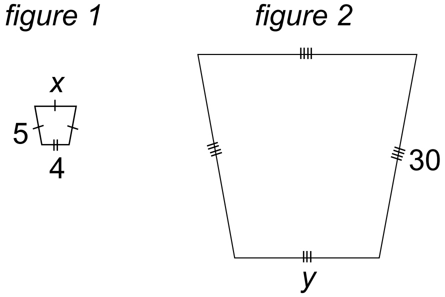
SF ________ ; x = ________ ; y = ________ P_1 ________

3.



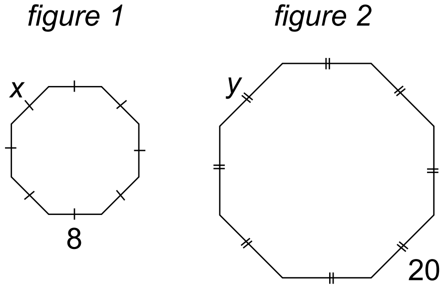
SF __________ ; x =__________ ; y = __________ P_2 __________

4.



SF __________ ; x =__________ ; y = __________ P_1 __________

5.



SF __________ ; x =__________ ; y = __________ P_1 __________

6.

figure 1

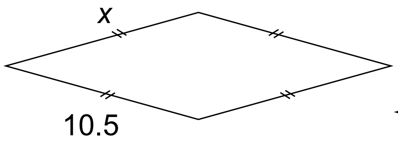
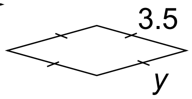


figure 2



SF ________ ; x = ________ ; y = ________ P_2 ________

7.

figure 1

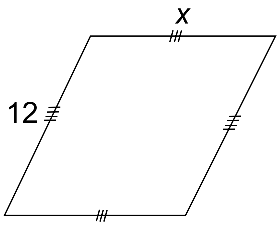
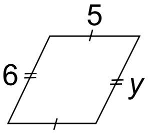


figure 2



SF ________ ; x = ________ ; y = ________ P_2 ________________

8.

figure 1

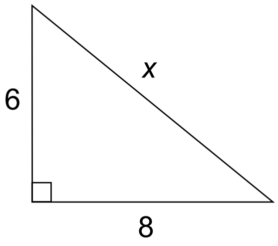
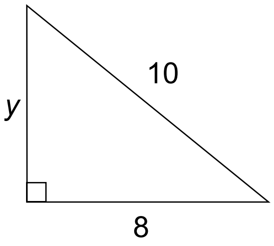


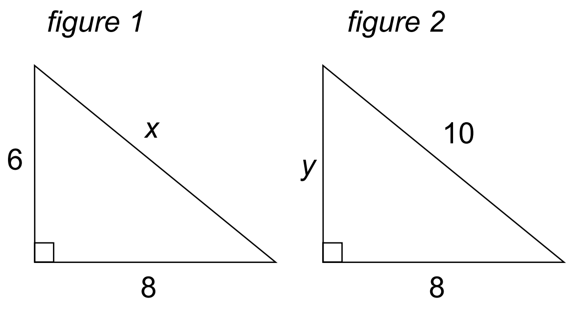
figure 2



SF ________ ; x = ________ ; y = ________ P_1 ________

Using Proportions to Find Heights

Look at the figures below. They are from number 8 in the previous practice.



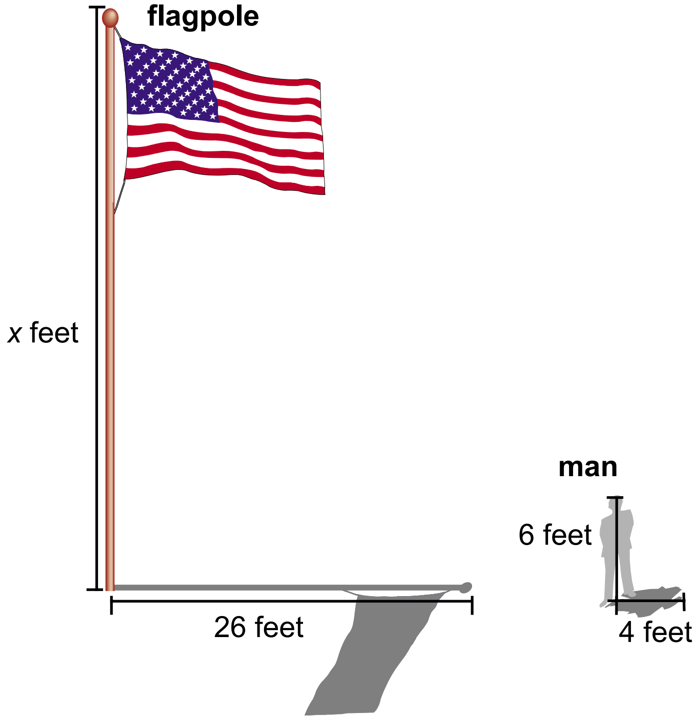
Here is what we know about *figure 1* and *figure 2* above.

- Their scale factor is $\frac{1}{1}$. This makes all the pairs of corresponding sides the same length.

- We already knew that their corresponding angles were the same measure because we knew that they were similar. This makes the triangles identical to each other.

Geometric figures that are *exactly* the same *shape* and *exactly* the same *size* are *congruent* to each other. The symbol for congruence, $\cong$, is a lot like the symbol for similar, but the equal sign, =, underneath it tells us that two things are *exactly* the same *size*.

We can use proportions to find the lengths of some items that would be difficult to measure. For instance, if we needed to know the height of a flagpole without having to inch our way up, we could use proportions. See the example on the following page.



A 6-foot man casts a 4-foot shadow at the same time a flagpole casts a 26-foot shadow. Find the **height (*h*)** of the flagpole.

To solve a problem like this, set up a proportion comparing corresponding parts.

$$\frac{\text{man's height}}{\text{man's shadow}} = \frac{\text{flagpole's height}}{\text{flagpole's shadow}}$$

$$\frac{6}{4} = \frac{x}{26}$$

$$4x = 6 \text{ x } 26$$

cross multiply

$$4x = 156$$

$$\frac{4x}{4} = \frac{156}{4}$$

divide both sides by 4

$$x = 39 \text{ feet}$$

Now try the following practice.

Practice

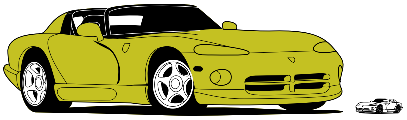
Use **proportions** *to solve the following. Refer to the previous pages as needed.*
Round to the nearest tenth. Show all your work.

1. A tree casts a 50-foot shadow at the same time a 4-foot fence post casts a 3-foot shadow. How tall is the tree?

   Answer: ____________ feet

2. If the scale factor for a miniature toy car and a real car is 1 to 32 and the windshield on the toy car is 2 inches long, how long is the windshield on the real car?

   Answer: ____________ inches



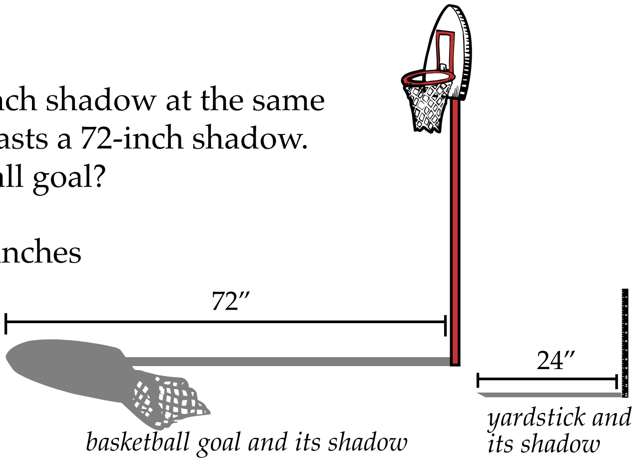
scale factor = $\frac{1}{32}$

3. The goal post on the football field casts an 18-foot shadow. The 4-foot water cooler casts a 5-foot shadow. How tall is the goal post?

Answer: ____________ feet

4. A yardstick casts a 24-inch shadow at the same time a basketball goal casts a 72-inch shadow. How tall is the basketball goal?

Answer: ____________ inches



5. A photo that is 4 inches by 6 inches needs to be enlarged so that the shorter sides are 6 inches. What will be the length of the enlargement?

Answer: ____________ inches

Practice

Use the list below to complete the following statements.

congruent (≅)
equiangular
equilateral
perimeter (*P*)
proportion
ratio
regular polygon
scale factor

1. A figure with all angles congruent is called

   ________________________ .

2. The comparison of two quantities is a ________________________ .

3. Figures or objects that are exactly the same shape and size are said to be ________________________ .

4. The ________________________ is the distance around a figure.

5. A figure with all sides congruent is called

   ________________________ .

6. A(n) ________________________ is a mathematical sentence stating that two ratios are equal.

7. The constant that is multiplied by the lengths of each side of a figure that produces an image that is the same shape as the original figure is the ________________________ .

8. A polygon that is both equilateral and equiangular is called a

   ________________________ .

Unit Review

Find the **value of the variable** *in the following. Check your answers.* **Show all your work.**

1. $\frac{x + 2}{x} = \frac{5}{3}$

2. $\frac{4}{3x + 1} = \frac{7}{5x - 2}$

3. $\frac{3x}{x + 7} = \frac{2}{3}$

4. $\frac{9x - 1}{7} = \frac{3x - 11}{2}$

Use **proportions** *to solve the following. Check your answers.* **Show all your work.**

5. The ratio of two integers is 9:7. The smaller integer is 448. Find the *larger* integer.

   Answer: ____________

6. The ratio of two integers is 6:11. The larger integer is 88. Find the *smaller* integer.

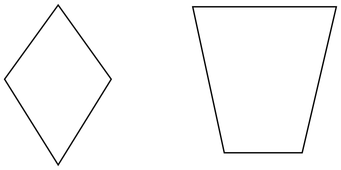
   Answer: ____________

7. The cost of 24 pounds of rice is $35. At the same rate, what would 5 pounds of rice cost? **Round to the nearest whole cent.**

   Answer: $ ____________

Look at each **pair of figures** *below. Determine if they are* **similar to each other***. Write* **yes** *if they are similar. Write* **no** *if they are* not *similar.*

____________ 8.



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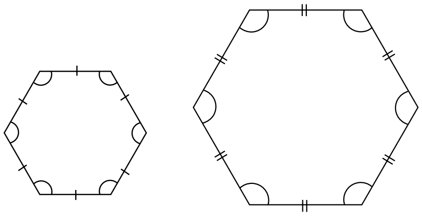
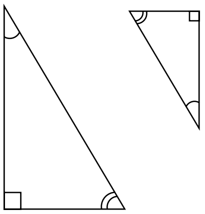
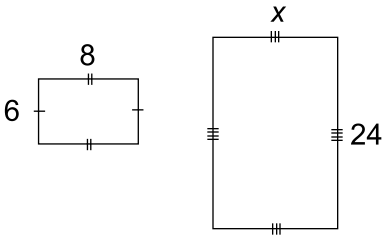


____________ 10.



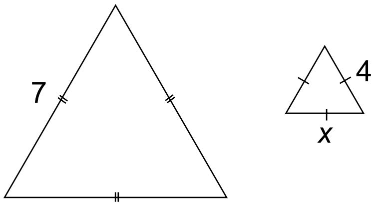
Each **pair of figures** *below is* **similar**. *Find the* **scale factor** *and* **value of the variable**.

11.



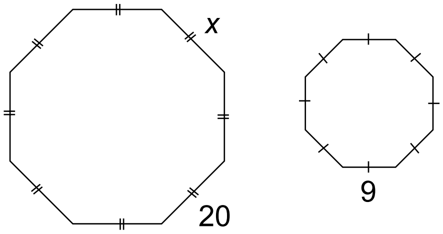
SF = ______________ ; x = ______________

12.



SF = ______________ ; x = ______________

13.

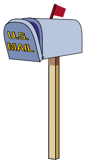


SF = ______________ ; x = ______________

Use **proportions** *to solve the following.* **Show all your work.**

14. A tree casts a 40-foot shadow at the same time a 6-foot post casts an 8-foot shadow. How tall is the tree?

Answer: ____________ feet



15. A 3.5-foot-tall mailbox casts a shadow of 5 feet at the same time a light pole casts a 20-foot shadow. How tall is the light pole?

Answer: ____________ feet

Unit 7: Exploring Relationships with Venn Diagrams

This unit introduces the concept of set theory and operations involving sets. It will also explore the relationship between sets and Venn diagrams, in addition to using set theory to solve problems.

Unit Focus

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Algebra Body of Knowledge

Standard 7: Quadratic Equations

- MA.912.A.7.1
  Graph quadratic equations with and without graphing technology.

- MA.912.A.7.2
  Solve quadratic equations over the real numbers by factoring, and by using the quadratic formula.
- MA.912.A.7.8
  Use quadratic equations to solve real-world problems.
- MA.912.A.7.10
  Use graphing technology to find approximate solutions of quadratic equations.

Discrete Mathematics Body of Knowledge

Standard 7: Set Theory

- MA.912.D.7.1
  Perform set operations such as union and intersection, complement, and cross product.
- MA.912.D.7.2
  Use Venn diagrams to explore relationships and patterns, and to make arguments about relationships between sets.

Vocabulary

Use the vocabulary words and definitions below as a reference for this unit.

braces { }	grouping symbols used to express sets
Cartesian cross product	a set of ordered pairs found by taking the x-coordinate from one set and the y-coordinate from the second set
complement	the set of elements left over when the elements of one set are deleted from another
coordinate grid or **plane**	a two-dimensional network of horizontal and vertical lines that are parallel and evenly spaced; especially designed for locating points, displaying data, or drawing maps
counting numbers (natural numbers)	the numbers in the set {1, 2, 3, 4, 5, …}
element or **member**	one of the objects in a set
empty set or **null set (ø)**	a set with no elements or members
even integer	any integer divisible by 2; any integer with the digit 0, 2, 4, 6, or 8 in the units place; any integer in the set {… , -4, -2, 0, 2, 4, …}
expression	a mathematical phrase or part of a number sentence that combines numbers, operation signs, and sometimes variables *Examples*: $4r^2$; $3x + 2y$; $\sqrt{25}$ An expression does *not* contain equal (=) or inequality (<, >, ≤, ≥, or ≠) signs.

finite set	a set in which a whole number can be used to represent its number of elements; a set that has bounds and is limited
infinite set	a set that is not finite; a set that has no boundaries and no limits
integers	the numbers in the set $\{\ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$
intersection ($\cap$)	those elements that two or more sets have in common
member or **element**	one of the objects in a set
natural numbers (counting numbers)	the numbers in the set $\{1, 2, 3, 4, 5, \ldots\}$
null set (ø) or **empty set**	a set with no elements or members
ordered pair	the location of a single point on a rectangular coordinate system where the first and second values represent the position relative to the x-axis and y-axis, respectively *Examples*: (x, y) *or* $(3, -4)$
pattern (relationship)	a predictable or prescribed sequence of numbers, objects, etc.; may be described or presented using manipulatives, tables, graphics (pictures or drawings), or algebraic rules (functions) *Example*: 2, 5, 8, 11 … is a pattern. Each number in this sequence is three more than the preceding number. Any number in this sequence can be described by the algebraic rule, $3n - 1$, by using the set of counting numbers for n.

point	a specific location in space that has no discernable length or width
positive integers	integers greater than zero
relation	a set of ordered pairs (x, y)
roster	a list of all the elements in a set
rule	a description of the elements in a set
set	a collection of distinct objects or numbers
union ($\cup$)	combination of the elements in two or more sets
Venn diagram	overlapping circles used to illustrate relationships among sets
x-coordinate	the first number of an ordered pair
y-coordinate	the second number of an ordered pair

Unit 7: Exploring Relationships with Venn Diagrams

Introduction

We will become more familiar with Venn diagrams as a mathematical tool while learning to use operations relative to set theory.

Lesson One Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Discrete Mathematics Body of Knowledge

Standard 7: Set Theory

- MA.912.D.7.1
  Perform set operations such as union and intersection, complement, and cross product.

Sets

Unit 1 discussed **sets**. A *set* is a collection of distinct objects or numbers. Each item in the set is called an **element** or **member** of the set. Sets are indicated by grouping symbols called **braces { }**.

A set can have a few *elements*, lots of elements, or *no* elements—called a **null set (ø)** or **empty set**. Sets like the **counting numbers**, also called the **natural numbers**—{1, 2, 3, 4, 5, …}—are **infinite sets** because they continue in the **pattern** and *never* end. *Patterns* are predictable. They have a prescribed sequence of numbers or objects.

Other sets with a *specified* number of elements are called **finite sets**. Some *finite sets* are *very* large; however, even very large sets with bounds and limits are finite sets.

Sets can usually be written in two different ways. One way is by **roster**. A *roster* is a list. You have probably heard of a football roster—a list of players on the team—or a class roster—a list of students in the class. Look at this set expressed in roster format.

{red, orange, yellow, blue, green, indigo, violet}

We could also name this set using the **rule** format. That means describing the set.

{the colors in the rainbow}

This is another way to indicate the set of colors listed above. So you see, there are two ways to express the same set.

Let’s look at some more examples.

{the set of vowels in the alphabet} means {a, e, i, o, u}

{2, 4, 6, 8, …} is the same as {the set of positive **even integers**}

Remember: Integers are the numbers in the set {… , -4, -3, -2, -1, 0, 1, 2, 3, 4, …} and **positive integers** are integers greater than zero.

Practice

Use the list below to complete the following statements.

braces
element or **member**
finite
infinite
null (ø) or **empty**
pattern
roster
rule

1. The set of counting numbers is ____________________ because it has *no boundaries*.

2. A set with *no elements* is called a(n) ____________________ set.

3. *Each item* in the set is called a(n) ____________________ of the set.

4. The *grouping symbols* used to indicate sets are called ____________________ .

5. A set whose elements are *described* is in ____________________ format.

6. A set with a *specified* number of elements, and a whole number can be used to represent its number of elements, is a ____________________ set.

7. A *list* of all the elements in a set, like the list of students in one class, is in ____________________ format.

8. A ____________________ is predictable, or it has a prescribed sequence of numbers or objects.

Practice

Express the following as sets in **roster format**.

1. integers *greater than* 3 and *less than* 11

   ____

2. counting numbers *less than* 6

   ____

3. colors in the American flag

   ____

4. planets in the solar system

   ____

5. courses on your schedule

   ____

Express the following as sets in **rule format**.

6. breakfast, lunch, dinner

   ____

7. Chevrolet, Ford, Chrysler, Buick

   ____

8. fork, spoon, knife, plate, glass

   ____

9. shoulder, wrist, elbow, hand, finger

____

10. table of contents, chapter, glossary, index, page

____

Lesson Two Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Discrete Mathematics Body of Knowledge

Standard 7: Set Theory

- MA.912.D.7.1
  Perform set operations such as union and intersection, complement, and cross product.
- MA.912.D.7.2
  Use Venn diagrams to explore relationships and patterns, and to make arguments about relationships between sets.

Unions and Intersections

When you combine all the elements in one set with all the elements in another set, we call this the **union ($\cup$)**. A *union* is like a “marriage” of elements in a set. The symbol for union looks a bit like the letter “u.”

A problem involving a union looks like the following.

$\{2, 3, 4, 5\} \cup \{2, 4, 6, 8\}$

This means that you should combine everything in the first set with all new elements from the second set.

$\{2, 3, 4, 5\} \cup \{2, 4, 6, 8\} = \{2, 3, 4, 5, 6, 8\}$

Look at other examples.

Example 1

$\{6, 7, 8, 10\} \cup \{5, 7, 8, 9\} = \{5, 6, 7, 8, 9, 10\}$

Note: You do *not* repeat any element even though it may have been in both sets.

Example 2

$\{\ldots, -3, -2, -1, 0\} \cup \{0, 1, 2, 3, \ldots\} = \{\text{the integers}\}$

This result can be expressed in rule format.

Example 3

$\{5, 7, 9, 11\} \cup \{\ \} = \{5, 7, 9, 11\}$

The *empty set* had nothing to add, so the answer is the same as the first set.

The intersection of two streets is the place where the streets cross each other. The intersection of two lines is also the **point** where they cross, or the *point(s)* they have in common. Likewise, when we take the intersection of two sets, we take only those elements that the two sets have in common. The symbol for **intersection ($\cap$)** looks like an upside-down union symbol.

An *intersection* problem would look like the following.

$\{2, 3, 4, 5\} \cap \{2, 4, 6, 8\}$

This means that you should include only those elements that the sets have in common.

$\{2, 3, 4, 5\} \cap \{2, 4, 6, 8\} = \{2, 4\}$

Look at these examples.

Example 1

$\{6, 7, 8, 10\} \cap \{5, 7, 8, 9\} = \{7, 8\}$

Note: The only elements that appears in both sets are 7 and 8.

Example 2

$\{\ldots, -3, -2, -1, 0\} \cap \{0, 1, 2, 3, \ldots\} = \{0\}$

The only element the sets have in common is 0.

Example 3

$\{5, 7, 9, 11\} \cap \{\ \} = \{\ \}$

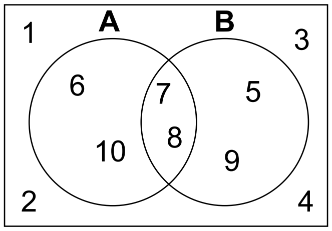
Since the empty set has no elements, it cannot have any elements in common with another set.

We can also use **Venn diagrams** to illustrate the union and intersection of sets. Unit 1 had a *Venn diagram* showing the relationships between sets of numbers.

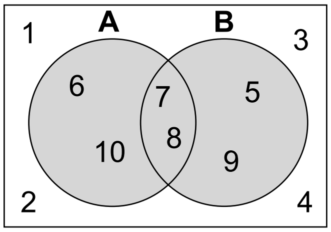
Example 1

Look at the examples below. The sets illustrated are using Venn diagrams.

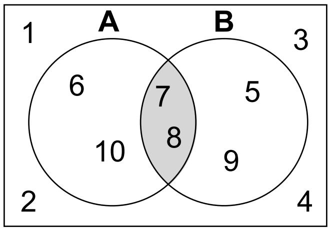
Set A = {6, 7, 8, 10} and set B = {5, 7, 8, 9}



The union of A and B ($A \cup B$) is both circles. Notice that there are numbers outside of set A and set B. Those are *not* part of the union *or* intersection.



The intersection of A and B ($A \cap B$) is only the football shape in the middle where the numbers that A and B have in common are located.



Look at these examples as well.

Example 2

$A = \{\ldots, -3, -2, -1, 0\}$, $B = \{0, 1, 2, 3, \ldots\}$

$A \cup B = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

$A \cap B = \{0\}$

Example 3

$A = \{5, 7, 9, 11\}$, $B = \{\ \}$

$A \cup B = \{5, 7, 9, 11\}$

$A \cap B = \{\ \}$

Your turn to try some.

Practice

Answer the following.

1. What is the union of {6, 7, 13} and {5, 6, 15}?

   ____

2. What is the union of {6, 7, 10} and {2}?

   ____

3. What is the union of {5, 7, 9} and {3, 7, 9}?

   ____

4. $\{2, 3, 4\} \cup \{1, 3, 5, 7\}$

   ____

5. $\{2, 4, 6, 8\} \cup \{1, 3, 5, 7\}$

   ____

6. $\{\ \} \cup \{5, 12, 15\}$

   ____

7. $\{1, 2, 3, 4\} \cup \{\ \}$

   ____

8. $\{1, 2, 3, \ldots, 10\} \cup \{2, 4, 6, 8, 10\}$

   ____

9. What is the intersection of {2, 8} and {1, 3, 9, 13}?

____

10. What is the intersection of {6, 7, 13,} and {5, 6, 15}?

____

11. What is the intersection of {3, 5, 9} and {3, 6, 9}?

____

12. $\{2, 3, 4\} \cap \{1, 3, 5, 7\}$

____

13. $\{2, 4, 6, 8\} \cap \{1, 3, 5, 7\}$

____

14. $\{\ \} \cap \{5, 12, 15\}$

____

15. $\{1, 2, 3, 4\} \cap \{\ \}$

____

16. $\{1, 2, 3, \ldots, 10\} \cap \{2, 4, 6, 8, 10\}$

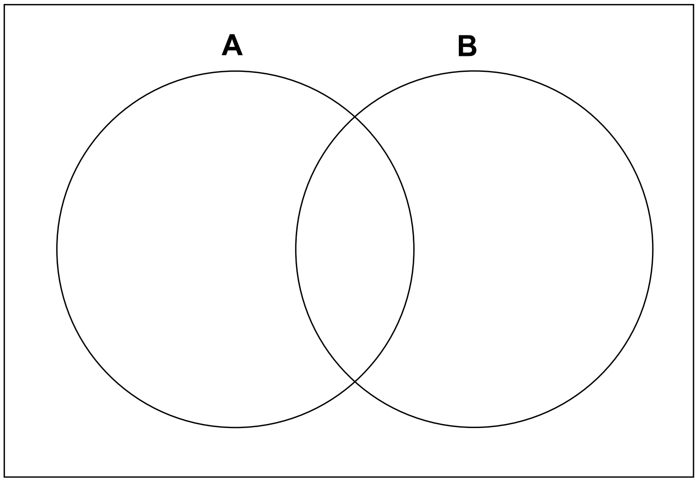
____

Practice

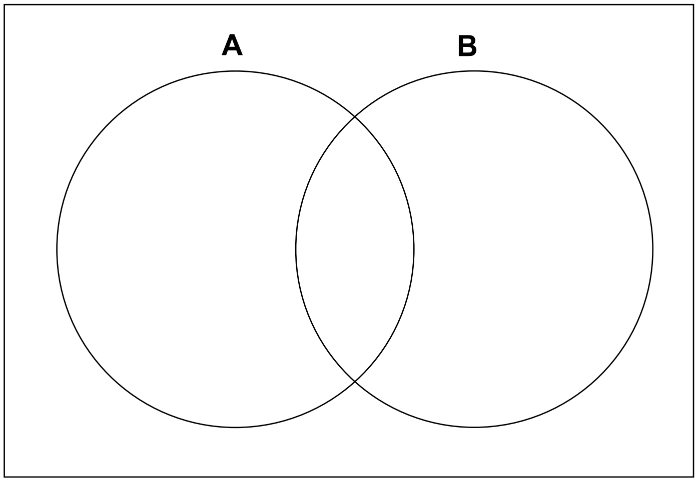
Use the **Venn diagrams** *below to illustrate the following* **sets**.

1. A = {1, 2, 3, 4, 5, 6}
   B = {4, 5, 6, 7, 8}

   a. A ∪ B

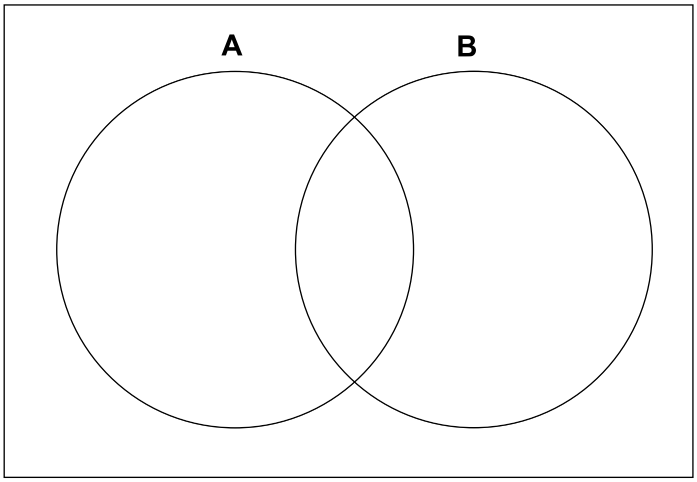


   b. A ∩ B

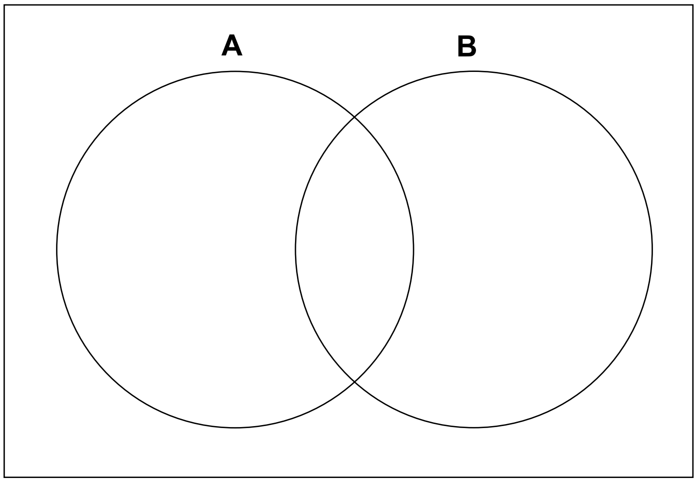


2. A = {1, 2, 3, 4, 5, 6, 7, 8}
   B = {2, 4, 6, 8, 10}

   a. $A \cup B$

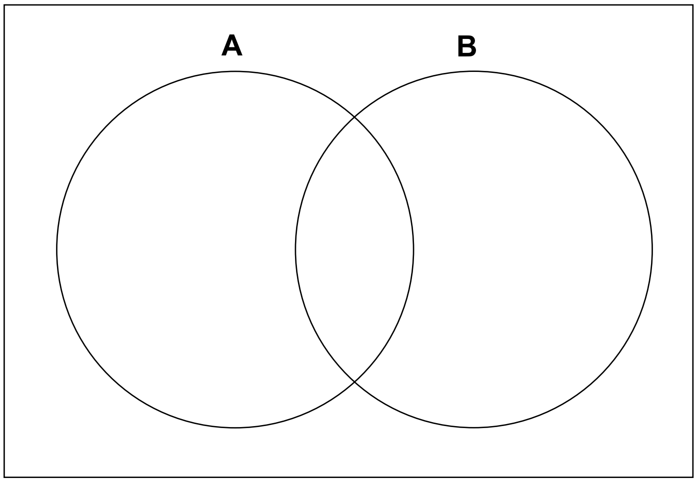


   b. $A \cap B$

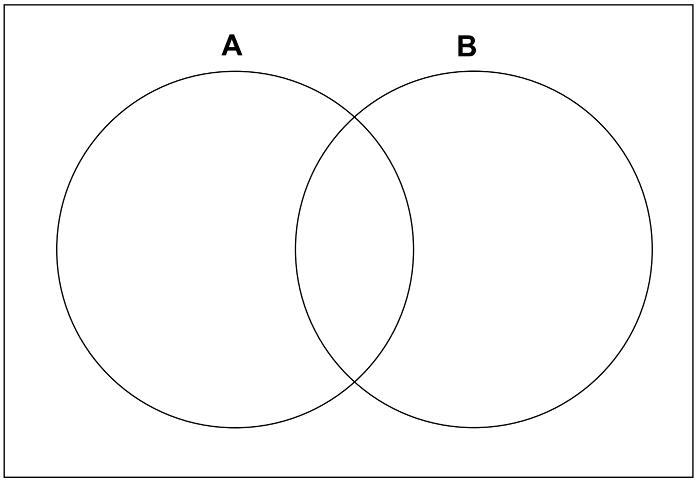


3. A = { }
B = {2, 4, 6, 8, 10}

a. $A \cup B$

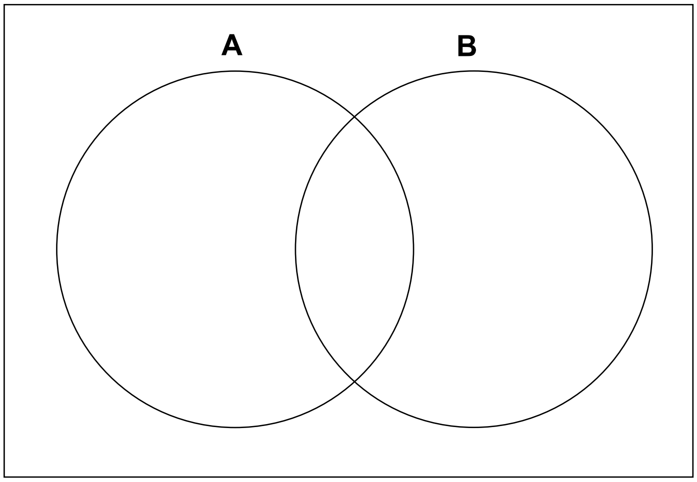


b. $A \cap B$

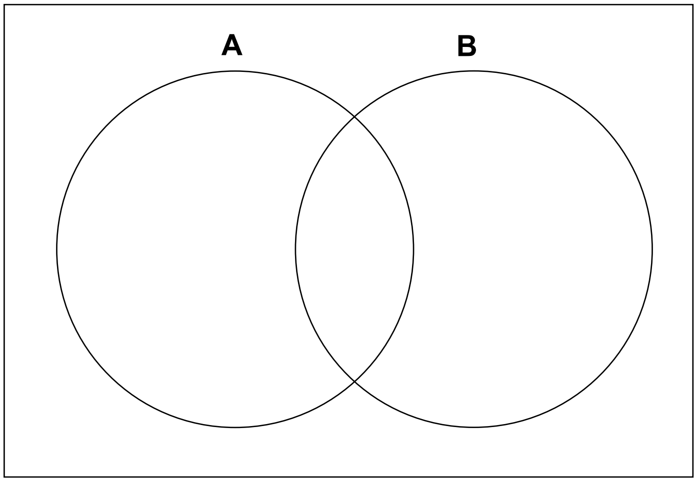


4. $A = \{3, 6, 9, 12, 15\}$
$B = \{2, 4, 6, 8, 10, 12\}$

a. $A \cup B$



b. $A \cap B$



Lesson Three Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.

- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.

- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Discrete Mathematics Body of Knowledge

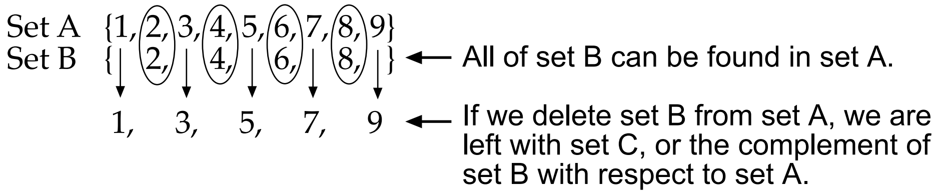
Standard 7: Set Theory

- MA.912.D.7.1
  Perform set operations such as union and intersection, complement, and cross product.

- MA.912.D.7.2
  Use Venn diagrams to explore relationships and patterns, and to make arguments about relationships between sets.

Complements

Look at set A {1, 2, 3, 4, 5, 6, 7, 8, 9} and set B {2, 4, 6, 8}. Do you see that all of the elements from set B can be found in set A? If we delete set B from set A we are left with the elements 1, 3, 5, 7, 9.



All of set B can be found in set A.

If we delete set B from set A, we are left with set C, or the complement of set B with respect to set A.

We could place these in a set and call it by another name, perhaps set C. We call set C the **complement** of set B with respect to set A. In other words, when we delete the elements of set B from set A we end up with set C.

Let's look at another example.

With respect to set R {red, orange, yellow, green, blue, indigo, violet}, find the *complement* of set S {red, yellow, blue}. We would delete red, yellow, and blue from set R and end up with a new set T {orange, green, indigo, violet}.

In symbols, this example looks like the following.

$$R - S = T.$$

The symbol for complement looks like a minus sign (–).

Practice

Answer the following.

1. With respect to A {1, 3, 6, 9, 12, 15, 18} find the complement of B {3, 12, 15}.

   ____

2. With respect to A {1, 3, 6, 9, 12, 15, 18} find the complement of C {6, 12, 18}.

   ____

3. {2, 4, 6, 7, 8} – {2, 6, 8}

   ____

4. {2, 4, 6, 8, 9, 13, 14, 16} – {6, 9, 14}

   ____

5. {integers} – {odd integers}

   ____

6. {letters of the alphabet} – {vowels}

   ____

7. Find the complement of {animals with four feet} with respect to {dogs, cats, fish, birds, mice, rabbits}.

   ____

8. {6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19} – {multiples of 3}

   ____

9. {integers} – {positive numbers}

____

10. {2, 4, 6, 8, 10} – {2, 4, 6, 8, 10}

____

11. {2, 4, 6, 8, 10} – { }

____

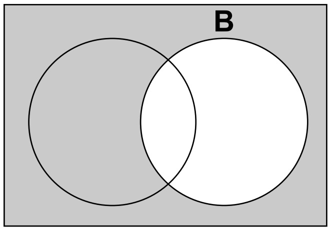
12. {students in your class} – {male students in your class}

____

Complements in Venn Diagrams

When talking about complements in Venn diagrams, we use a slightly different notation.

The figure below represents the complement of B.



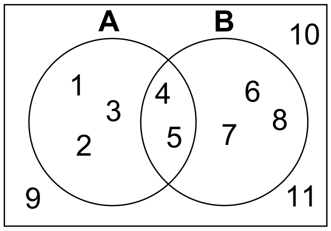
We use the symbol $\overline{B}$ to indicate that we are deleting all the elements of set B from the diagram and shading everything except what is in set B.

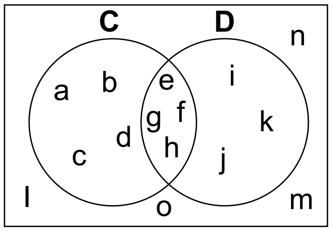
Practice

Use the **Venn diagrams** *below to give each set in* **roster format***.*

Remember: *Roster format* is a list of all the elements in a set.

Note: Elements listed outside the circles but inside the rectangles are part of the sets.





1. $\overline{A}$

____

2. $\overline{B}$

____

3. $\overline{C}$

____

4. $\overline{D}$

____

Lesson Four Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Discrete Mathematics Body of Knowledge

Standard 7: Set Theory

- MA.912.D.7.1
  Perform set operations such as union and intersection, complement, and cross product.

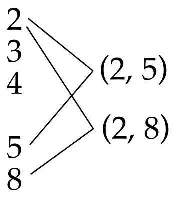
Cartesian Cross Products

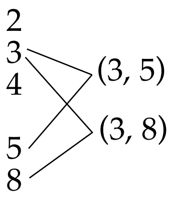
Another operation we can do with sets involves **Cartesian cross products**. A *Cartesian cross product* is a set of **ordered pairs** found by taking the ***x*-coordinate** from one set and the ***y*-coordinate** from the second set. The Cartesian coordinate system is named after the mathematician René Descartes (1596-1650). We use his work every time we graph on a **coordinate grid** or **plane**. Keeping that in mind, you will find it no surprise that Cartesian cross products have something to do with graphing.

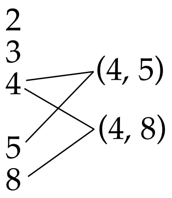
To find a Cartesian cross product we must have two sets.

Let's let A = {2, 3, 4} and B = {5, 8}.

The **expression** in symbols looks like A X B. The X almost looks like a large multiplication sign. However, don't be fooled. We are not going to multiply. We are going to create a **relation**, which is another name for a set of *ordered pairs*.







So, A X B = {(2, 5), (2, 8), (3, 5), (3, 8), (4, 5), (4, 8)}

Notice that in the newly created set, every element is an ordered pair (x, y). Also see that each number in the x position came from set A and each number in the y position came from set B.

Let's look at another one.

{3, 5} X {1, 2, 3} = {(3, 1), (3, 2), (3, 3), (5, 1), (5, 2), (5, 3)}

Notice that the resulting set is a *relation* because every element is an ordered pair.

It's time for you to try.

Practice

Answer the following.

1. {1, 2} X {4, 6}

   ____

2. {3, 4, 7, 8} X {2, 5}

   ____

3. {1, 5, 9} X {3, 6, 9}

   ____

4. {2, 4} X {1, 3}

   ____

5. {2, 4} X {2, 4}

   ____

6. {6, 8} X {4, 5, 7}

   ____

Practice

Use the list below to complete the following statements.

braces
element or **member**
finite
intersection (∩)
null (ø) or **empty set**
relation
roster
rule
set
union (∪)

1. The combining of the elements in two or more sets is called the __________________ of the sets.

2. A set of ordered pairs is called a __________________ .

3. A __________________ is a list of the elements in a set.

4. The set of elements that two or more sets have in common is called the __________________ of the sets.

5. A __________________ is a collection of distinct objects or numbers.

6. A description of the elements in a set is called a __________________ .

7. The symbols used to express a set are called __________________ .

8. A set with no elements is called a(n) __________________ .

9. An item in a set is called a(n) __________________ .

10. A set with a specified number of elements is called a __________________ set.

Lesson Five Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Discrete Mathematics Body of Knowledge

Standard 7: Set Theory

- MA.912.D.7.1
  Perform set operations such as union and intersection, complement, and cross product.

- MA.912.D.7.2
  Use Venn diagrams to explore relationships and patterns, and to make arguments about relationships between sets.

Using Venn Diagrams for Three Categories

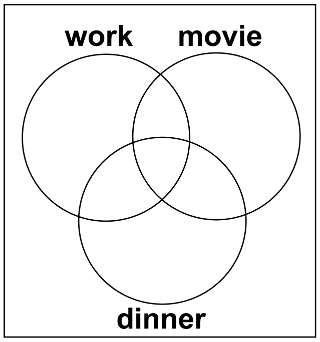
We can use Venn diagrams to solve problems that might otherwise seem impossible. Here is an example.

> A group of Leon High School seniors answered a questionnaire about their plans for the weekend. In the group, 20 planned to work, 19 planned to see a movie, while 28 were planning to go out to dinner. Exactly 7 seniors planned to do all three. Another 12 seniors were planning to do dinner only. There are 2 seniors who were going to work and go to a movie but not go out to dinner, and 15 seniors were going to work and go out to dinner.

Next, we will use Venn diagrams to answer the following.

1. How many seniors answered the questionnaire?
2. How many seniors were going to dinner and a movie, but not work?
3. How many seniors were going to dinner or a movie?
4. How many seniors were only planning to work?

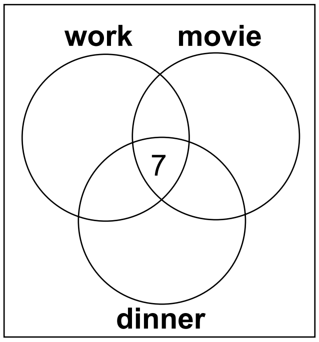
The first thing we will do is set up a Venn diagram for the *three* categories of plans. Notice that the three circles overlap.



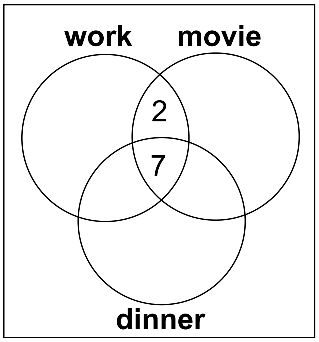
Then we read back through the statements to fill in the different sections of the diagram. Try to find the *middle* information first and then work your way to the outside.

Pay careful attention to the wording. When the word "and" is used that indicates the *intersection* of two sets. The word "or" means *union*.

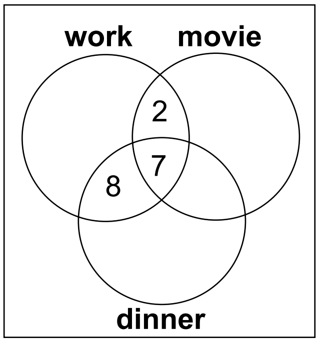
Let's fill this in step by step.



Exactly **7** seniors planned to do *all three activities*. The 7 goes in the *middle*.

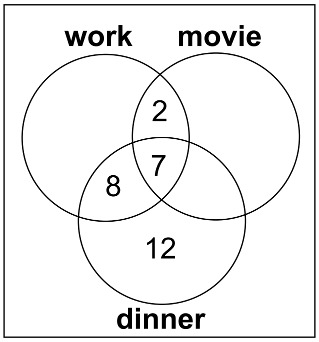


There are **2** seniors who were going to work *and* to a movie, but **not** dinner.



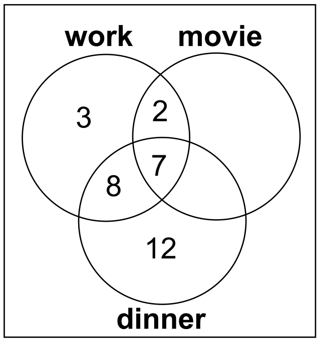
There are **15** who were going to work **and** going out to dinner. *And* means *intersection*, which means the football shape where work and dinner *overlap*.

Since that football shape already contains 7, we subtract 15 – 7 = **8** to fill in the rest of the football shape.



There are **12** who were planning to do dinner *only*. That means they are in the dinner circle but *not* in any of the *overlapping* parts.

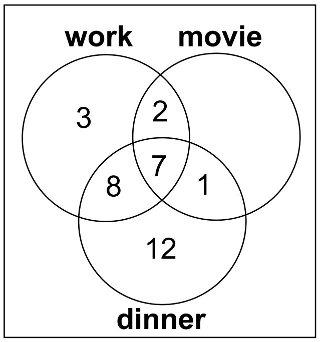
Now go back to the *broader clues*.



There are **20** who planned to work. This means the *entire work circle must contain 20 people*.

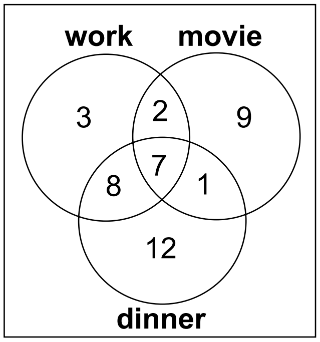
There are already 7 + 8 + 2 in the work circle.

So 20 – (7 + 8 + 2) = 20 – 17 = 3. There are **3** who are *only* going to work.



There are **19** who planned to see a *movie*, but there are two spaces for the rest of the students. So let's look at those who were going to dinner. There are **28** *dinner folks*. Our diagram shows 12 + 8 + 7 already in the dinner circle.

So the empty space in the dinner circle will be 28 – (12 + 8 + 7) = **1**.



Now we can go back to the *moviegoers*. There are **19** who planned to see a movie.

So 19 – (2 + 7 + 1) = **9**.

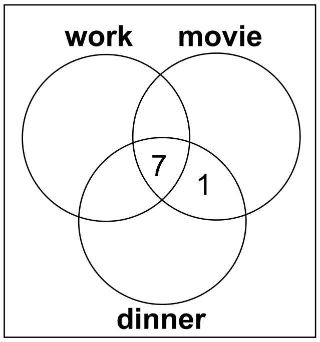
Now we have everything filled in and can answer the questions.

1. How many seniors answered the questionnaire?

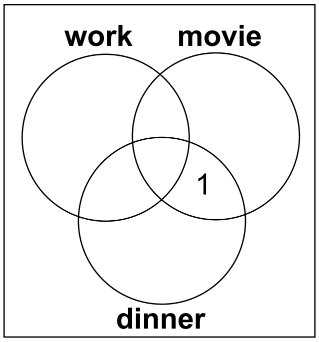
   Count each number in the diagram only once and add them together.

   3 + 2 + 9 + 8 + 7 + 1 + 12 = **42**

2. How many seniors were going to dinner and a movie, but *not* work?

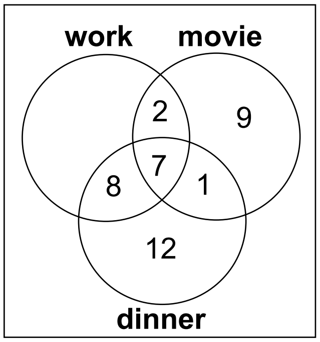


Dinner **and** movie = 7 + 1 = 8
(in football shape)



Delete those in the work part of the football shape
8 – 7 = **1**

3. How many seniors were going to dinner **or** a movie?

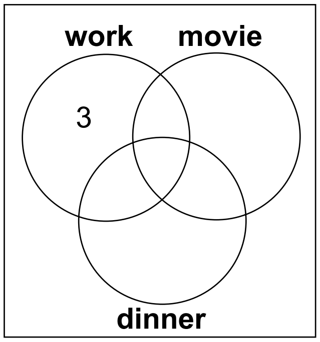


Dinner **or** a movie

Remember: This means union. Be careful *not* to count anyone twice!

2 + 9 + 8 + 7 + 1 + 12 = **39**

4. How many seniors were only planning to work?



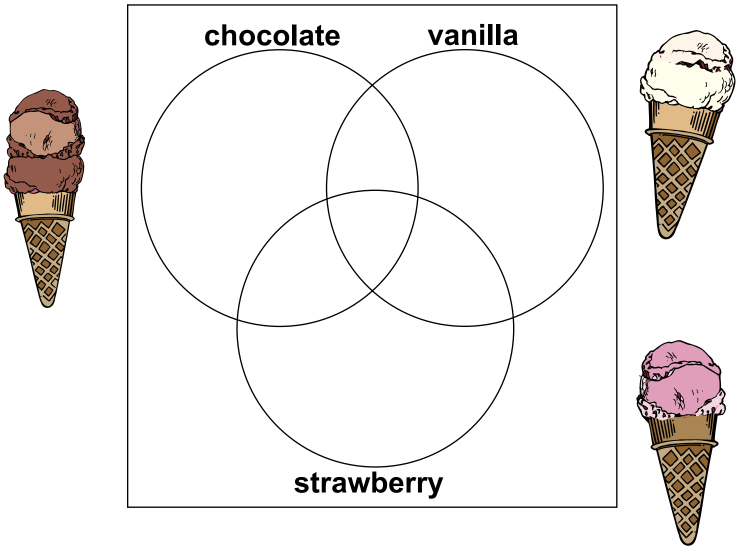
There were **3** students who were in the work circle *without* overlapping into the other circles.

So **3** students planned to work only.

Practice

Use the **Venn diagram** *below to answer the following.*

Jen and Berry's Ice Cream store had 110 customers yesterday. There were 62 customers who bought chocolate ice cream, 38 who chose vanilla, and 41 who chose strawberry ice cream. Another 13 chose chocolate and strawberry. Then 20 chose strawberry *only*, 16 chose chocolate and vanilla, and 7 chose *all three*.



1. How many bought *no* ice cream?

2. How many chose vanilla *only*?

3. How many chose chocolate *or* vanilla or both?

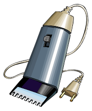
4. How many chose strawberry *or* vanilla or both, but *not* chocolate?

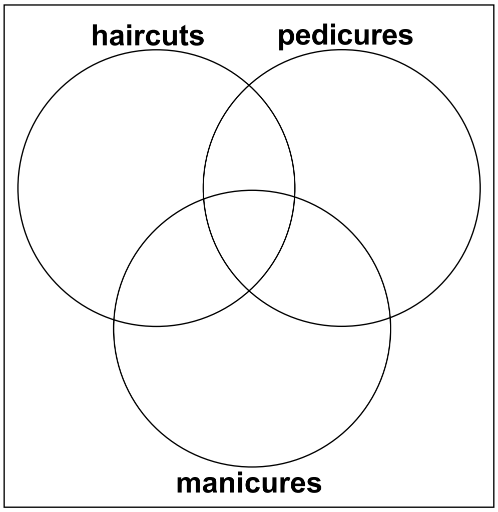
Practice

Use the **Venn diagram** *below to answer the following.*

Svetlana owns a day spa. At the end of the day, the tabulation indicated that clients visited for the following reasons: Haircuts, 62; pedicures, 28; manicures, 41; *all three*, 5; haircut *and* pedicure, 13; manicure *and* haircut *only*, 6; manicure *only*, 25.

Note: Assume everyone who visited the spa had one of the procedures.





1. How many had manicures *and* pedicures?

____

2. How many had manicures *or* pedicures or both?

____

3. How many had haircuts *and* manicures?

____

4. How many had haircuts *and* pedicures but *not* manicures?

____

5. How many had *only* a pedicure?

____

6. How many clients visited the salon on this day?

____

Unit Review

Answer the following.

1. Express the set of integers *greater than* 5 and *less than* 12 in *roster* format.

2. Express the set containing eyes, eyebrows, nose, mouth, and chin in *rule* format.

3. $\{6, 8, 10\} \cup \{10, 12, 14\}$

4. $\{6, 8, 10\} \cap \{10, 12, 14\}$

5. $\{1, 3, 6\} \cup \{1, 2, 3, 4\}$

6. $\{1, 3, 6\} \cap \{1, 2, 3, 4\}$

7. $\{5, 7, 8\} \cup \{\ \}$

8. $\{5, 7, 8\} \cap \{\ \}$

9. $\{1, 2, 3, 4\} \cup \{5, 6, 7, 8\}$

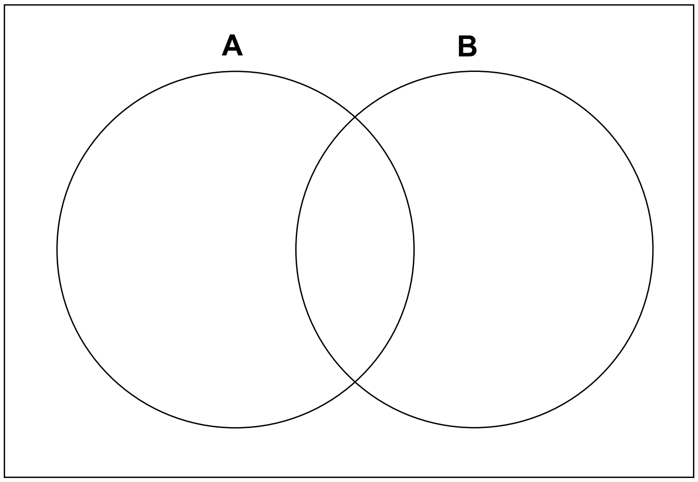
10. {1, 2, 3, 4} $\cap$ {5, 6, 7, 8}

____

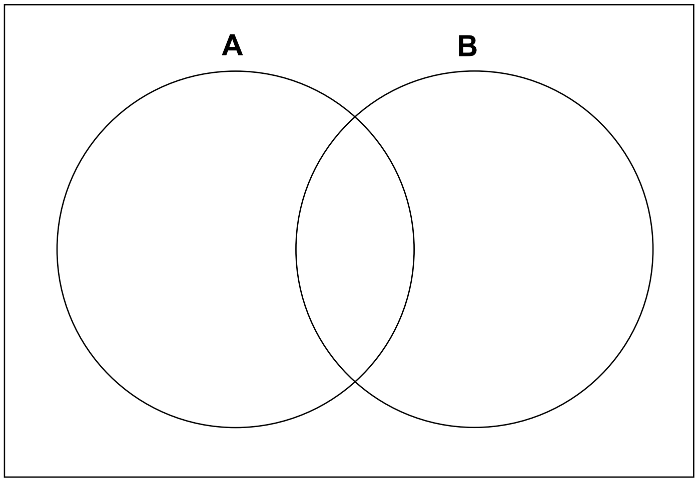
Use the **Venn diagrams** *below to illustrate the following* **sets**.

A = {2, 4, 6, 9, 12}   B = {2, 4, 5, 6, 7, 8, 10}

11. A $\cup$ B



12. A $\cap$ B



Answer the following.

13. With respect to A {5, 10, 15, 20, 25, 30} find the complement of B {10, 20, 30}

____

14. {2, 4, 6, 8, 10} – {2, 4}

____

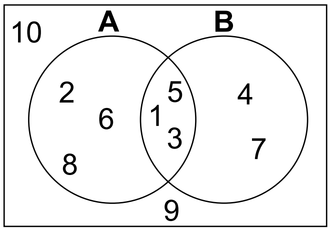
15. {1, 2, 3, 4, 5} – { }

____

16. {1, 2, 3, 4} – {1, 2, 3, 4}

____

17. Use the Venn diagram below to give a set in roster format for $\overline{A}$.



____

Answer the following.

18. {3, 5} X {2, 4, 6}

____

19. {1, 2, 5} X {0, 4}

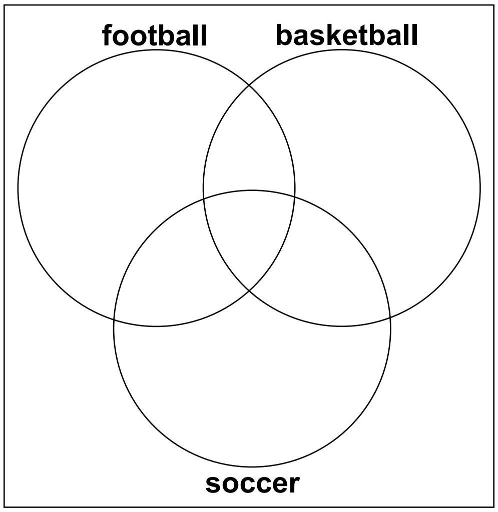
____

20. {4, 8} X {2, 6, 8}

____

Use the **Venn diagram** *below to answer the following.*

Of 74 boys in a school, the numbers out for a sport or sports were as follows: football, 48; basketball, 20; soccer, 30; football and soccer, 10; basketball and football, 11; soccer and basketball, 8; all three, 3.



21. How many were not out for any sport?

____

22. How many were out for football but not soccer?

____

23. How many were out for soccer and basketball but not football?

____

24. How many play basketball only?

____

25. How many play football or soccer?

____

Unit 8: Is There a Point to This?

This unit uses algebraic concepts along with the rules related to radical expressions to explore the coordinate plane.

Unit Focus

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Algebra Body of Knowledge

Standard 1: Real and Complex Number Systems

- MA.912.A.1.8
  Use the zero product property of real numbers in a variety of contexts to identify solutions to equations.

Standard 3: Linear Equations and Inequalities

- MA.912.A.3.7
  Rewrite equations of a line into slope-intercept form and standard form.

- MA.912.A.3.8
  Graph a line given any of the following information: a table of values, the x- and y-intercepts, two points, the slope and a point, the equation of the line in slope-intercept form, standard form, or point-slope form.

- MA.912.A.3.9
  Determine the slope, x-intercept, and y-intercept of a line given its graph, its equation, or two points on the line.

- MA.912.A.3.10
  Write an equation of a line given any of the following information: two points on the line, its slope and one point on the line, or its graph. Also, find an equation of a new line parallel to a given line, or perpendicular to a given line, through a given point on the new line.

Standard 5: Rational Expressions and Equations

- MA.912.A.5.1
  Simplify algebraic ratios.

Standard 6: Radical Expressions and Equations

- MA.912.A.6.1
  Simplify radical expressions.

- MA.912.A.6.2
  Add, subtract, multiply and divide radical expressions (square roots and higher).

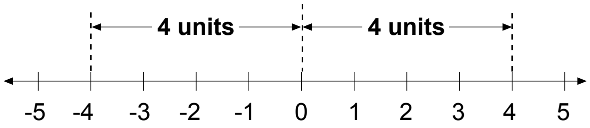
Geometry Body of Knowledge

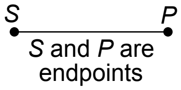
Standard 1: Points, Lines, Angles, and Planes

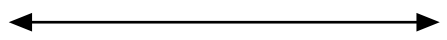
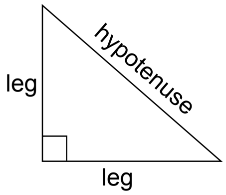
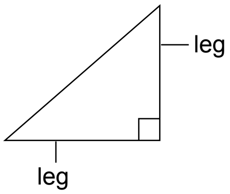
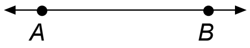
- MA.912.G.1.4
  Use coordinate geometry to find slopes, parallel lines, perpendicular lines, and equations of lines.

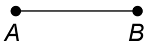
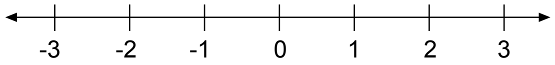
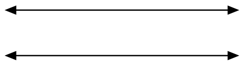
Vocabulary

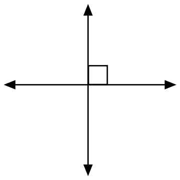
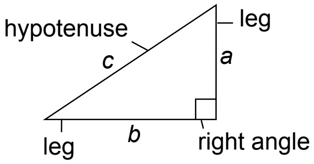
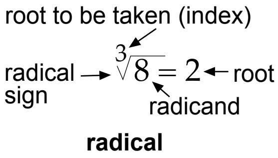
Use the vocabulary words and definitions below as a reference for this unit.

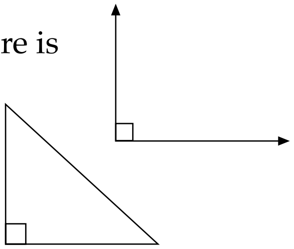
absolute value	a number's distance from zero (0) on a number line; distance expressed as a positive value *Example*: The absolute value of both 4, written $\lvert 4 \rvert$, and negative 4, written $\lvert -4 \rvert$, equals 4. 
common denominator	a common multiple of two or more denominators *Example*: A common denominator for $\frac{1}{4}$ and $\frac{5}{6}$ is 12.
constant	a quantity that always stays the same
coordinate grid or **plane**	a two-dimensional network of horizontal and vertical lines that are parallel and evenly spaced; especially designed for locating points, displaying data, or drawing maps
coordinate plane	the plane containing the x- and y-axes
coordinates	numbers that correspond to points on a coordinate plane in the form (x, y), or a number that corresponds to a point on a number line
degree (°)	common unit used in measuring angles

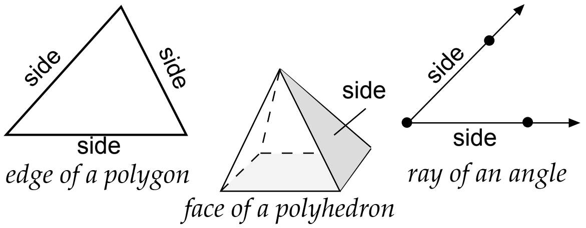
denominator	the bottom number of a fraction, indicating the number of equal parts a whole was divided into *Example*: In the fraction $\frac{2}{3}$ the denominator is 3, meaning the whole was divided into 3 equal parts.
distance	the length of a segment connecting two points
endpoint	either of two points marking the end of a line segment 
equation	a mathematical sentence stating that the two expressions have the same value *Example*: $2x = 10$
expression	a mathematical phrase or part of a number sentence that combines numbers, operation signs, and sometimes variables *Examples*: $4r^2$; $3x + 2y$; $\sqrt{25}$ An expression does *not* contain equal (=) or inequality ($<$, $>$, $\leq$, $\geq$, or $\neq$) signs.
factor	a number or expression that divides evenly into another number; one of the numbers multiplied to get a product *Examples*: 1, 2, 4, 5, 10, and 20 are factors of 20 and $(x + 1)$ is one of the factors of $(x^2 - 1)$.
formula	a way of expressing a relationship using variables or symbols that represent numbers

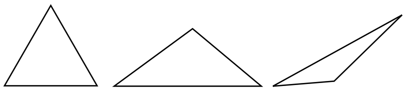
graph	a drawing used to represent data *Example*: bar graphs, double bar graphs, circle graphs, and line graphs
graph of a point	the point assigned to an ordered pair on a coordinate plane
horizontal	parallel to or in the same plane of the horizon 
hypotenuse	the longest side of a right triangle; the side opposite the right angle 
integers	the numbers in the set {… , -4, -3, -2, -1, 0, 1, 2, 3, 4, …}
intersect	to meet or cross at one point
leg	in a right triangle, one of the two sides that form the right angle 
length (*l*)	a one-dimensional measure that is the measurable property of line segments
line (↔)	a collection of an infinite number of points forming a straight path extending in opposite directions having unlimited length and no width 
linear equation	an algebraic equation in which the variable quantity or quantities are raised to the zero or first power and the graph is a straight line *Example*: $20 = 2(w + 4) + 2w$; $y = 3x + 4$

line segment (—)	a portion of a line that consists of two defined endpoints and all the points in between *Example*: The line segment *AB* is between point *A* and point *B* and includes point *A* and point *B*. 
midpoint (of a line segment)	the point on a line segment equidistant from the endpoints
negative integers	integers less than zero
negative numbers	numbers less than zero
number line	a line on which ordered numbers can be written or visualized 
numerator	the top number of a fraction, indicating the number of equal parts being considered *Example*: In the fraction $\frac{2}{3}$, the numerator is 2.
ordered pair	the location of a single point on a rectangular coordinate system where the first and second values represent the position relative to the *x*-axis and *y*-axis, respectively *Examples*: (x, y) *or* $(3, -4)$
parallel (||)	being an equal distance at every point so as to never intersect
parallel lines	two lines in the same plane that are a constant distance apart; lines with equal slopes 

perpendicular (⊥)	two lines, two line segments, or two planes that intersect to form a right angle
perpendicular lines	two lines that intersect to form right angles 
point	a specific location in space that has no discernable length or width
positive numbers	numbers greater than zero
product	the result of multiplying numbers together *Example*: In 6 x 8 = 48, the product is 48.
Pythagorean theorem	the square of the hypotenuse (c) of a right triangle is equal to the sum of the square of the legs (a and b), as shown in the equation $c^2 = a^2 + b^2$ 
radical	an expression that has a root (square root, cube root, etc.) *Example*: $\sqrt{25}$ is a radical Any root can be specified by an index number, b, in the form $\sqrt[b]{a}$ (e.g., $\sqrt[3]{8}$).  A radical without an index number is understood to be a square root.
radical expression	a numerical expression containing a radical sign *Examples*: $\sqrt{25}$ $2\sqrt{25}$

radical sign ($\sqrt{\ }$)	the symbol ($\sqrt{\ }$) used before a number to show that the number is a *radicand*
radicand	the number that appears within a radical sign *Example*: In $\sqrt{25}$, 25 is the radicand.
reciprocals	two numbers whose product is 1; also called *multiplicative inverses* *Examples:* 4 and $\frac{1}{4}$ are reciprocals because $\frac{4}{1}$ x $\frac{1}{4}$ = 1; $\frac{3}{4}$ and $\frac{4}{3}$ are reciprocals because $\frac{3}{4}$ x $\frac{4}{3}$ = 1; zero (0) has no multiplicative inverse
right angle	an angle whose measure is exactly 90° 
right triangle	a triangle with one right angle
rise	the vertical change on a graph between two points
root	an equal factor of a number *Examples*: In $\sqrt{144} = 12$, 12 is the square root. In $\sqrt[3]{125} = 5$, 5 is the cube root.
run	the horizontal change on a graph between two points

side	the edge of a polygon, the face of a polyhedron, or one of the rays that make up an angle *Example*: A triangle has three sides. 
simplest radical form	an expression under the radical sign that contains no perfect squares greater than 1, contains no fractions, and is not in the denominator of a fraction *Example*: $\sqrt{27} = \sqrt{9 \bullet 3} = \sqrt{9} \bullet \sqrt{3} = 3\sqrt{3}$
simplify a fraction	write fraction in lowest terms or simplest form
slope	the ratio of change in the vertical axis (y-axis) to each unit change in the horizontal axis (x-axis) in the form $\frac{\text{rise}}{\text{run}}$ or $\frac{\Delta y}{\Delta x}$; the constant, m, in the linear equation for the slope-intercept form $y = mx + b$
slope-intercept form	a form of a linear equation, $y = mx + b$, where m is the slope of the line and b is the y-intercept
square (of a number)	the result when a number is multiplied by itself or used as a factor twice *Example*: 25 is the square of 5.
square root	a positive real number that can be multiplied by itself to produce a given number *Example*: The square root of 144 is 12 or $\sqrt{144} = 12$.

standard form (of a linear equation)	$ax + by + c = 0$, where a, b, and c are integers and $a > 0$
sum	the result of adding numbers together *Example*: In $6 + 8 = 14$, the sum is 14.
triangle	a polygon with three sides 
value (of a variable)	any of the numbers represented by the variable
variable	any symbol, usually a letter, which could represent a number
vertical	at right angles to the horizon; straight up and down 
***x*-axis**	the horizontal number line on a rectangular coordinate system
***x*-coordinate**	the first number of an ordered pair
***x*-intercept**	the value of x at the point where a line or graph intersects the x-axis; the value of y is zero (0) at this point
***y*-axis**	the vertical number line on a rectangular coordinate system
***y*-coordinate**	the second number of an ordered pair
***y*-intercept**	the value of y at the point where a line or graph intersects the y-axis; the value of x is zero (0) at this point

Unit 8: Is There a Point to This?

Introduction

We will explore the relationships that exist between points, segments, and lines on a coordinate plane. Utilizing the formulas for finding distance, midpoint, slope, and equations of lines, we can identify the ways in which points and lines are related to each other.

Lesson One Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Algebra Body of Knowledge

Standard 6: Radical Expressions and Equations

- MA.912.A.6.1
  Simplify radical expressions.

- MA.912.A.6.2
  Add, subtract, multiply and divide radical expressions (square roots and higher).

Geometry Body of Knowledge

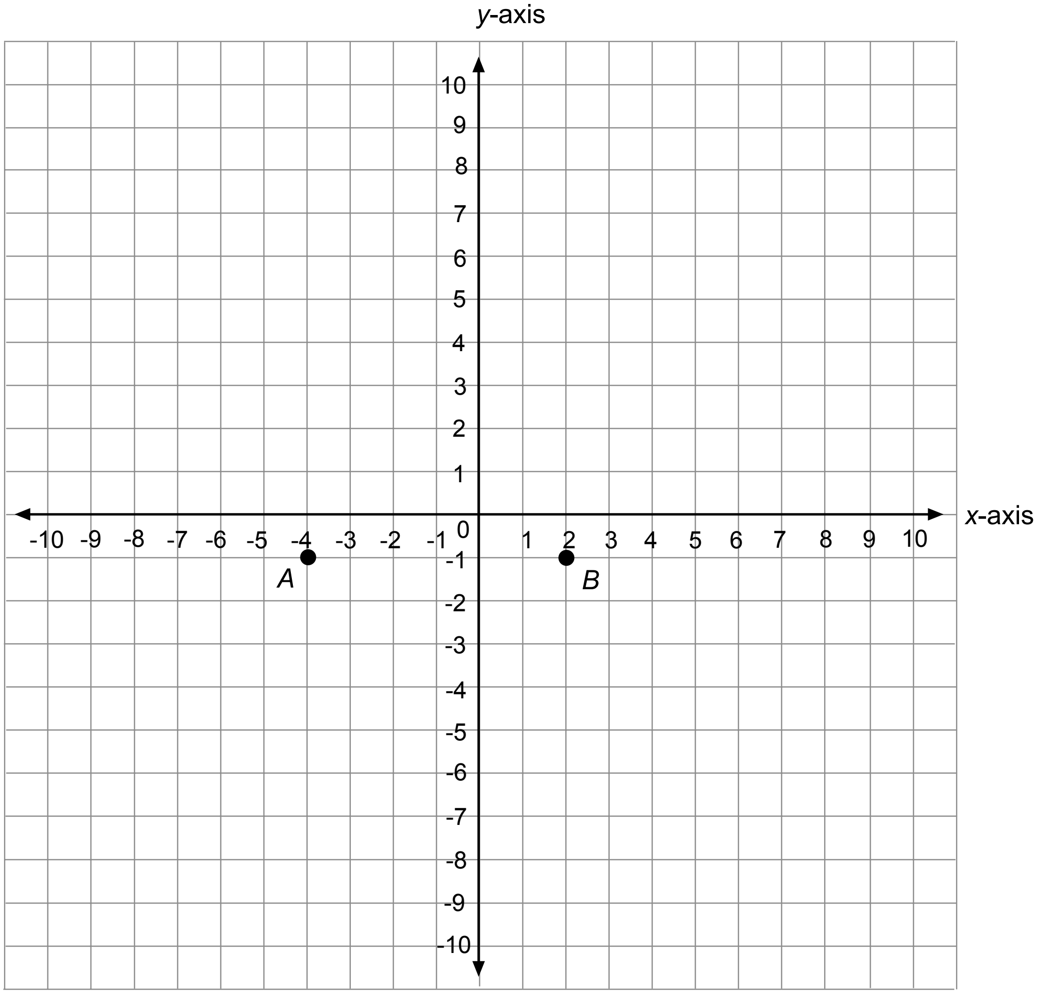
Standard 1: Points, Lines, Angles, and Planes

- MA.912.G.1.4
  Use coordinate geometry to find slopes, parallel lines, perpendicular lines, and equations of lines.

Distance

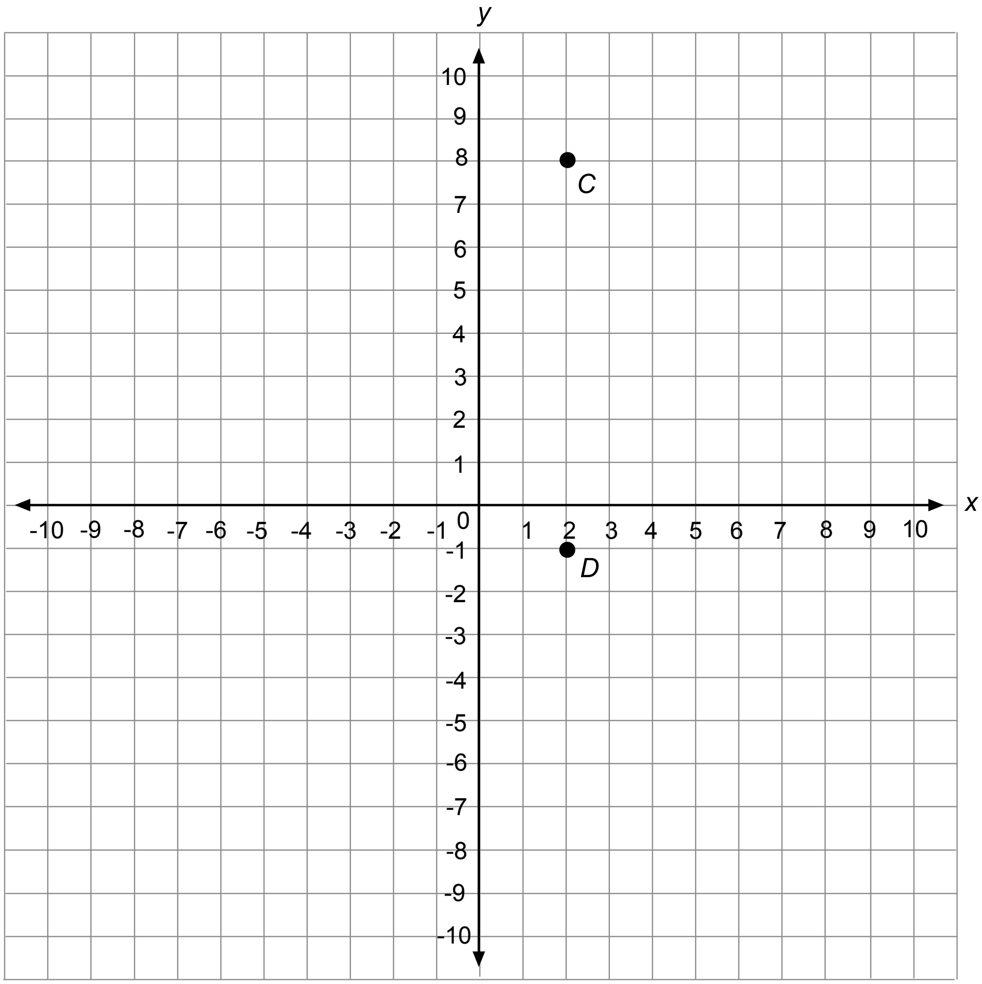
Look at the following **coordinate grids** or **planes**. The horizontal number line on a *rectangular coordinate system* is the ***x*-axis**. The vertical line on a coordinate system is the ***y*-axis**. We can easily find the **distance** between the given **graphs of the points** below. The *graph of a point* is the **point** assigned to an **ordered pair** on a *coordinate plane*.

Graph of Points *A* and *B*



Because the *points* on the **graph** above are on the same **horizontal (⟷) line**, we can count the spaces from one *point* to the other. So, the *distance* from *A* to *B* is 6.

Graph of Points *C* and *D*



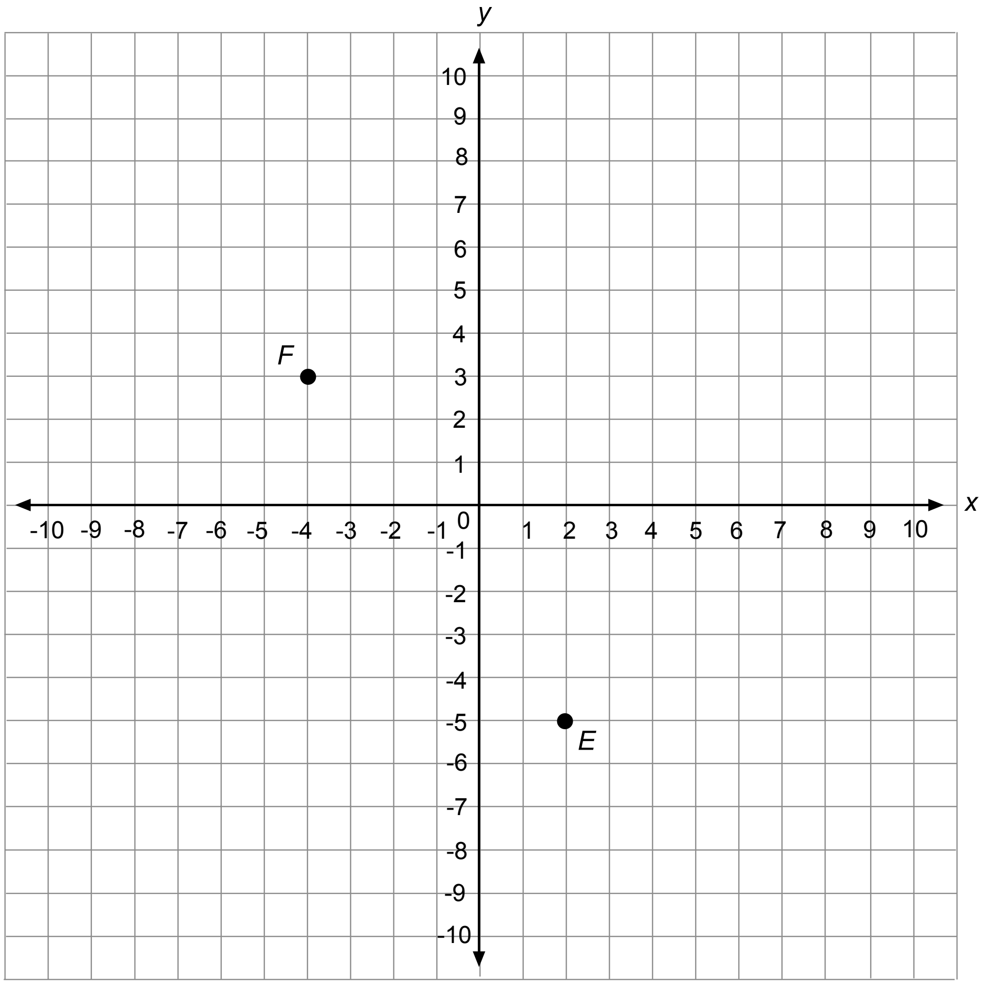
Because the points on the graph above are on the same **vertical (↕)** *line*, we can count the spaces from one point to the other. So, the distance from *C* to *D* is 9.

Remember: Distance is always a **positive number**. Even when you back your car down the driveway, you have covered a *positive* distance. If you get a **negative number**, simply take the **absolute value** of the number.

In many instances, the points we need to identify to find the distance between are not on the same *horizontal* or *vertical* line. Because we would have to count points on a *diagonal*, we would not get an accurate measure of the distance between those points. We will examine two methods to determine the distance between any two points.

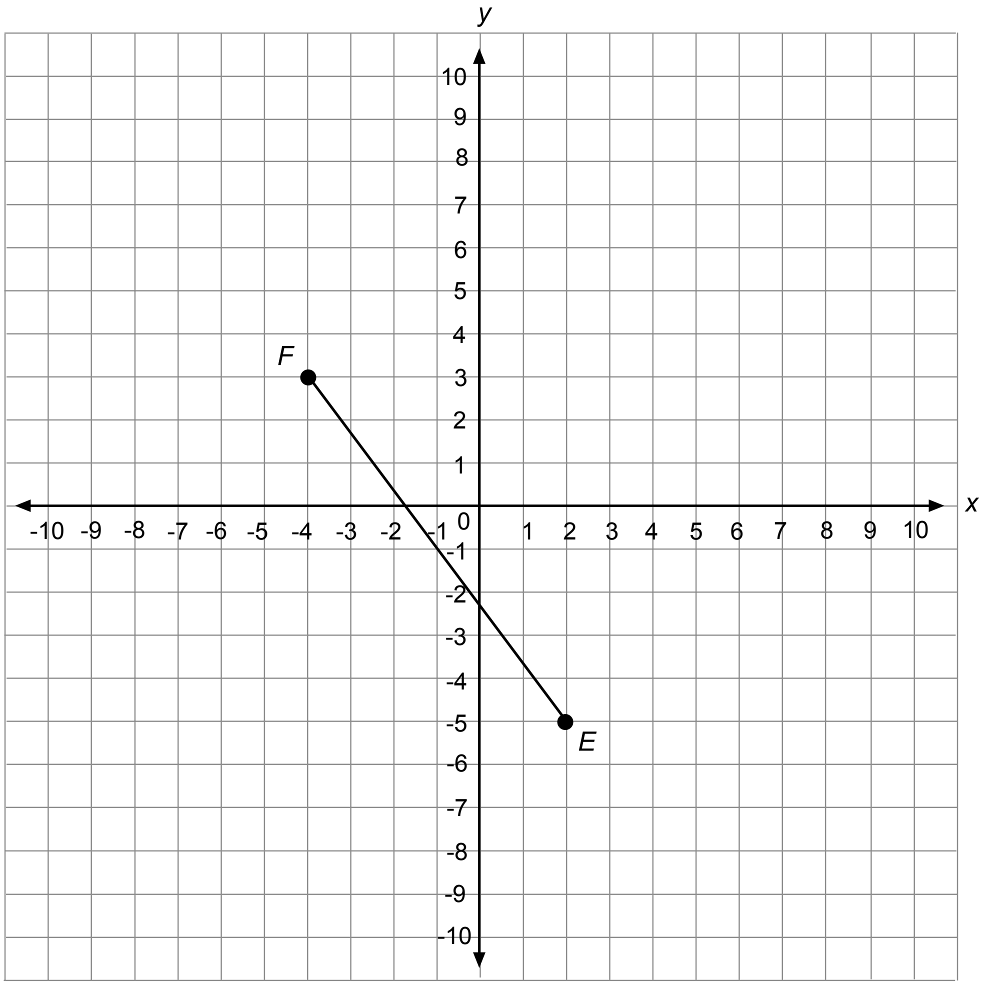
Look at the graph below. We want to find the distance between point *E* (2, -5) and *F* (-4, 3).

Graph of Points *E* and *F*

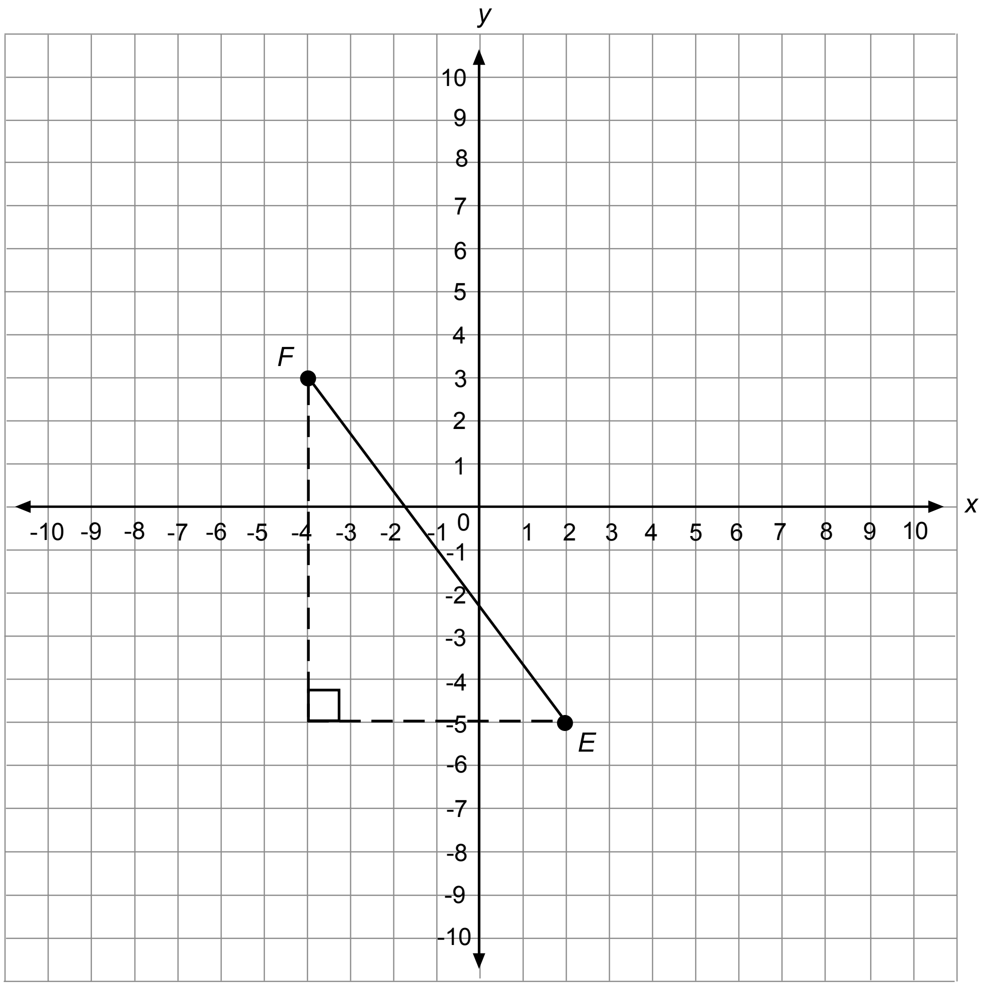


Notice that the distance between *E* and *F* looks like the **hypotenuse** of a **right triangle**.

Graph of Points *E* and *F*



Let's sketch the rest of the **triangle** and see what happens.


Graph of Points *E* and *F*

By completing the sketch of the *triangle*, we see that the result is a *right triangle* with one horizontal **side** and one vertical *side*. We can count to find the **lengths (*l*)** of these two sides, and then use the **Pythagorean theorem** to find the distance from *E* to *F*.

Remember: The *Pythagorean theorem* is the **square** of the *hypotenuse* (*c*) of a right triangle and is equal to the **sum** of the *squares* of the **legs** (*a* and *b*), as shown in the **equation** $a^2 + b^2 = c^2$.

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 6^2 + 8^2 &= c^2 \\ 36 + 64 &= c^2 \\ 100 &= c^2 \\ \sqrt{100} &= c \\ 10 &= c \end{aligned}$$

Remember:

- The opposite of *squaring* a number is called *finding the* **square root**. For example, the *square root* of 100, or $\sqrt{100}$, is 10.
- The square root of a number is shown by the symbol $\sqrt{}$, which is called a **radical sign** or *square root sign*.
- The number underneath is called a **radicand**.

$$\text{radical sign} \rightarrow \sqrt{100} \leftarrow \text{radicand}$$

radical

- The **radical** is an **expression** that has a **root**. A *root* is an equal **factor** of a number.
- $\sqrt{100}$ is a **radical expression**. It is a numerical *expression* containing a *radical sign*.

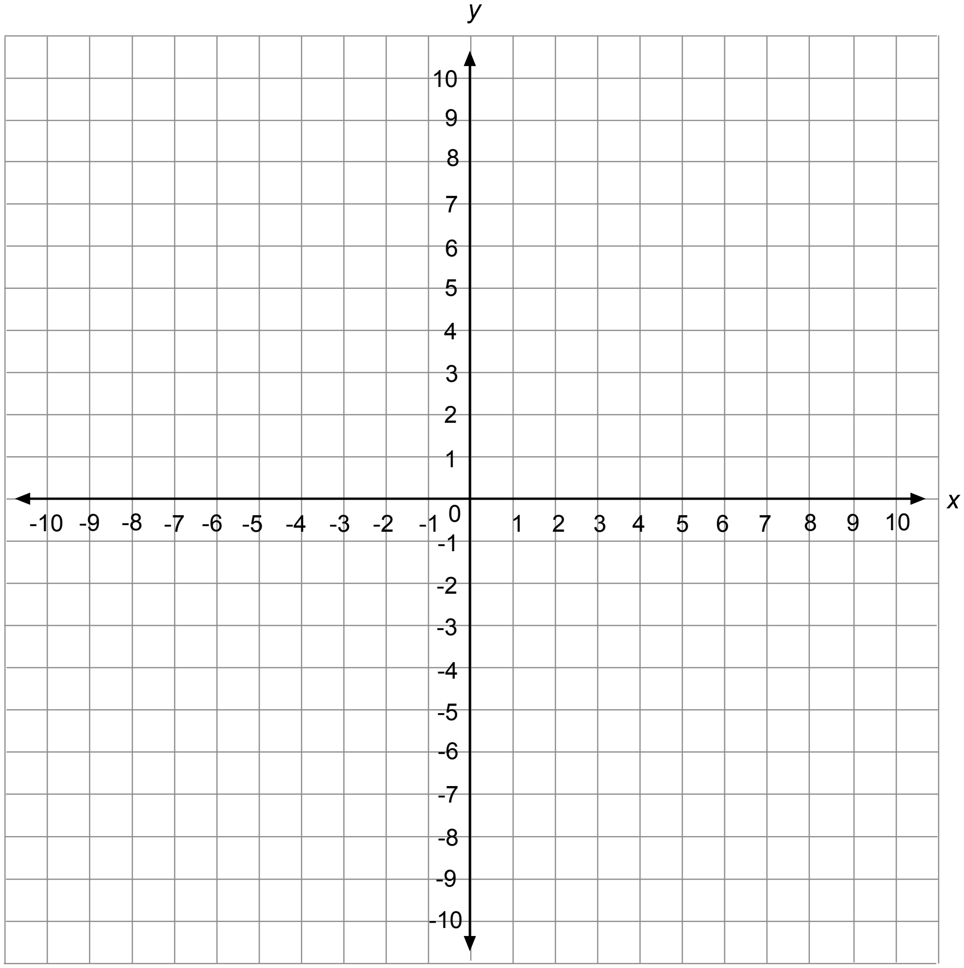
$$\begin{aligned} \sqrt{100} &= 10 \text{ because } 10^2 = 100 \\ \sqrt{9} &= 3 \text{ because } 3^2 = 9 \\ \sqrt{121} &= 11 \text{ because } 11^2 = 121 \end{aligned}$$

Practice

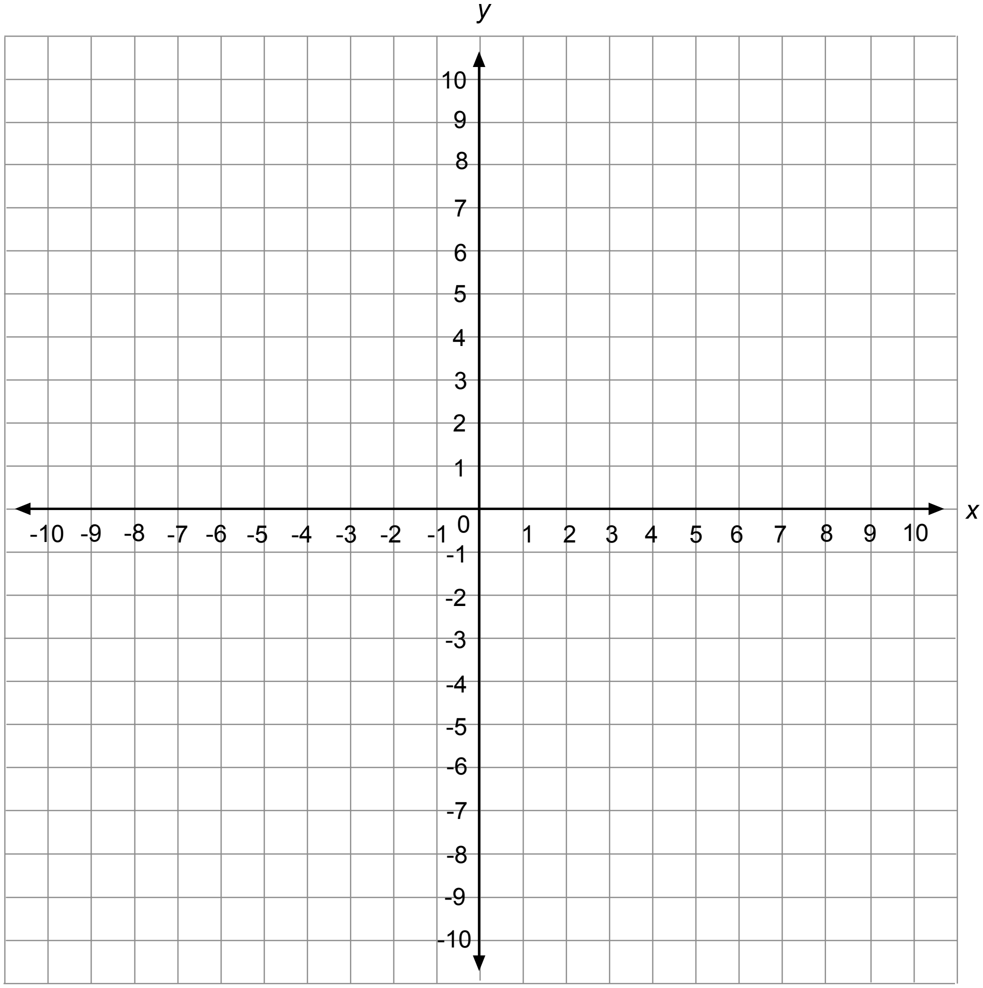
For the following:

- plot the two points
- draw the hypotenuse
- complete the triangle
- use the Pythagorean theorem to find the distance between the given points
- show all your work
- leave answers in **simplest radical form**.

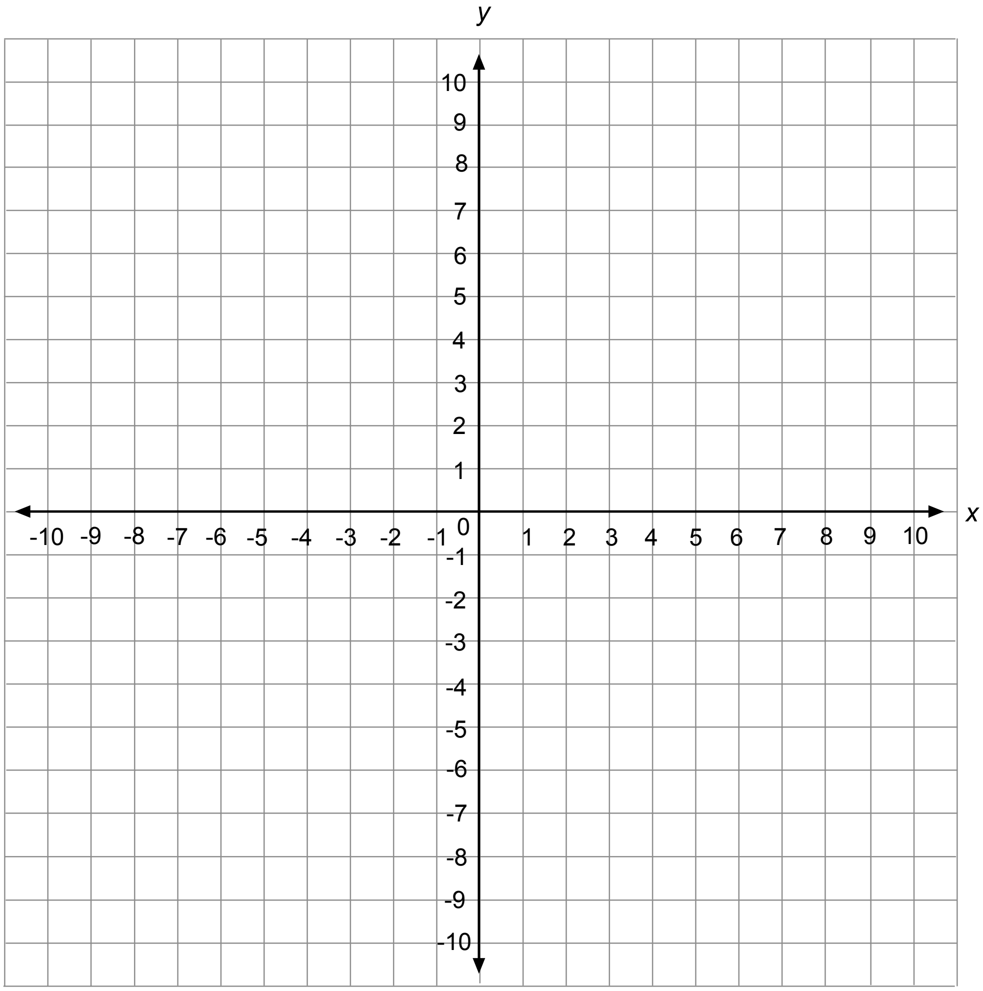
1. (3, 4), (-2, 6)



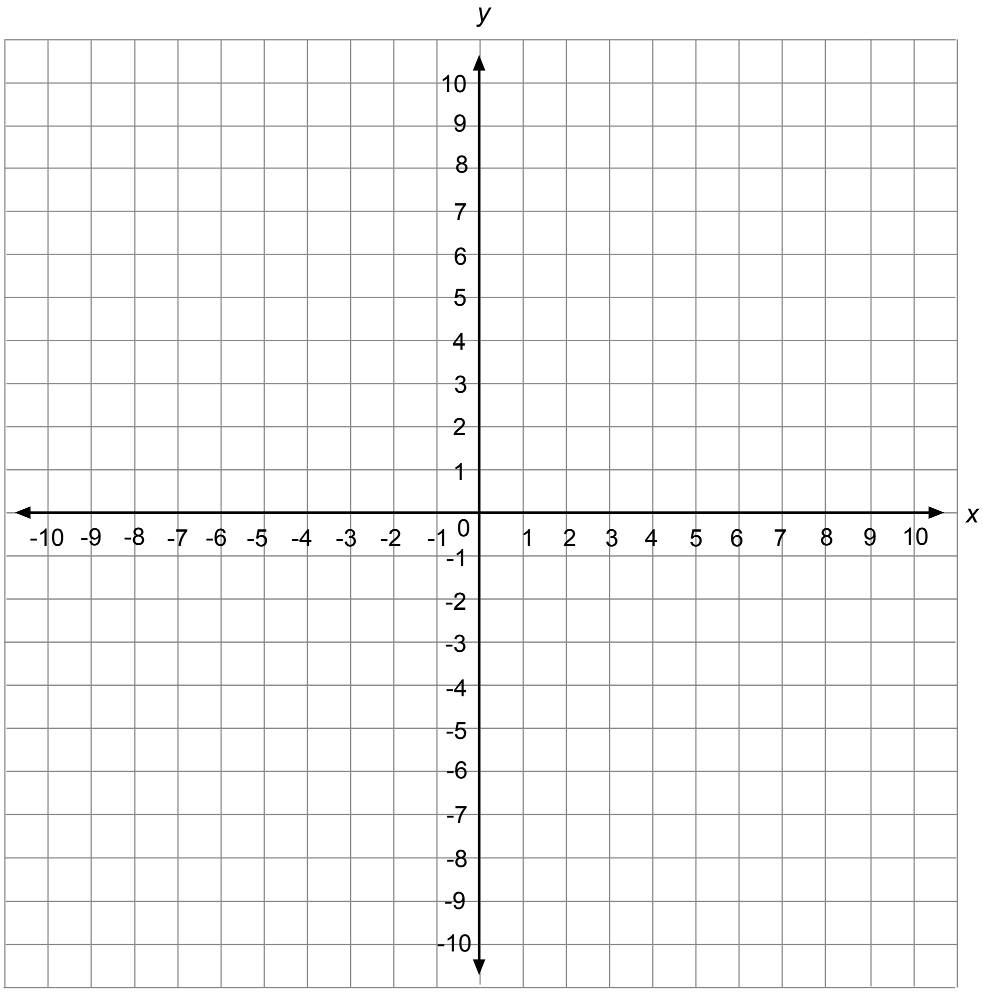
2. (3, -3), (6, 4)



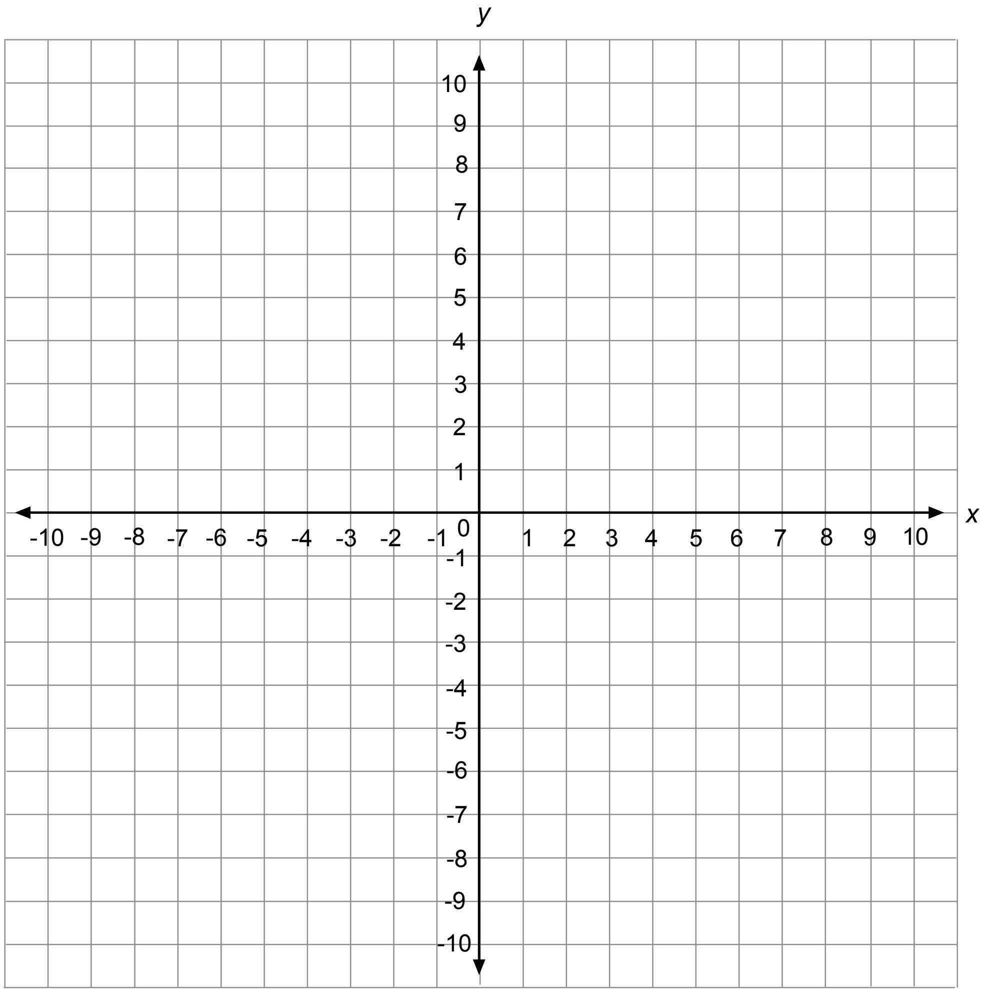
3. (-5, 0), (2, 3)



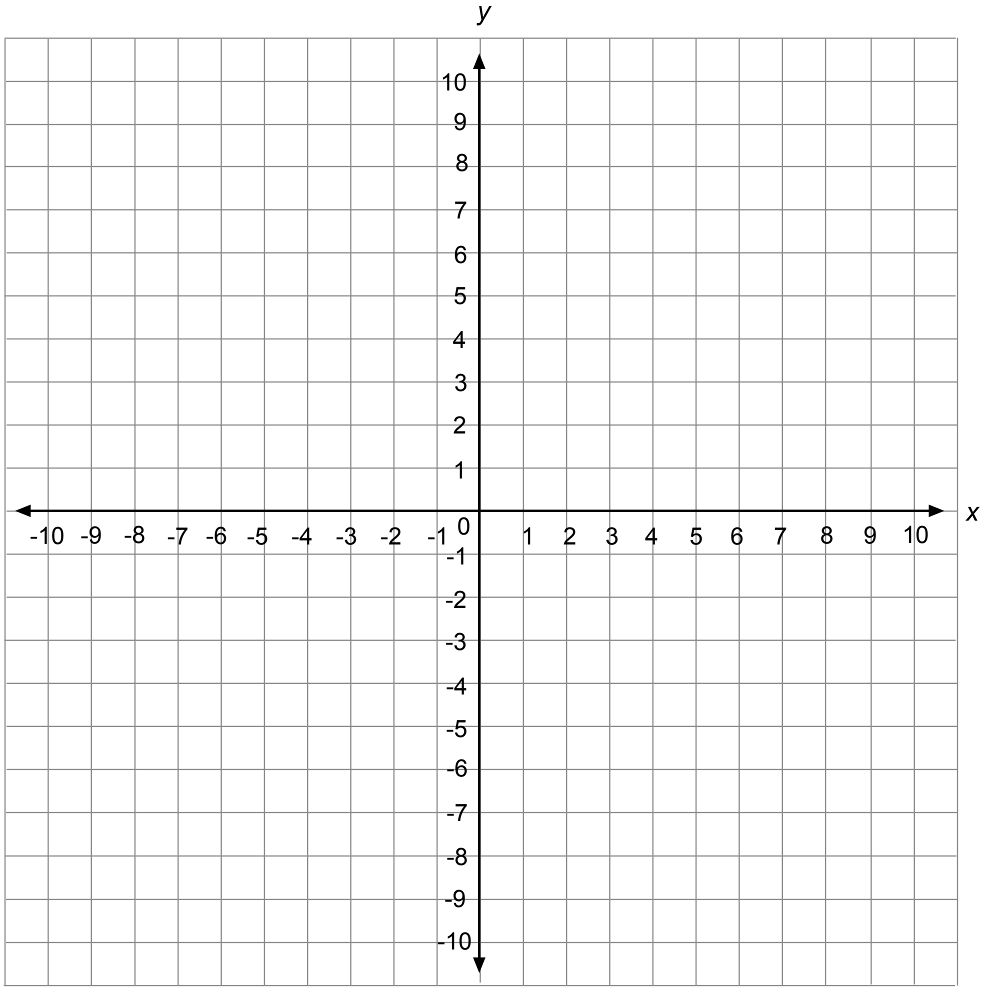
4. (4, -3), (-3, 4)



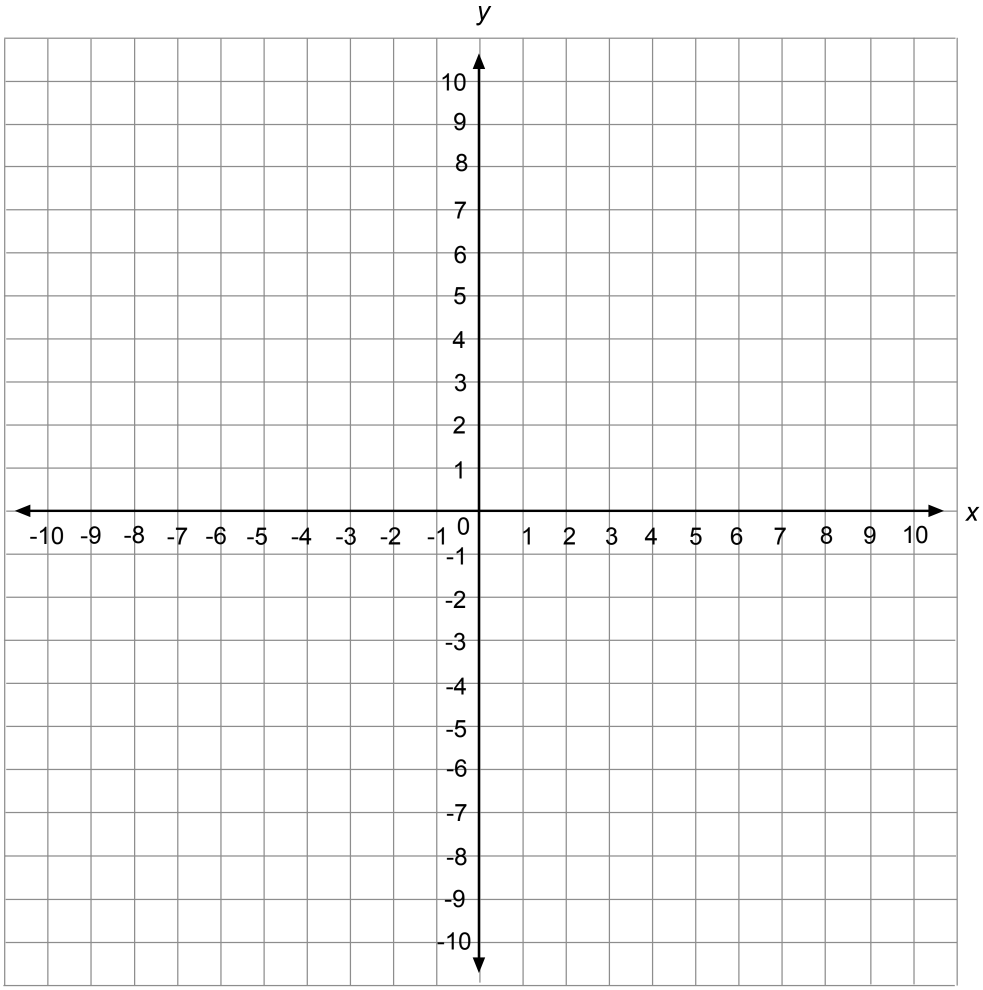
5. (0, 2), (-5, 7)



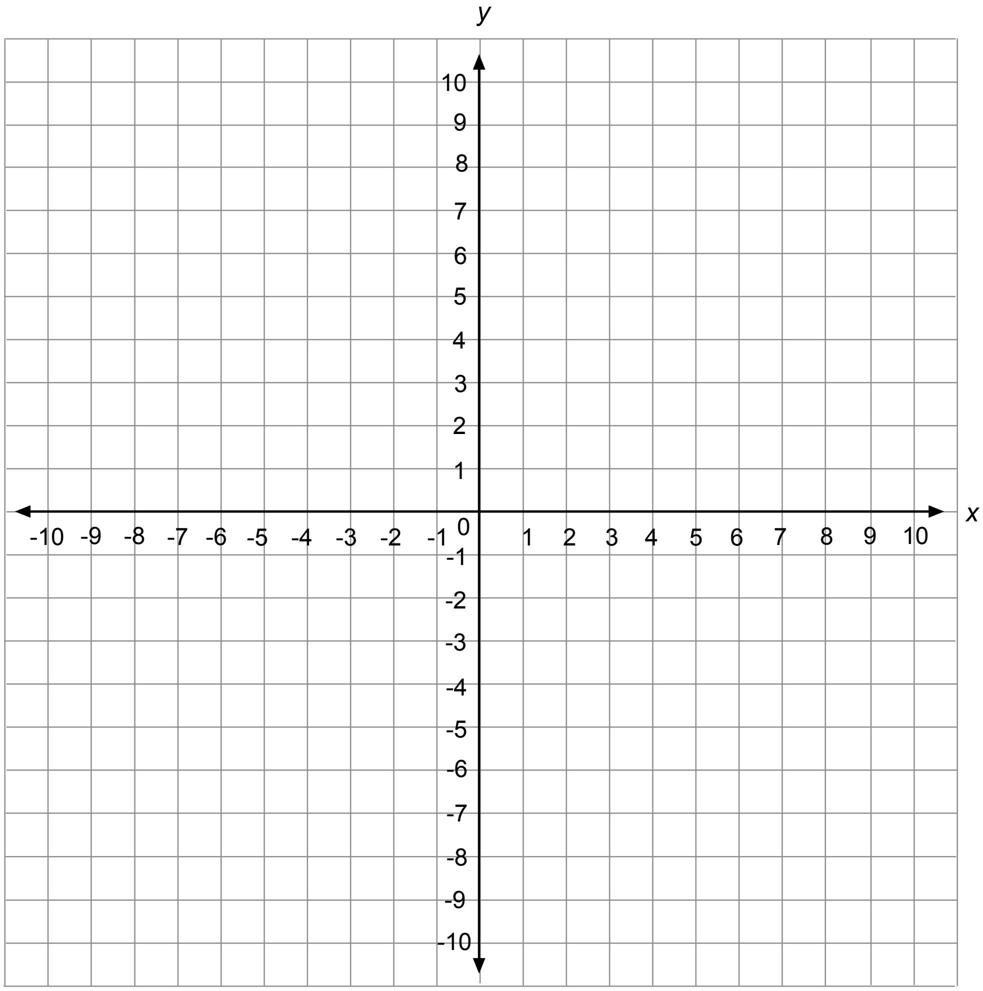
6. (2, 2), (-1, -2)



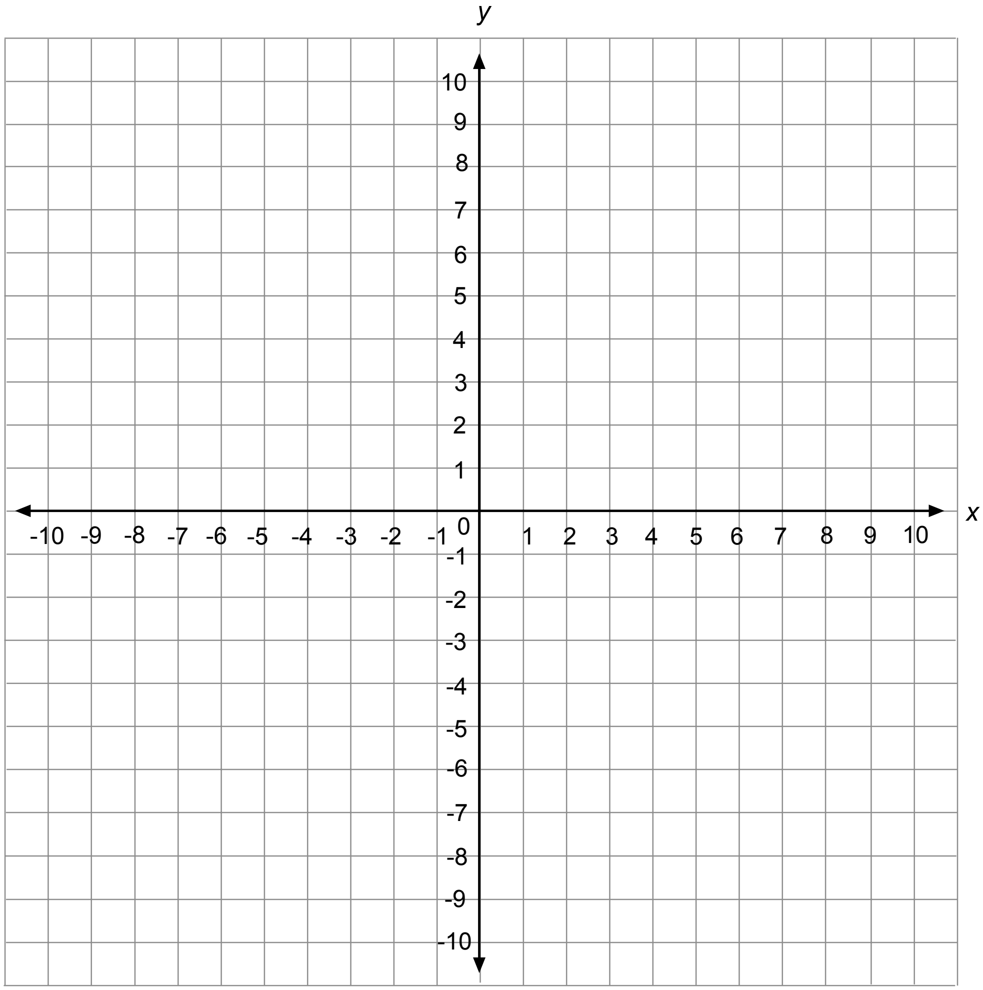
7. (0, 0), (-4, 4)



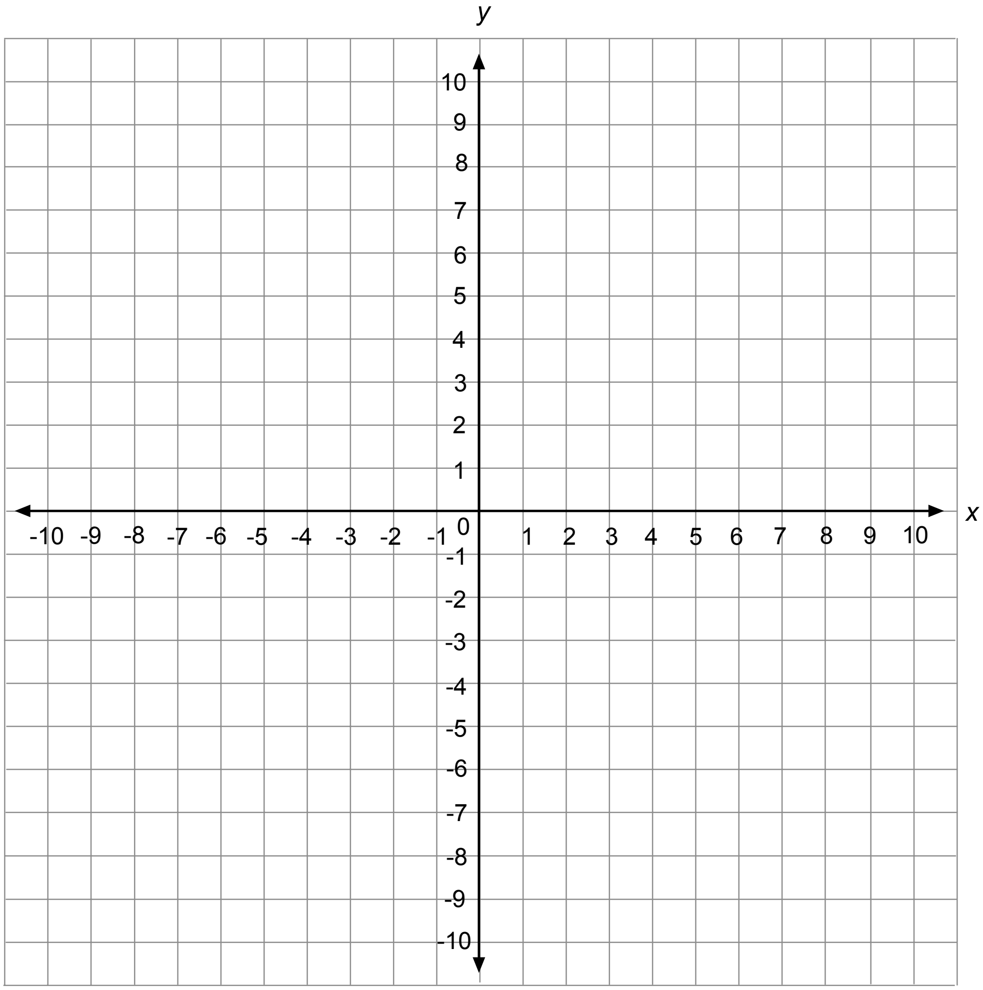
8. (3, 5), (-2, -7)



9. (6, -7), (-2, 8)



10. (-4, 6), (5, -6)



Practice

Use the list below to write the correct term for each definition on the line provided.

absolute value
coordinate grid or **plane**
distance
graph (of a point)
horizontal
negative numbers
positive numbers
vertical
x-axis
y-axis

________________________ 1. parallel to or in the same plane of the horizon

________________________ 2. the length of a segment connecting two points

________________________ 3. at right angles to the horizon; straight up and down

________________________ 4. numbers less than zero

________________________ 5. a number's distance from zero (0) on a number line

________________________ 6. numbers greater than zero

________________________ 7. the vertical number line on a rectangular coordinate system

________________________ 8. the point assigned to an ordered pair on a coordinate plane

________________________ 9. the horizontal number line on a rectangular coordinate system

________________________ 10. a two-dimensional network of horizontal and vertical lines that are parallel and evenly spaced

Practice

Match each definition with the correct term. Write the letter on the line provided.

_______ 1. a one-dimensional measure that is the measurable property of line segments

_______ 2. the longest side of a right triangle; the side opposite the right angle

_______ 3. the square of the hypotenuse (c) of a right triangle is equal to the sum of the square of the legs (a and b)

_______ 4. the edge of a polygon

_______ 5. a polygon with three sides

_______ 6. a triangle with one right angle

_______ 7. the result of adding numbers together

_______ 8. in a right triangle, one of the two sides that form the right angle

_______ 9. the result when a number is multiplied by itself or used as a factor twice

A. hypotenuse

B. leg

C. length (l)

D. Pythagorean theorem

E. right triangle

F. side

G. square (of a number)

H. sum

I. triangle

Using the Distance Formula

Sometimes, it is inconvenient to *graph* when finding the distance. So, another method we often use to find the distance between two points is the distance **formula**.

The distance *formula* is as follows.

distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The little 1s and 2s that are *subscripts* to the *x*'s and *y*'s signify that they come from different *ordered pairs*.

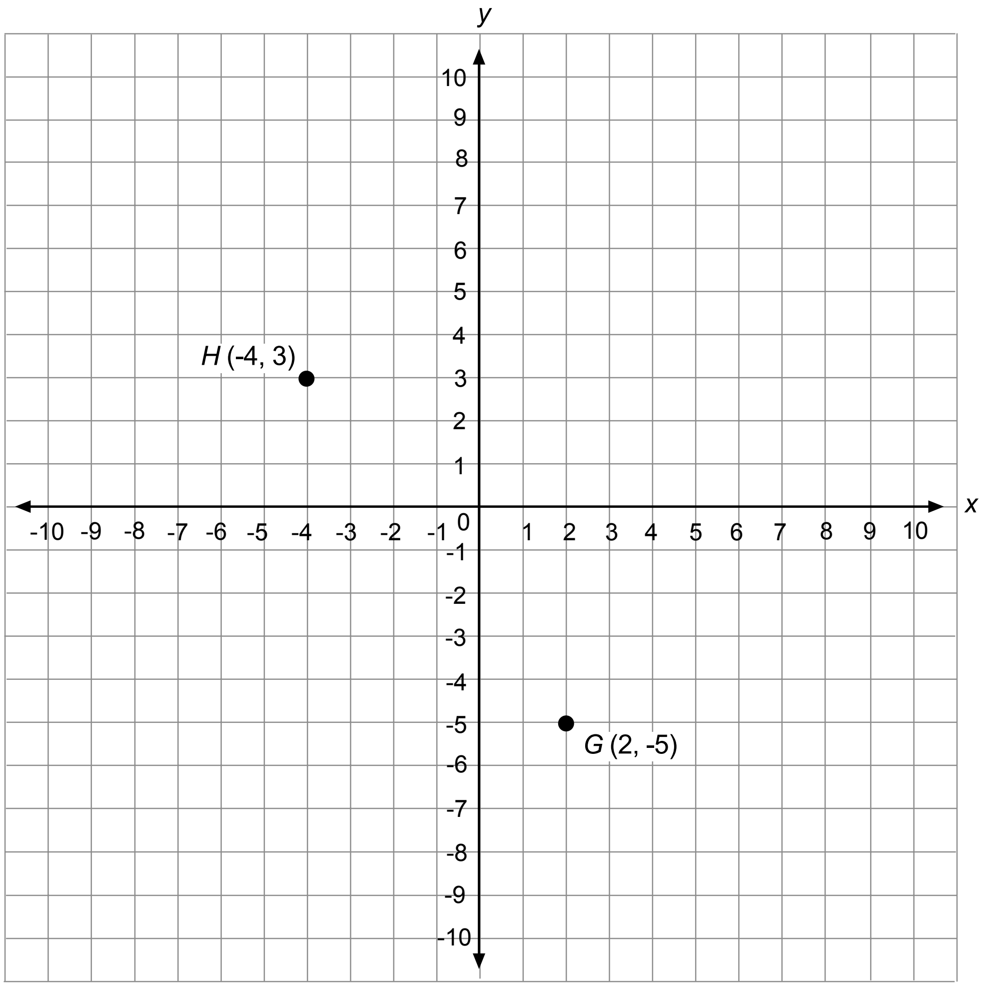
Example

(x_1, y_1) is one *ordered pair* and (x_2, y_2) is another ordered pair.

Note: Be consistent when putting the **values** into the formulas.

Let's look at the same example of *G* (2, -5) and *H* (-4, 3), and use the distance formula. See the graph on the following page.

Graph of Points *G* and *H*



$x_1 = 2$
$y_1 = \text{-}5$
$x_2 = \text{-}4$
$y_2 = 3$

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\sqrt{(\text{-}4 - 2)^2 + (3 - \text{-}5)^2} =$$

$$\sqrt{(\text{-}6)^2 + (8)^2} =$$

$$\sqrt{36 + 64} =$$

$$\sqrt{100} =$$

$$10$$

Compare the numbers in the distance formula to the numbers used in the *Pythagorean theorem*.

$$
\begin{array}{rcrcl}
a^2 & + & b^2 & = & c^2 \\
6^2 & + & 8^2 & = & c^2 \\
36 & + & 64 & = & c^2 \\
 & & 100 & = & c^2 \\
 & & \sqrt{100} & = & c \\
 & & 10 & = & c
\end{array}
$$

You should always get the same answer using either method.

Practice

Use the distance formula to solve the following. **Show all your work.** *Leave answers in* **simplest radical form**.

distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

1. (3, 4), (-2, 6)

2. (3, -3), (6, 4)

3. (-5, 0), (2, 3)

4. (4, -3), (-3, 4)

5. (0, 2), (-5, 7)

6. (2, 2), (-1, -2)

7. (0, 0), (-4, 4)

8. (3, 5), (-2, -7)

9. (6, -7), (-2, 8)

10. (-4, 6), (5, -6)

✓ **Check yourself:** Compare your answers to the practice on pages 517-526. Do they match? If not, rework until both sets of practice answers match.

Practice

Use your **favorite of the two methods** *shown on pages 529-531. One method uses the* **distance formula** *and the other method uses the* **Pythagorean theorem**. *Find the* **distance between each pair of points** *below using either method. Refer to the examples on pages 529-531 as needed.*

Show all your work. *Leave answers in* **simplest radical form**.

distance formula	Pythagorean theorem
$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	$a^2 + b^2 = c^2$

1. (0, 0), (-3, 4)

2. (5, -6), (6, -5)

3. (-5, -8), (3, 7)

4. (-2, -8), (0, 0)

5. (6, 6), (-3, -3)

6. (-1, 2), (5, 10)

Lesson Two Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Algebra Body of Knowledge

Standard 5: Rational Expressions and Equations

- MA.912.A.5.1
  Simplify algebraic ratios.

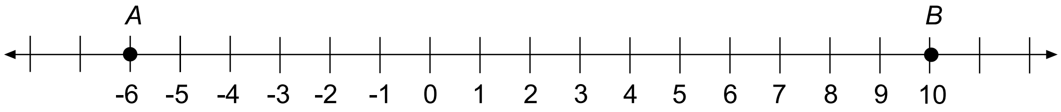
Midpoint

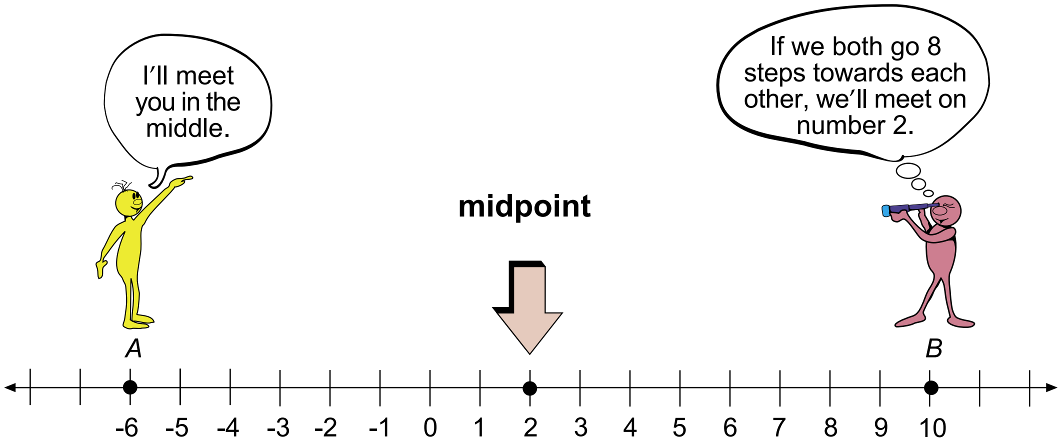
Sometimes it is necessary to find the *point* that is exactly in the middle of two given **endpoints**. We call this the **midpoint (of a line segment)**. What we are actually trying to find are the **coordinates** of that point, which is like the *address* of the point, or its *location* on a coordinate plane or a **number line**.

Finding the Midpoint of a Line Segment Using a Number Line

You can find the *midpoint* of a **line segment (—)**, also called a *segment*, in a couple of different ways. One way is to use a *number line*.

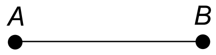
On a number line, you can find the midpoint of a *line segment* by counting in from both *endpoints* until you reach the middle.



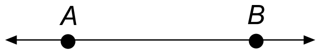


how to use a number line to find the midpoint of a line segment

Remember: If we draw a *line segment* from one point to another, we can call it line segment *AB* or *segment AB*. See a representation of line segment *AB* ($\overline{AB}$) below. The symbol (—) drawn over the two uppercase letters describes a line segment. The symbol has no arrow because the line segment has a definite beginning and end called endpoints. *A* and *B* are endpoints of the line segment *AB* ($\overline{AB}$).



On the other hand, the symbol (⟷) drawn over two uppercase letters describes a line. The symbol has arrows because a line has no definite beginning or end. *A* and *B* are points on the line *AB* ($\overleftrightarrow{AB}$).



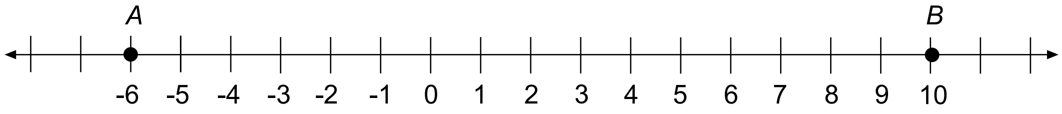
Method One Midpoint Formula

Another way to find the midpoint of a line segment is to use the Method One midpoint formula below. To do this, add the two endpoints together and divide by two.

Method One midpoint formula

$$\frac{a+b}{2}$$

$$\begin{aligned} \frac{a+b}{2} &= \\ \frac{-6+10}{2} &= \\ \frac{4}{2} &= \\ 2 & \end{aligned}$$



Therefore, for points *A* and *B* on the number line, the midpoint is

$$\frac{-6+10}{2} = \frac{4}{2} = 2.$$

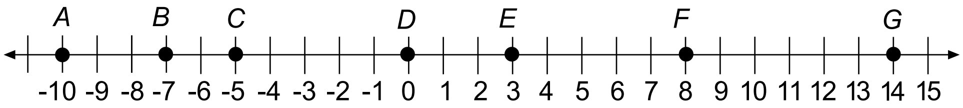
Practice

Find the **coordinate of the midpoint** *for each* **pair of points** *on the* **number line** *below. Use either of the methods below from pages 538-539.*

- **Use the number line and count in from both endpoints** *of a line segment until you reach the middle to determine the midpoint.*
- **Use the Method One midpoint formula** *and add the two endpoints together, then divide by two.* **Show all your work.**

Method One midpoint formula

$$\frac{a + b}{2}$$



Refer to previous pages as needed.

1. *A* and *C*

2. *B* and *E*

3. *A* and *E*

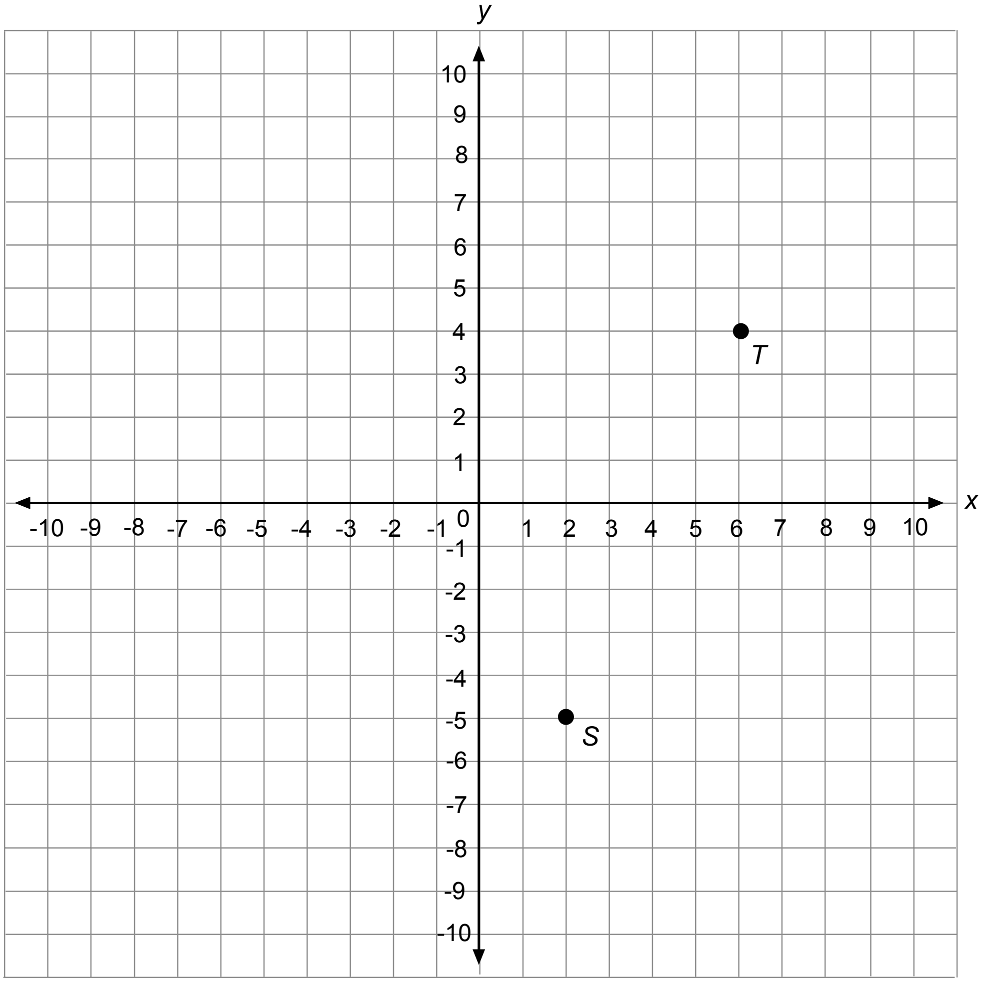
4. *D* and *G*

5. *A* and *G*

Method Two Midpoint Formula

Do you think the process may change a bit when we try to find the midpoint of points *S* and *T* as seen on the graph below?

Graph of Points *S* and *T*



When the points are on a **coordinate plane**, or the plane containing the *x*- and *y*-axes, we have to think in two dimensions to find the *coordinates* of the midpoint. The midpoint will have an ***x*-coordinate** and a ***y*-coordinate** (*x*, *y*). To find the midpoint on a coordinate plane, we simply use the Method Two midpoint formula twice—once to find the *x-coordinate* and again to find the *y-coordinate*.

Method Two midpoint formula

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Let's see how this works.

We see that point *S* has coordinates (2, -5), and *T* is located at (6, 4). Use the Method Two midpoint formula to find the exact location of the midpoint of $\overline{ST}$.

$$\text{midpoint of } \overline{ST} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) =$$

find the *average* of the *x*-values, then the average of the *y*-values

$$\left(\frac{2 + 6}{2}, \frac{-5 + 4}{2}\right) =$$

add the *x*'s then the *y*'s

$$\left(\frac{8}{2}, \frac{-1}{2}\right) =$$

now **simplify each fraction**

$$\left(4, \frac{-1}{2}\right) =$$

Practice

Find the **midpoint** *of the* **coordinates** *for each* **segment** *whose endpoints are given. Use the* **Method Two midpoint formula** *below.* **Show all your work.** *Refer to pages 542-543 as needed.*

Method Two midpoint formula

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

1. (2, 8), (-4, 2)

2. (0, 0), (-3, -4)

3. (1, 2), (4, 3)

4. (-3, -5), (9, 0)

5. (-4, 6), (3, 3)

6. (5, -6), (-5, 6)

7. (6, 6), (-4, -4)

8. (5, 5), (-5, -5)

9. (8, -4), (10, 9)

10. (6, 8), (-3, 5)

Practice

Match each definition with the correct term. Write the letter on the line provided.

_______ 1. a portion of a line that consists of two defined endpoints and all points in between

_______ 2. the plane containing the x- and y-axes

_______ 3. write fraction in lowest terms or simplest form

_______ 4. the second number of an ordered pair

_______ 5. the number paired with a point on the number line

_______ 6. numbers that correspond to points on a coordinate plane in the form (x, y), or a number that corresponds to a point on a number line

_______ 7. the point on a line segment equidistant from the endpoints

_______ 8. a line on which ordered numbers can be written or visualized

A. coordinate plane

B. coordinates

C. line segment (—)

D. midpoint (of a line segment)

E. number line

F. simplify a fraction

G. x-coordinate

H. y-coordinate

Lesson Three Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.

- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.

- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Algebra Body of Knowledge

Standard 3: Linear Equations and Inequalities

- MA.912.A.3.9
  Determine the slope, x-intercept, and y-intercept of a line given its graph, its equation, or two points on the line.

Geometry Body of Knowledge

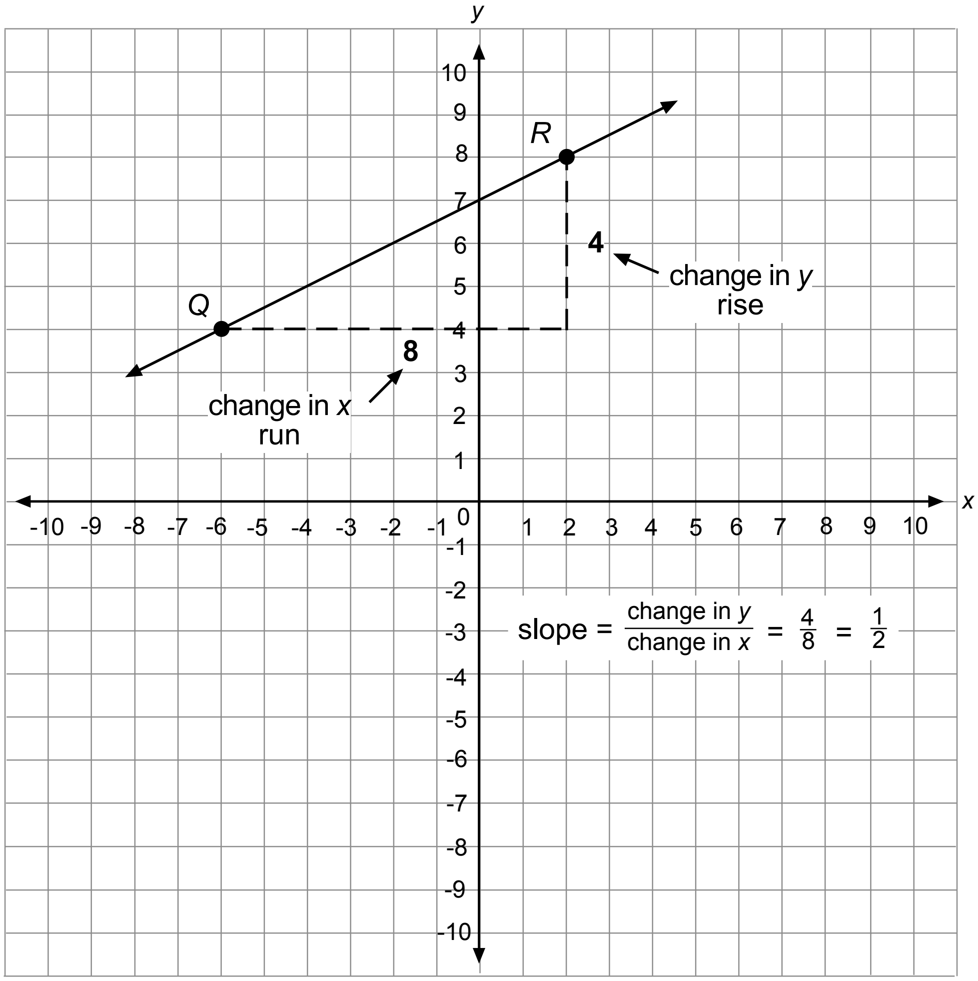
Standard 1: Points, Lines, Angles, and Planes

- MA.912.G.1.4
  Use coordinate geometry to find slopes, parallel lines, perpendicular lines, and equations of lines.

Slope

Slope can be thought of as the slant of a line. It is often defined as $\frac{\text{rise}}{\text{run}}$, which means the change in the *y*-values (**rise**) on the *vertical* axis, divided by the change in the *x*-values (**run**) on the *horizontal* axis. In the figure below we can count to find the *slope* between points *Q* (-6, 4) and *R* (2, 8).

Graph of Points *Q* and *R*



$$\text{slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{4}{8} = \frac{1}{2}$$

slope of a line

However, we can also use the *slope formula* to determine the slope of a line without having to see a graph of the two points of the line.

slope formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Remember: m is always used to represent slope.

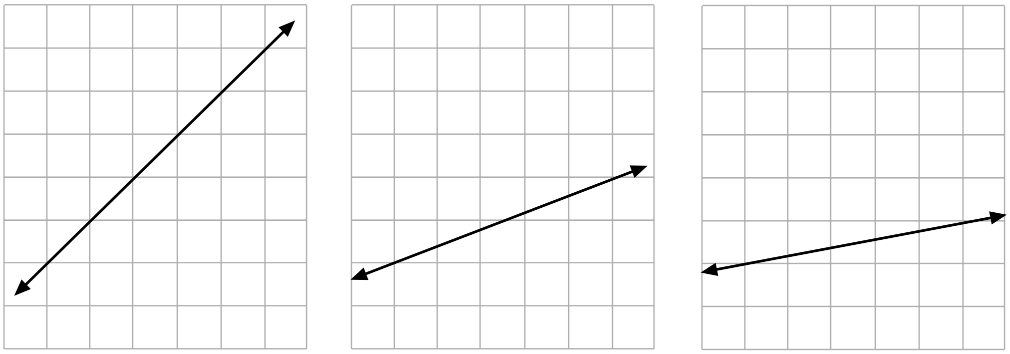
However, we must know the coordinates of two points on a line so that we can use the formula. Refer to points Q and R on the previous page. The coordinates of Q are (-6, 4) and the coordinates of R are (2, 8). Let's see how this works in the slope formula.

$x_1 = -6$
$x_2 = 2$
$y_1 = 4$
$y_2 = 8$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 4}{2 - -6} = \frac{4}{8} = \frac{1}{2}$$

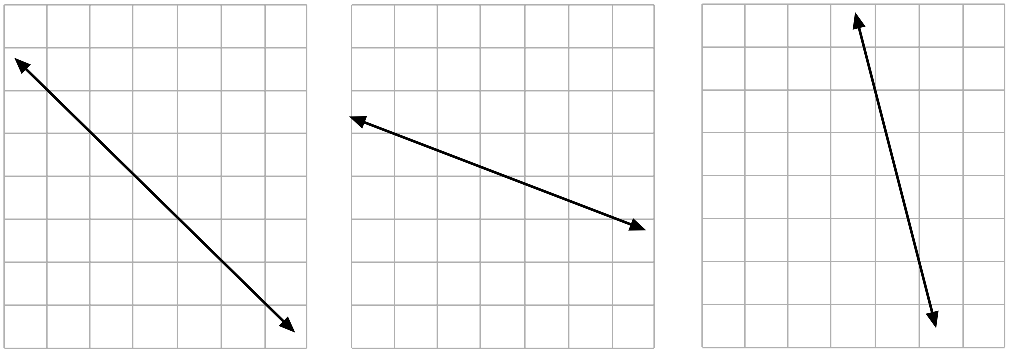
When the slope of a line is *positive*, the line will *rise* from left to right.

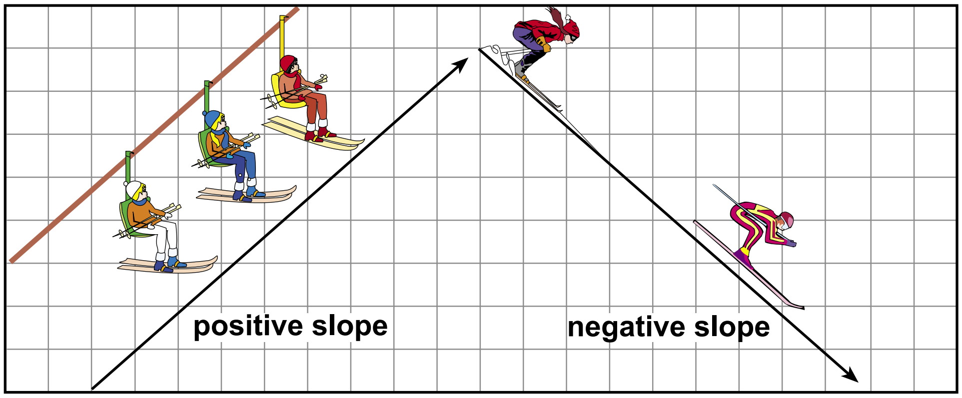
Examples



When the slope of a line is *negative*, the line will *fall* from left to right.

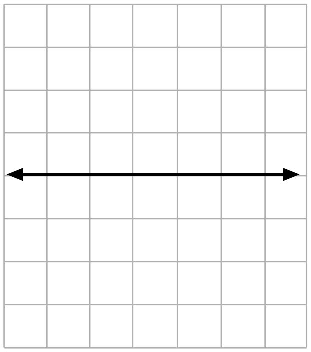
Examples



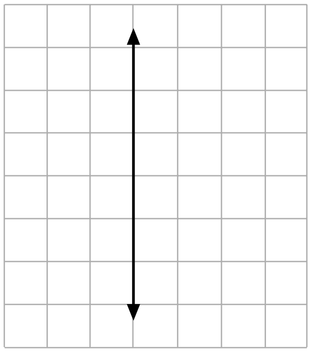


slope

When the slope has a zero in the **numerator** ($\frac{0}{x}$), the line will be *horizontal* and have a slope of 0.



When the slope has a zero in the **denominator** ($\frac{y}{0}$), the line will be *vertical* and have *no* slope at all. We sometimes say that the slope of a vertical line is *undefined*.



Practice

Use the **slope formula** *below to find the slope of each* **line passing through points** *listed below.* **Simplify the answer.** *Then determine whether the* **line** *is* **rising**, **falling**, **horizontal**, *or* **vertical**. *Write the answer on the line provided.* **Show all your work.**

slope formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Remember:

$\frac{0}{x}$ = a line that is horizontal with a **zero** (0) **slope**

$\frac{y}{0}$ = a line that is vertical with **no slope**

______________________________ 1. (2, 8), (-4, 2)

______________________________ 2. (0, 0), (-3, -4)

______________________________ 3. (1, 2), (4, 3)

____________________________ 4. (3, -6), (3, 4)

____________________________ 5. (-3, -5), (9, 0)

____________________________ 6. (-4, 6), (3, 3)

____________________________ 7. (4, 2), (-5, 2)

________________________________ 8. (5, -6), (-5, 6)

________________________________ 9. (6, 6), (-4, -4)

________________________________ 10. (6, 7), (6, -4)

________________________________ 11. (5, 5), (-5, -5)

______________________________ 12. (0, 4), (0, 9)

______________________________ 13. (8, -4), (10, -9)

______________________________ 14. (6, 5), (-3, 8)

______________________________ 15. (4, 5), (8, 16)

Lesson Four Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.

- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.

- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Algebra Body of Knowledge

Standard 1: Real and Complex Number Systems

- MA.912.A.1.8
  Use the zero product property of real numbers in a variety of contexts to identify solutions to equations.

Standard 3: Linear Equations and Inequalities

- MA.912.A.3.7
  Rewrite equations of a line into slope-intercept form and standard form.

- MA.912.A.3.8
  Graph a line given any of the following information: a table of values, the x- and y-intercepts, two points, the slope and a point, the equation of the line in slope-intercept form, standard form, or point-slope form.

- MA.912.A.3.9
  Determine the slope, x-intercept, and y-intercept of a line given its graph, its equation, or two points on the line.

Geometry Body of Knowledge

Standard 1: Points, Lines, Angles, and Planes

- MA.912.G.1.4
  Use coordinate geometry to find slopes, parallel lines, perpendicular lines, and equations of lines.

Equations of Lines

An *equation* of a line can be expressed in several ways. Mathematicians sometimes use the format $ax + by = c$. This is called **standard form (of a linear equation)**. In the *standard form*, **linear equations** have the following three rules.

1. a, b, and c are **integers**, or the numbers in the set
   {… , -4, -3, -2, -1, 0, 1, 2, 3, 4, …}

2. a cannot be a **negative integer**

3. a and b cannot both be equal to 0

Linear Equations in Standard Form

- $ax + by = c$
- x and y are **variables**
- a, b, and c are **constants** for the given equation

You can graph a line fairly easily by using standard form.

Follow this example.

$$3x + 2y = 12$$

If we replace x with 0 we get the following.

$$\begin{aligned} 3x + 2y &= 12 \\ 3(0) + 2y &= 12 \\ 0 + 2y &= 12 \\ 2y &= 12 \\ y &= 6 \end{aligned}$$

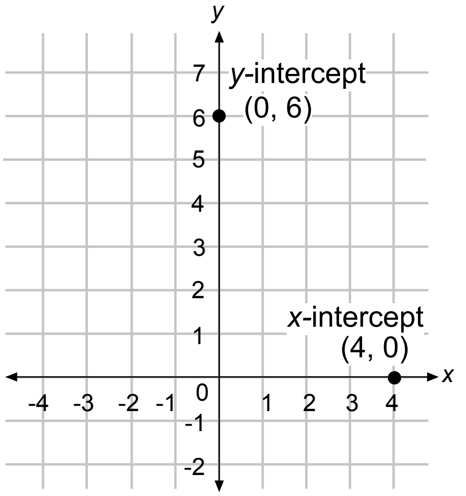
This tells us that the point (0, 6) is on the graph of the line $3x + 2y = 12$. In fact (0, 6) is called the ***y*-intercept** of the line. It is the point where the line crosses the y-axis.

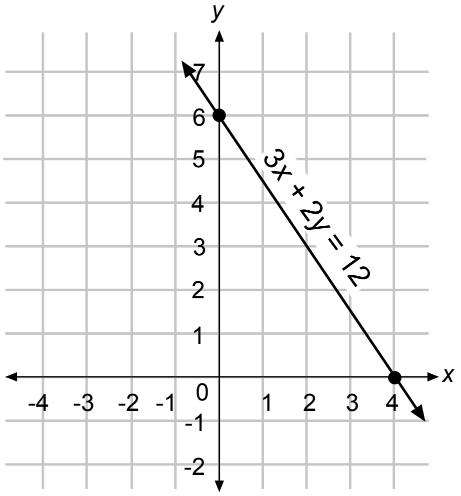
Remember that you must have two points to decide exactly where the line goes on the coordinate plane. So, we repeat the process, but this time replace y with 0.

$$\begin{aligned} 3x + 2y &= 12 \\ 3x + 2(0) &= 12 \\ 3x + 0 &= 12 \\ 3x &= 12 \\ x &= 4 \end{aligned}$$

This tells us that the point (4, 0) is also on the line. Did you guess that this is called the ***x*-intercept**?

So, if we plot the two points (0, 6) and (4, 0), we can draw a line connecting them.





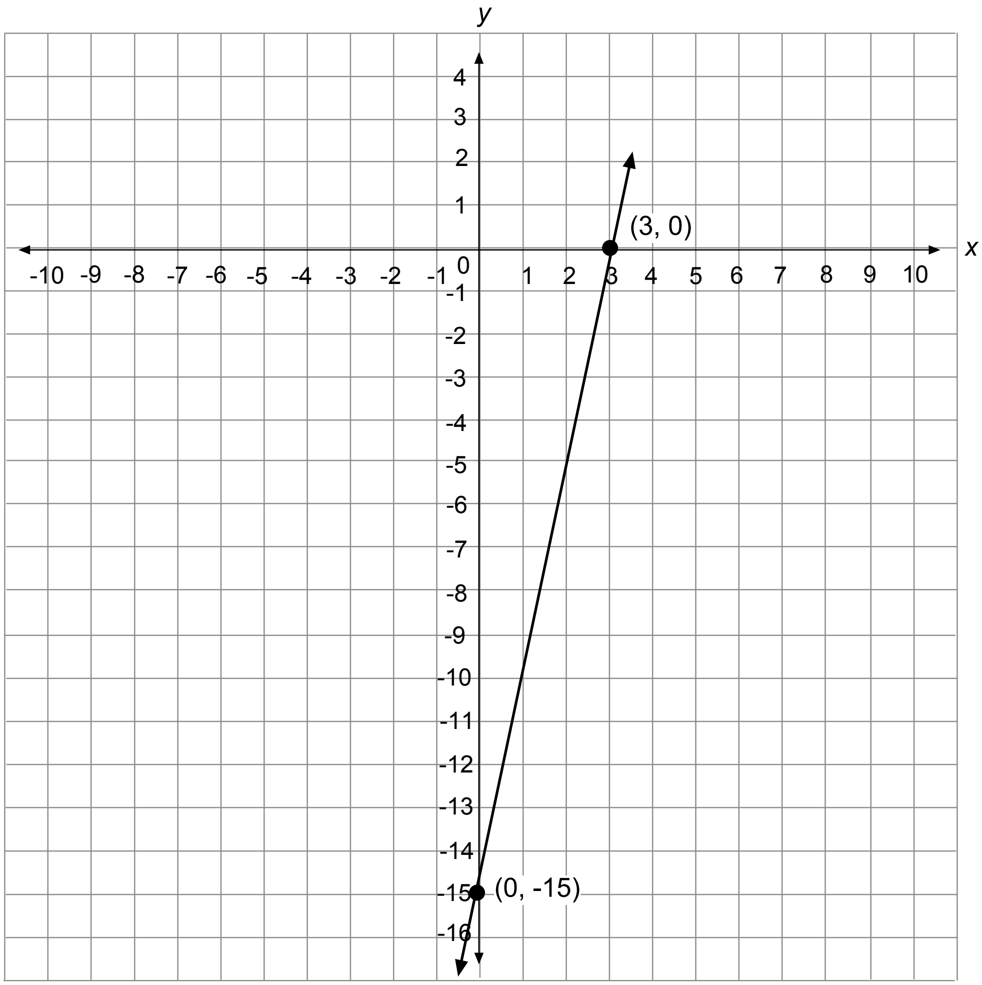
Did you notice that we could find the slope of the line above either by using the slope formula with the x- and y-intercepts ($m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 6}{4 - 0} = \frac{-6}{4} = \frac{-3}{2}$) or by counting rise and run from the graph?

Let's try another example.

$5x - y = 15$

If $x = 0$,	If $y = 0$,
$$\begin{aligned} 5x - y &= 15 \\ 5(0) - y &= 15 \\ 0 - y &= 15 \\ -y &= 15 \\ y &= -15 \end{aligned}$$	$$\begin{aligned} 5x - y &= 15 \\ 5x - y(0) &= 15 \\ 5x - 0 &= 15 \\ 5x &= 15 \\ x &= 3 \end{aligned}$$
(0, -15) y-intercept	(3, 0) x-intercept

Graph of 5*x* – *y* = 15



Your turn.

Practice

Use the equations in **standard form** *to find the* **y-intercepts**, *find the* **x-intercepts**, *and* **graph the lines** *of the following.*

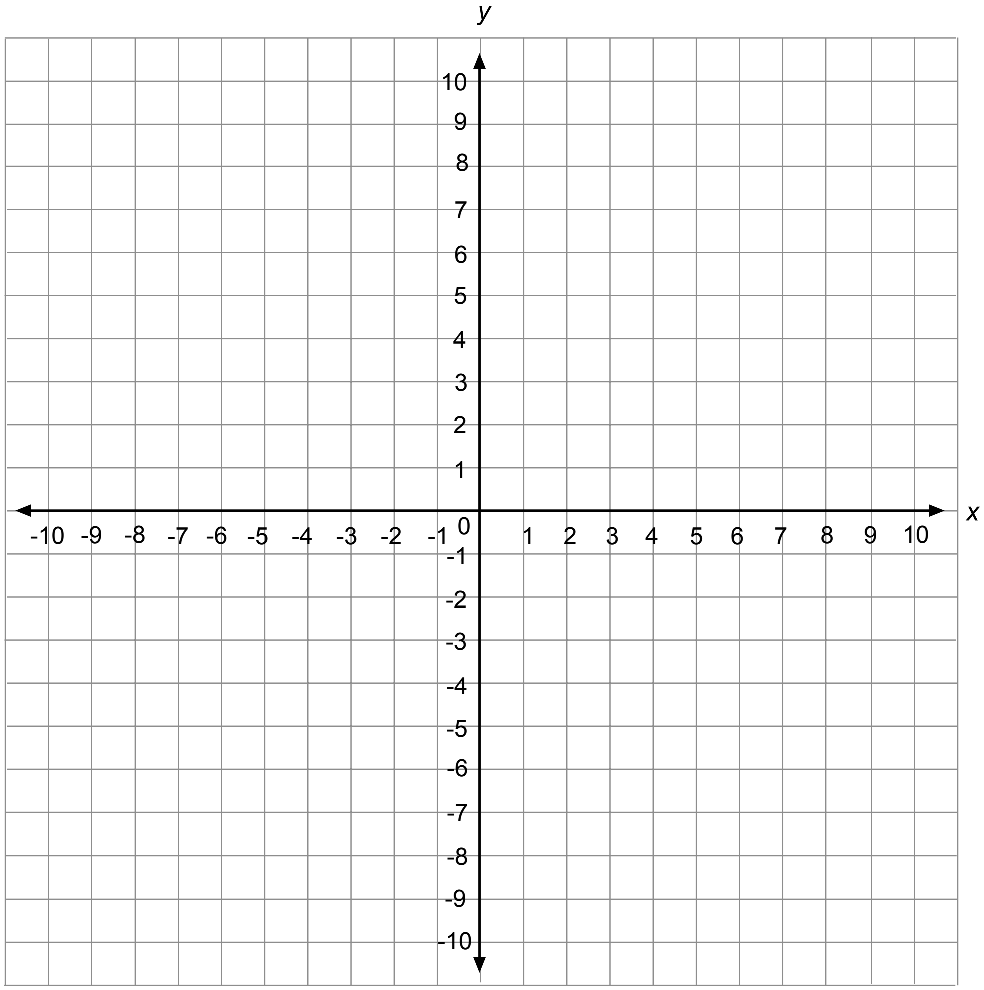
1. $2x + 5y = 10$

   a. y-intercept = ____________

   b. x-intercept = ____________

   c. graph

Graph of $2x + 5y = 10$



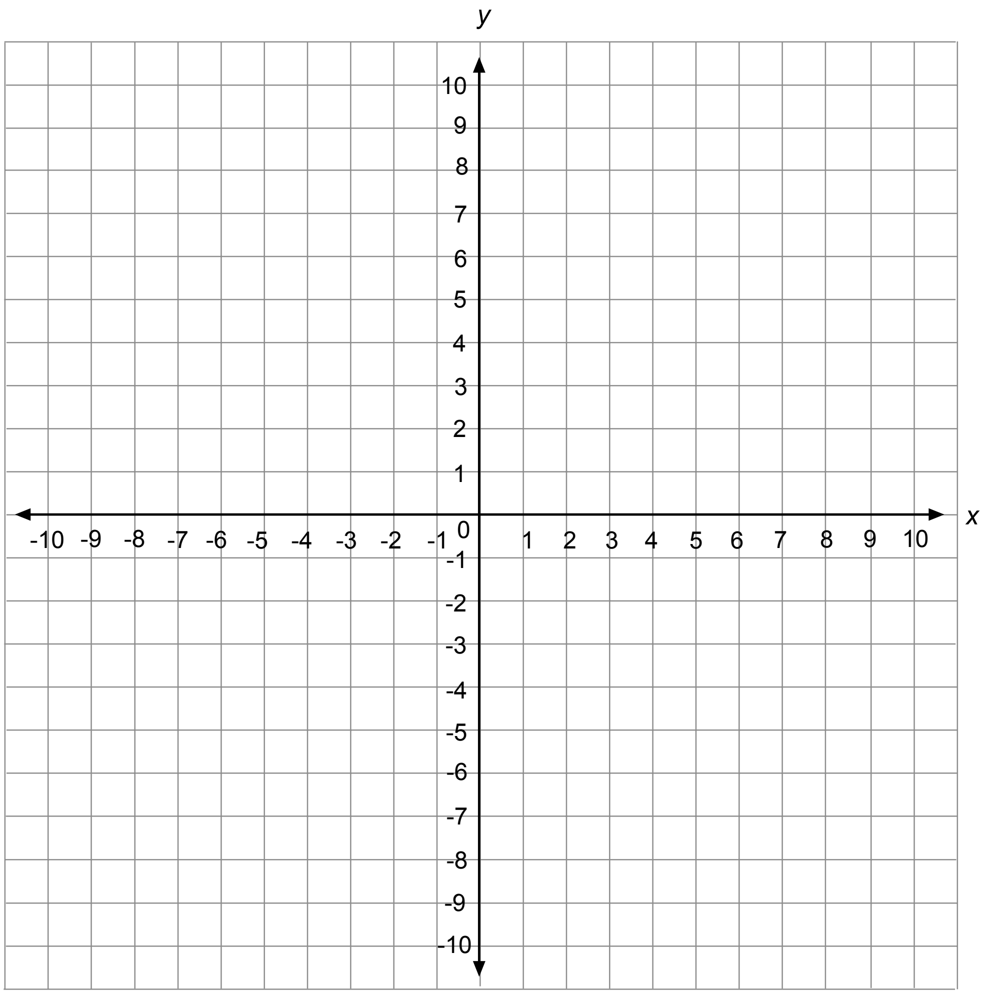
2. $8x - 3y = 24$

a. y-intercept = ____________

b. x-intercept = ____________

c. graph

Graph of $8x - 3y = 24$



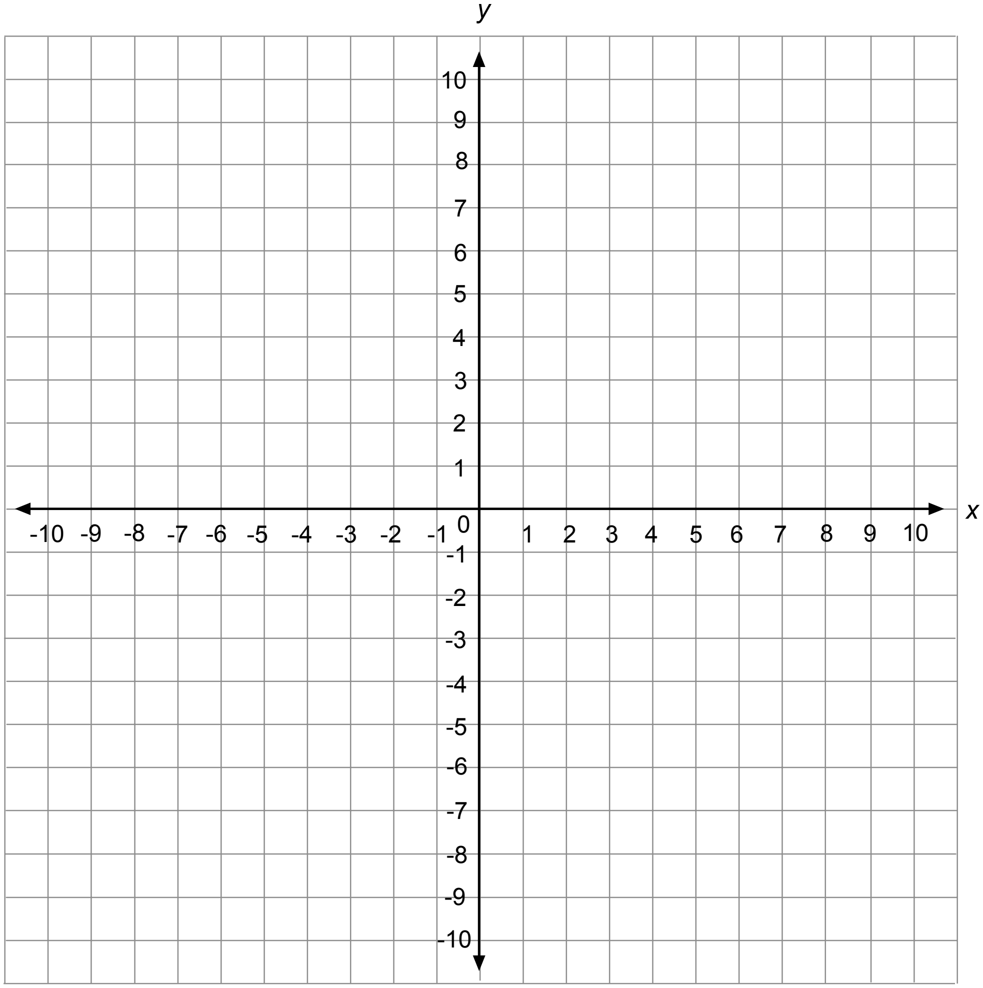
3. $3x - 8y = 24$

a. y-intercept = ____________

b. x-intercept = ____________

c. graph

Graph of $3x - 8y = 24$



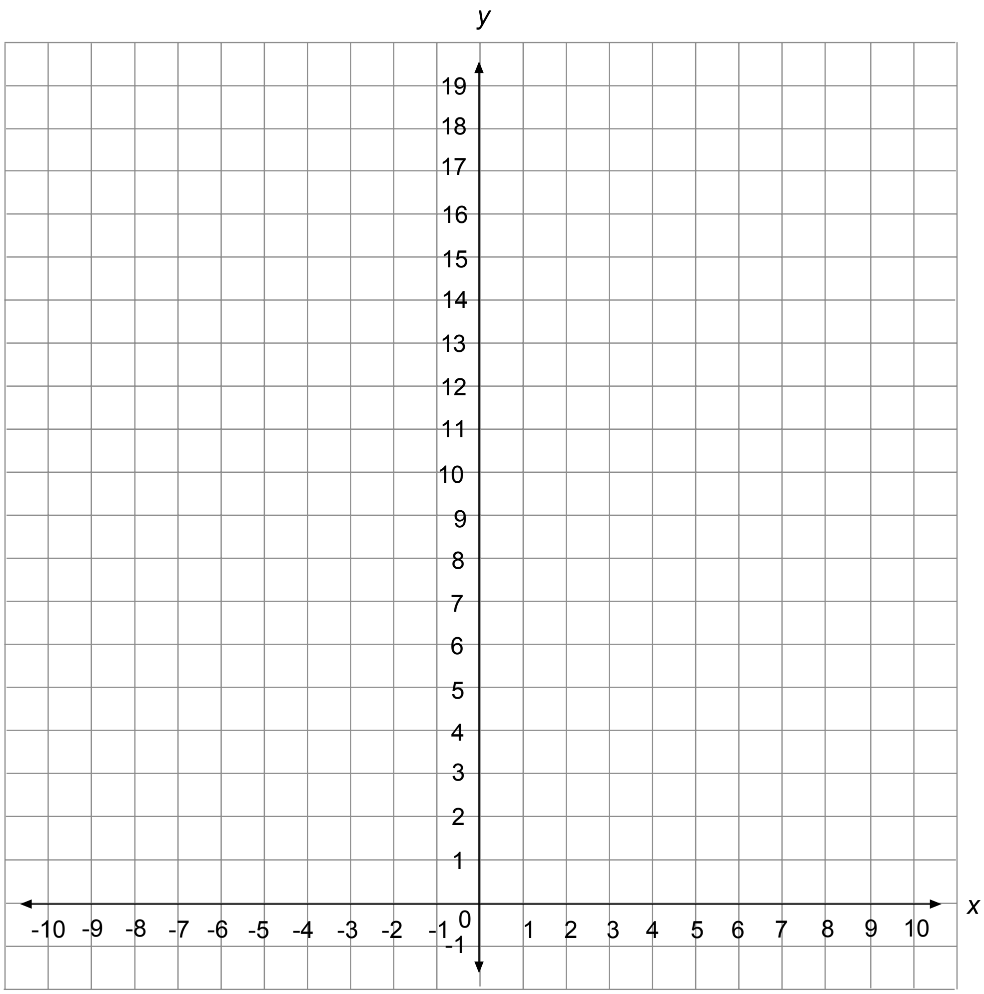
4. $6x + y = 18$

a. y-intercept = ____________

b. x-intercept = ____________

c. graph

Graph of 6*x* + *y* = 18



5. $4x = 8$

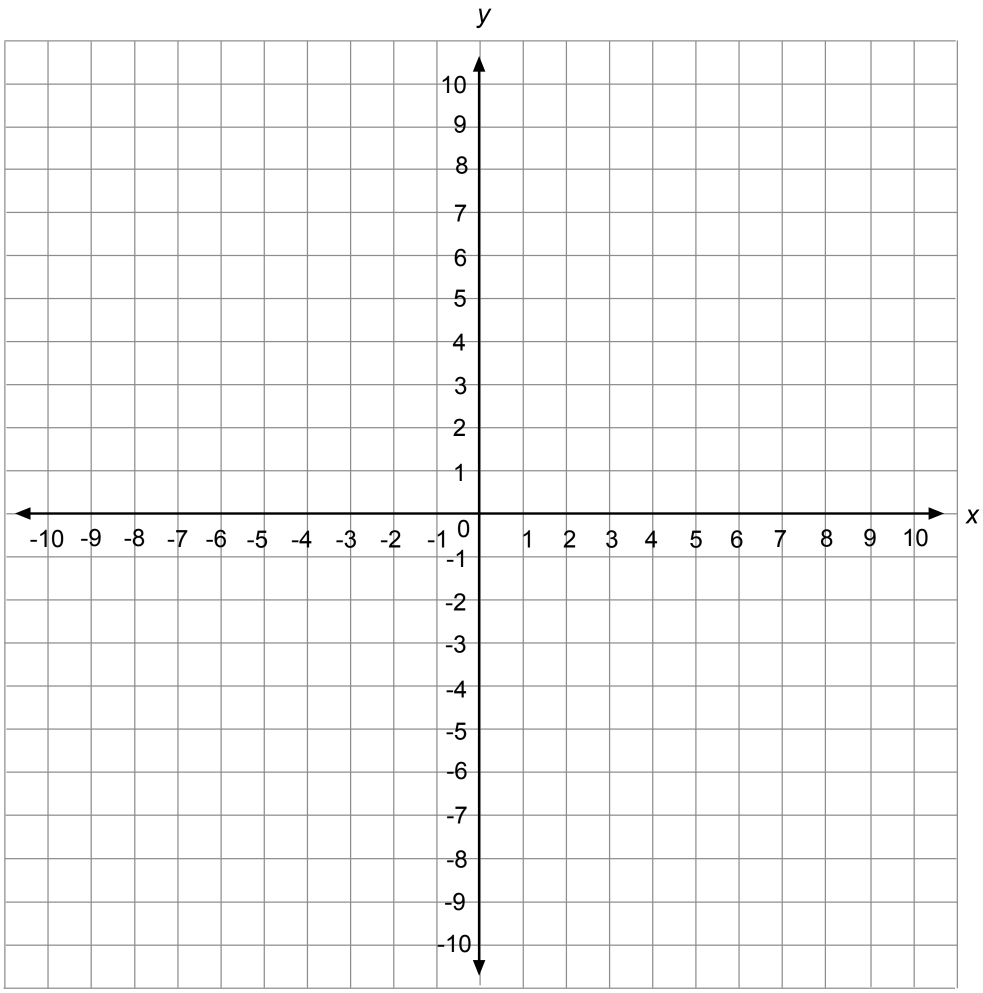
Hint: If there is *no* y-intercept, the line is *vertical*.
If there is *no* x-intercept, the line is *horizontal*.

a. y-intercept = ____________

b. x-intercept = ____________

c. graph

Graph of 4*x* = 8



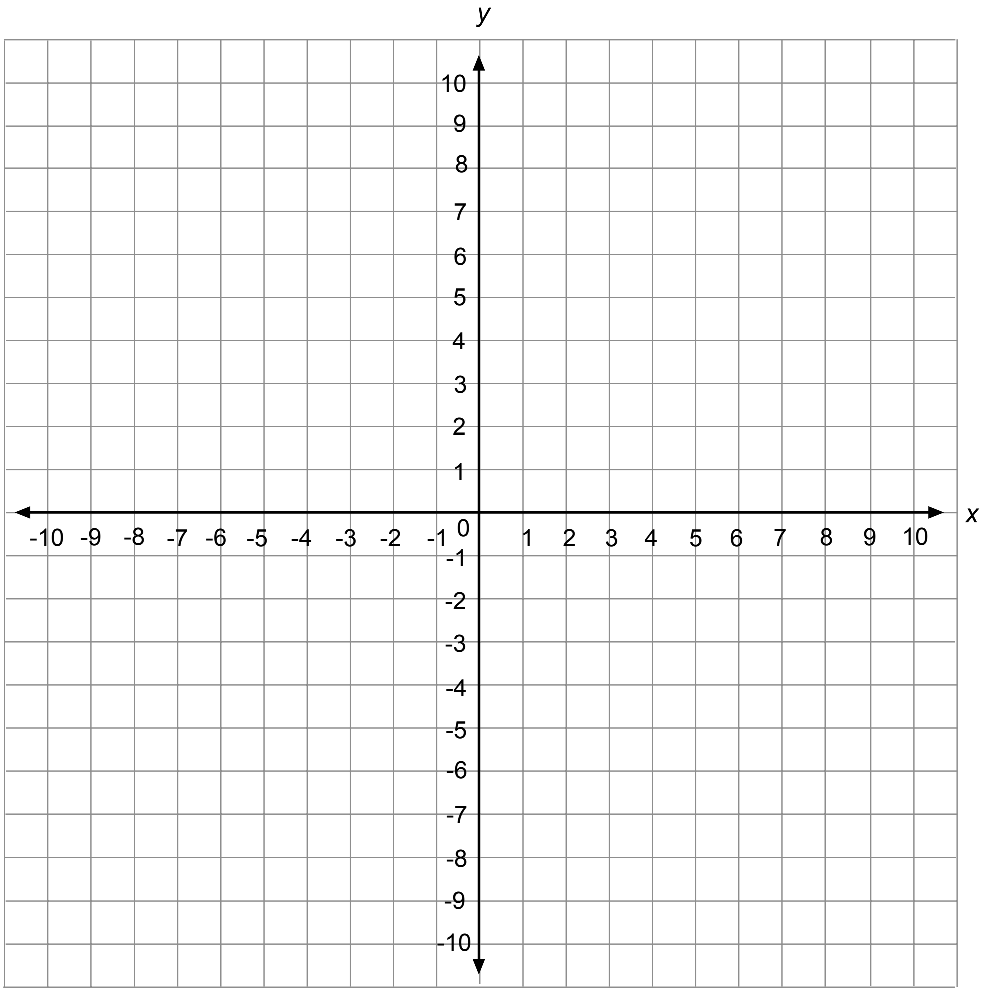
6. $4y = 8$

a. y-intercept = ____________

b. x-intercept = ____________

c. graph

Graph of $4y = 8$



Slope-Intercept Form

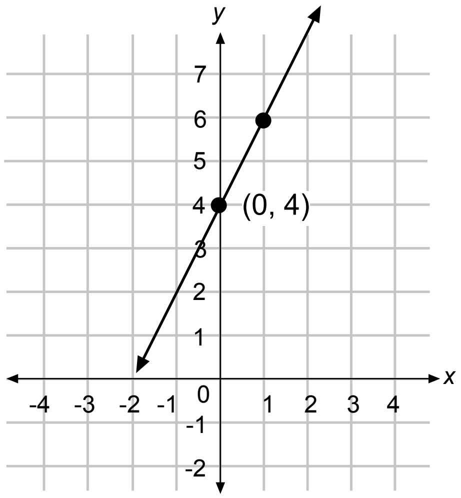
Many students prefer to use the **slope-intercept form** for the equation of a line. An equation in this form tells you the slope of a line and where it crosses the y-axis. The generic format looks like the following.

$$y = mx + b$$

m is the slope b is the y-intercept

So if $y = 2x + 4$, this line crosses the y-axis at 4 and has a slope of 2.

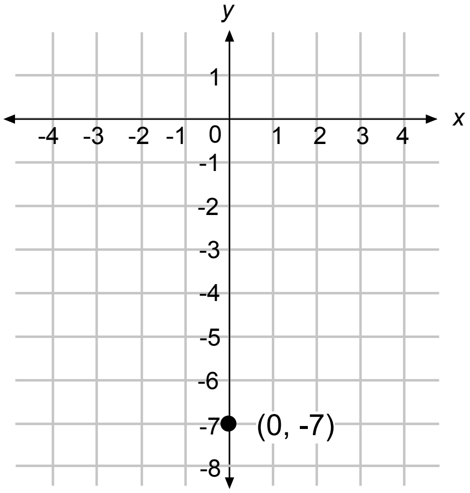
To graph this line, plot a point at (0, 4) and count the rise and run of the slope ($\frac{2}{1}$) from that point and draw a line.



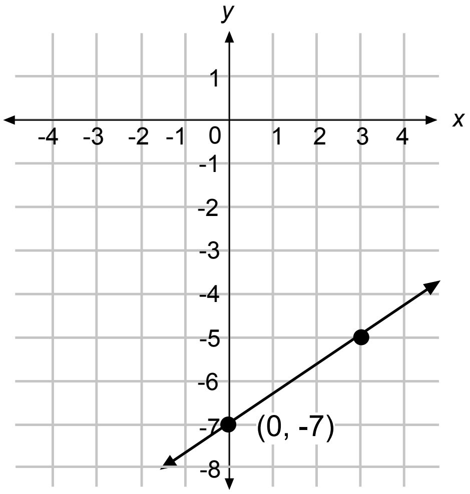
For the equation of the line

$$y = \frac{2}{3}x - 7,$$

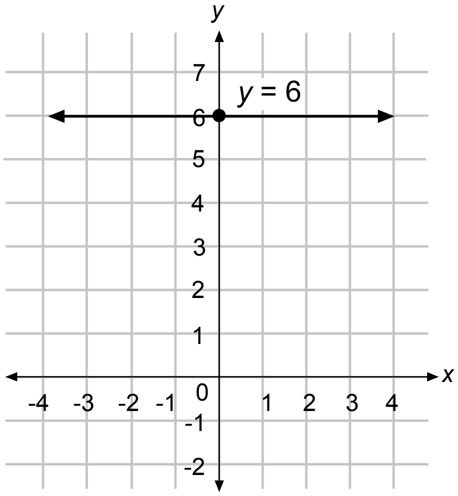
the y-intercept is -7.



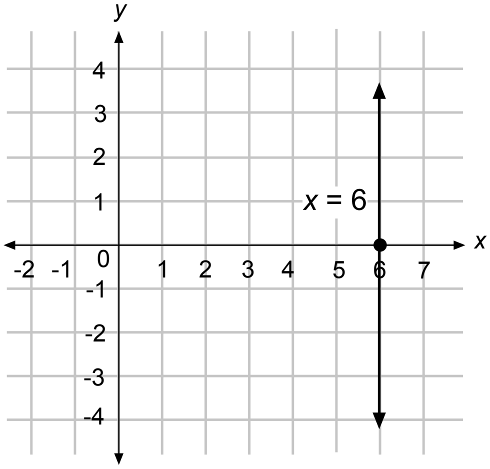
The slope is $\frac{2}{3}$, so the graph looks like the following.



Remember: If the equation looks like $y = 6$, the line is *horizontal* and has *zero* slope. If the equation looks like $x = 6$, the line is *vertical* and has *no* slope. Look at the graphs below.



y = 6
line is horizontal
zero slope



x = 6
line is vertical
no slope
(sometimes referred to as undefined)

Practice

Use the equations to find the **y-intercepts**, *find the* **slopes**, *and* **graph the lines** *of the following.*

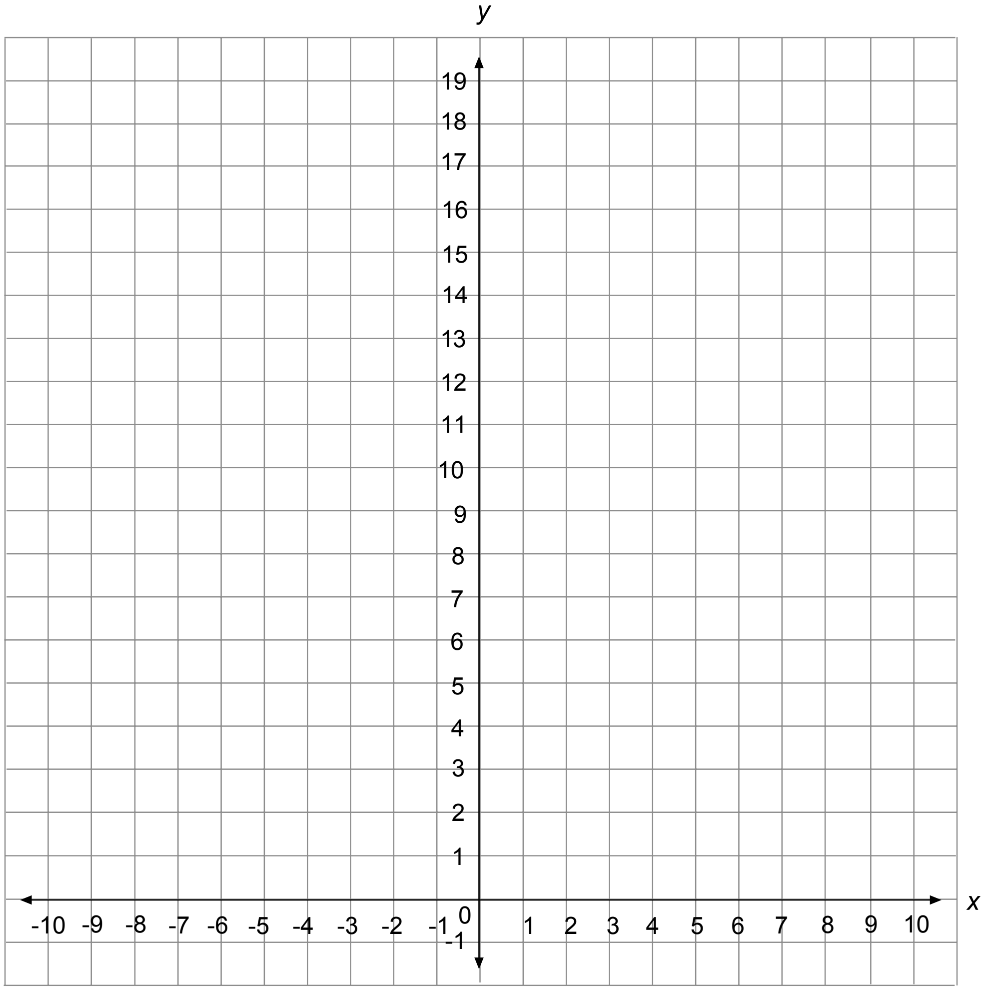
1. $y = 5x + 7$

   a. y-intercept = ____________

   b. slope = ____________

   c. graph

Graph of $y = 5x + 7$



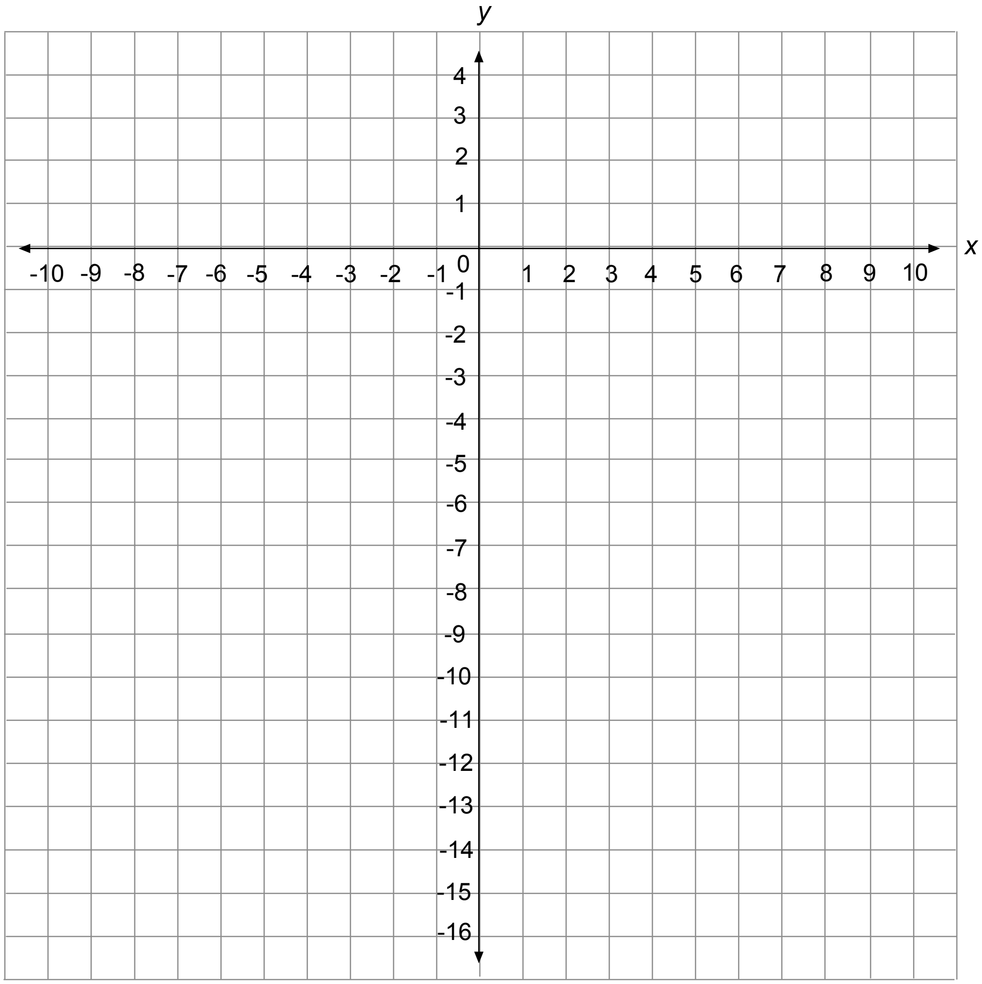
2. $y = -3x - 9$

   a. y-intercept = ____________

   b. slope = ____________

   c. graph

Graph of $y = -3x - 9$



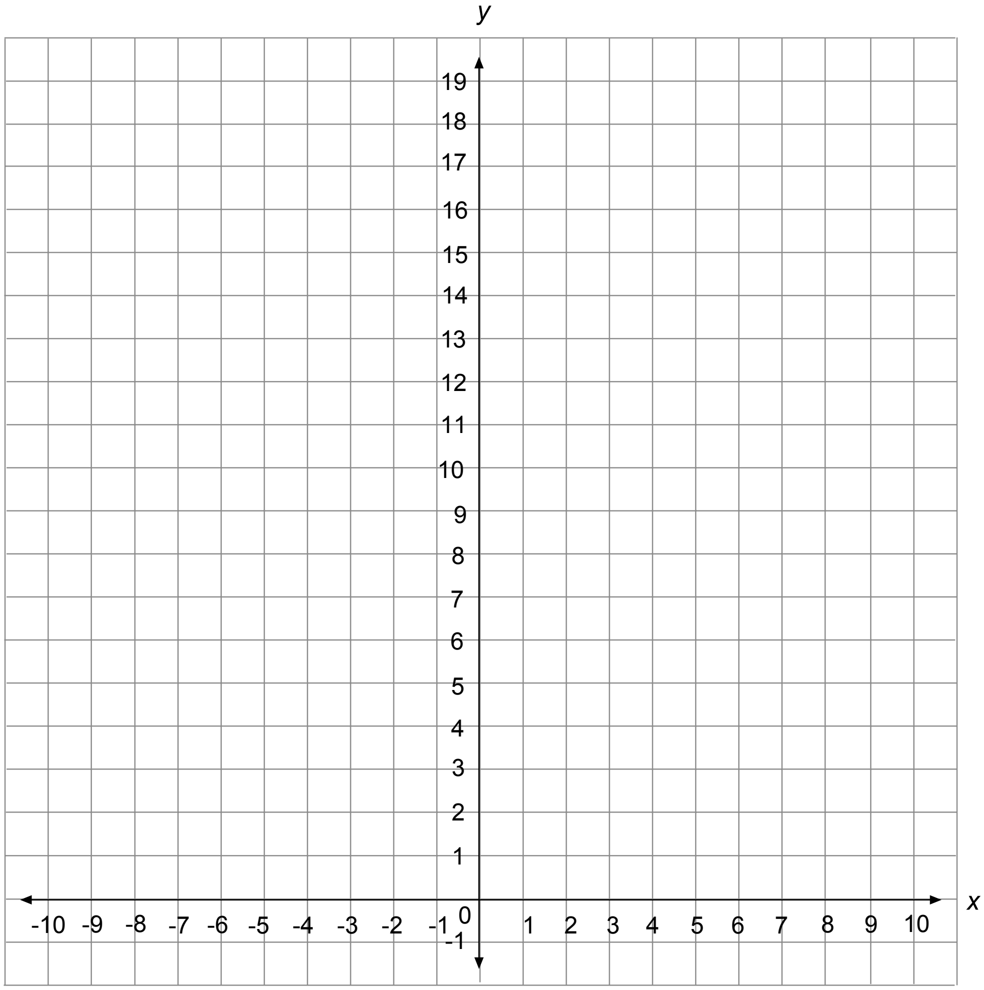
3. $y = \frac{5}{7}x + 11$

a. y-intercept = ____________

b. slope = ____________

c. graph

Graph of $y = \frac{5}{7}x + 11$



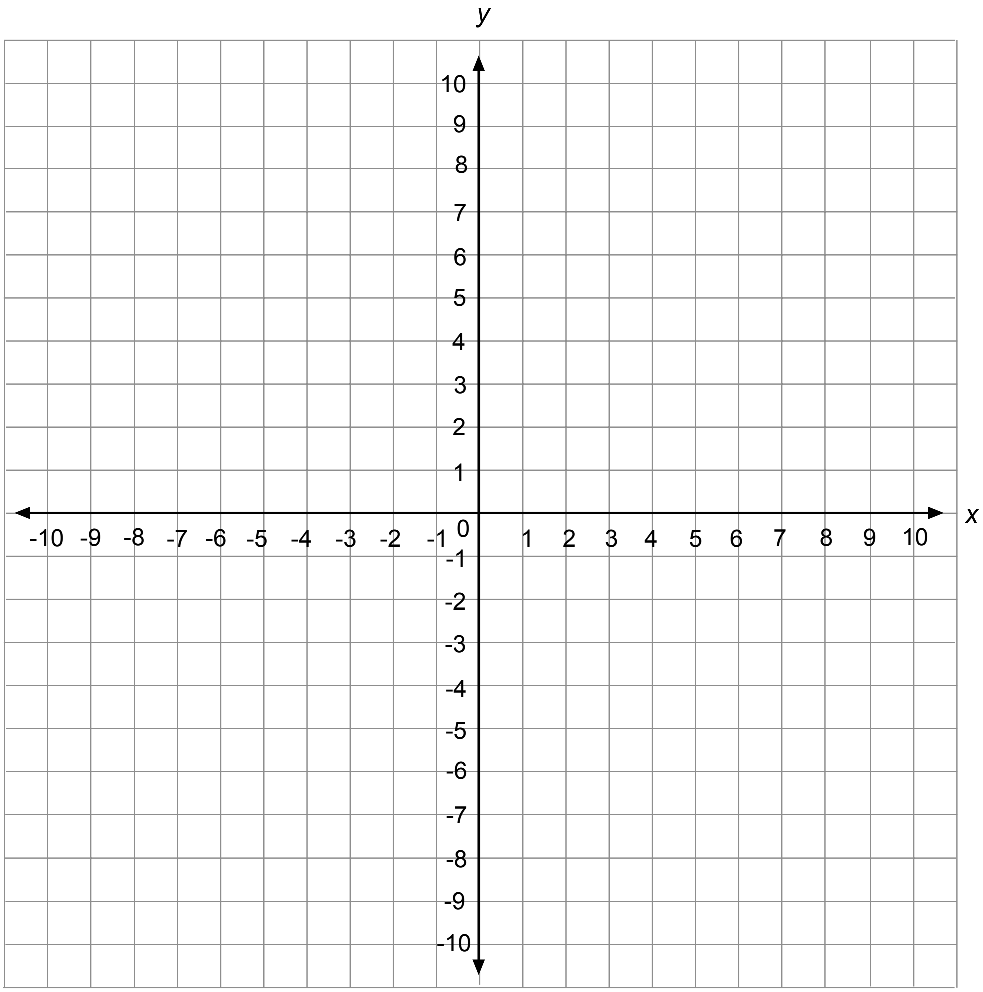
4. $x = -7$

a. y-intercept = ____________

b. slope = ____________

c. graph

Graph of x = -7



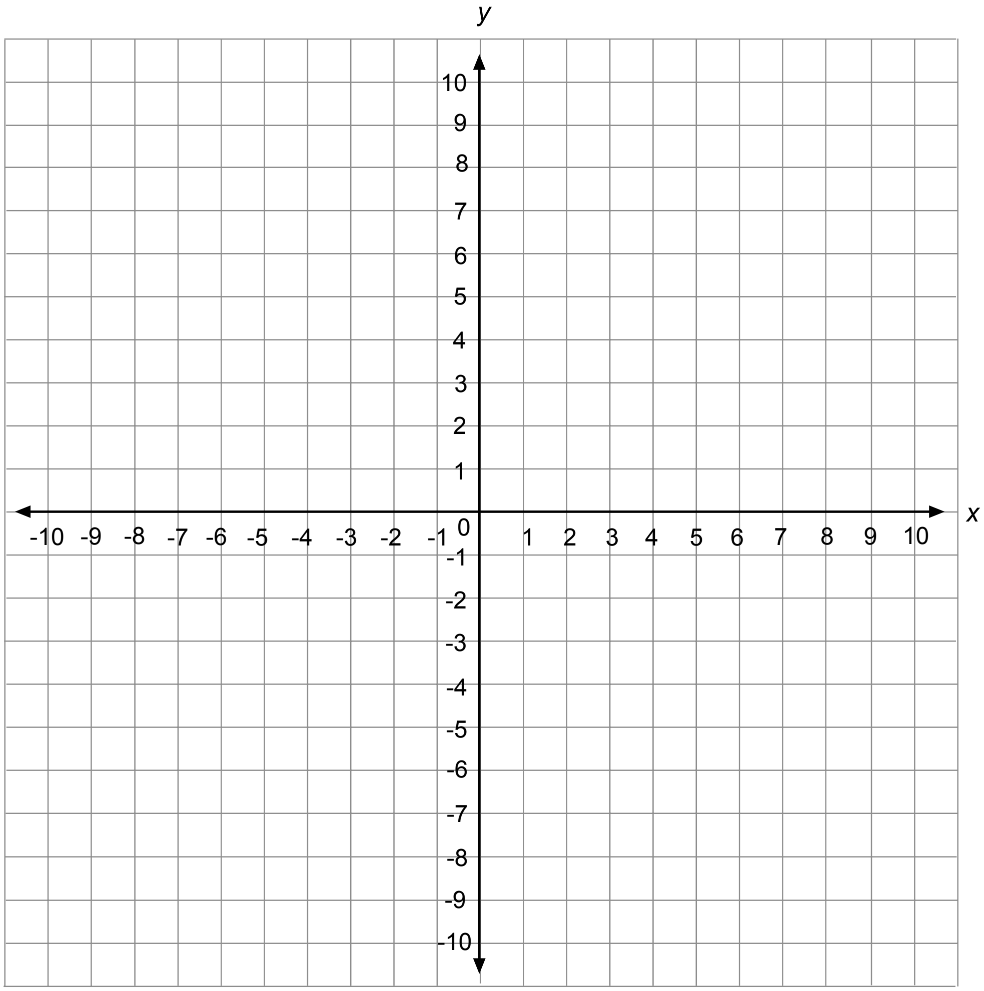
5. $y = \frac{2}{3}x - 4$

a. y-intercept = ____________

b. slope = ____________

c. graph

Graph of $y = \frac{2}{3}x - 4$



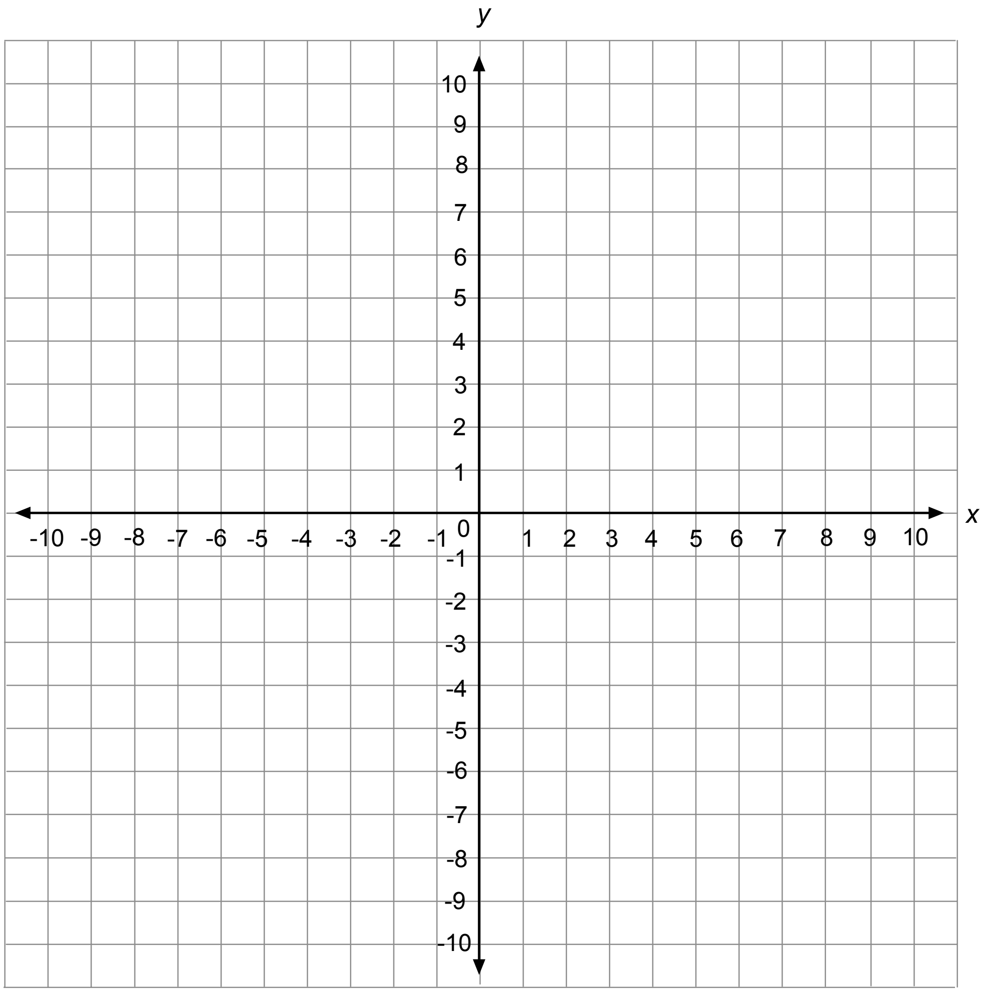
6. $y = -4x + 2$

a. y-intercept = _____________

b. slope = _____________

c. graph

Graph of $y = -4x + 2$



Transforming Equations into Slope-Intercept Form

Sometimes it is necessary to transform an equation into the *slope-intercept form* so that we can readily identify the slope or the y-intercept or both.

Follow these examples. Remember, we want it to be in the $y = mx + b$ format.

Example 1

$$\begin{aligned} 6x - 3y &= 12 \\ -3y &= -6x + 12 && \longleftarrow \text{ subtract } 6x \text{ from both sides} \\ y &= 2x - 4 && \longleftarrow \text{ divide both sides by -3} \end{aligned}$$

Now we can easily see that the slope is 2 and the y-intercept is -4.

Example 2

$$\begin{aligned} x + \tfrac{2}{3}y &= 8 \\ \tfrac{2}{3}y &= -x + 8 && \longleftarrow \text{ subtract } x \text{ from each side} \\ (\tfrac{3}{2})\tfrac{2}{3}y &= -(\tfrac{3}{2})x + (\tfrac{3}{2})8 && \longleftarrow \text{ multiply both sides by } \tfrac{3}{2} \\ y &= -\tfrac{3}{2}x + 12 && \longleftarrow \text{ simplify} \end{aligned}$$

slope = $-\frac{3}{2}$ y-intercept = 12

Practice

Express in **slope-intercept form**, *find the* **y-intercepts**, *find the* **slopes**, *and* **graph the lines** *of the following.*

1. $5x + 3y = -18$

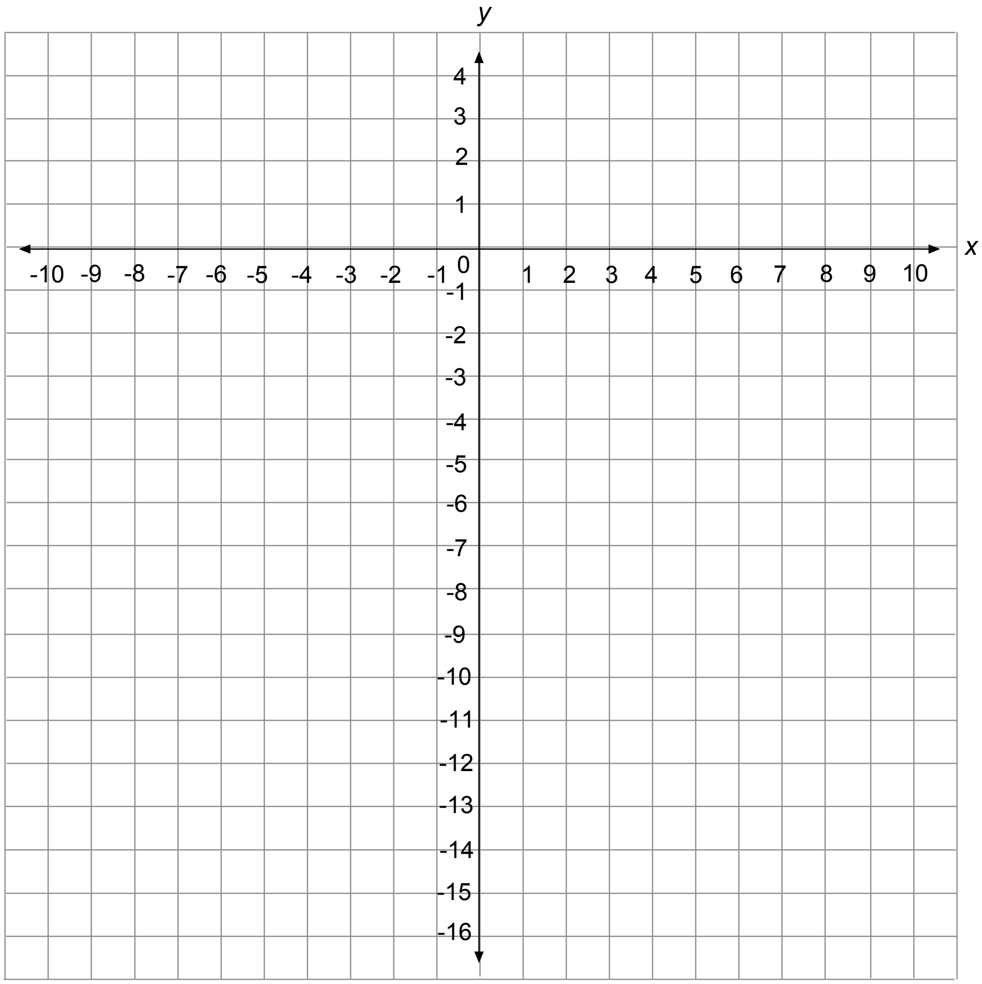
   a. slope-intercept form = ______________________________

   b. y-intercept = _______________

   c. slope = _______________

   d. graph

Graph of $5x + 3y = -18$



2. $2x + y = 8$

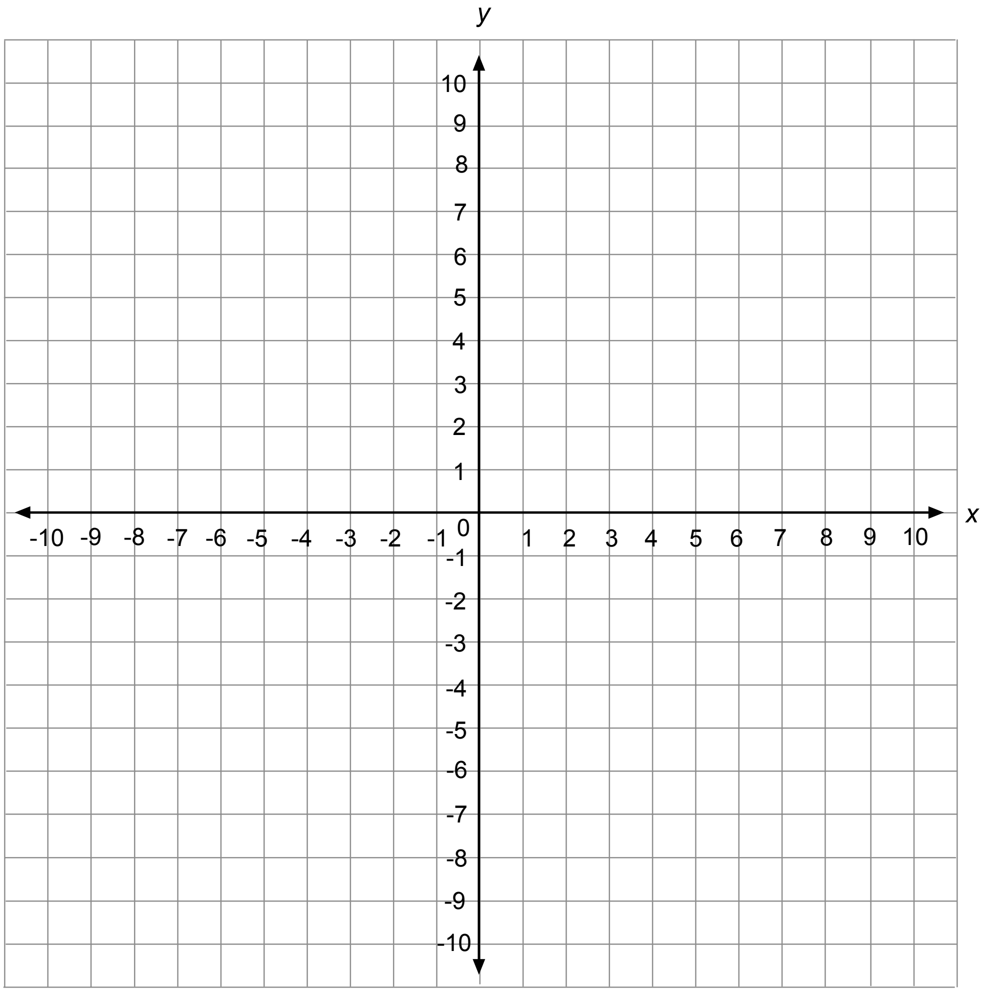
a. slope-intercept form = __________________________

b. y-intercept = _____________

c. slope = _____________

d. graph

Graph of 2*x* + *y* = 8



3. $3x + 3y = 6$

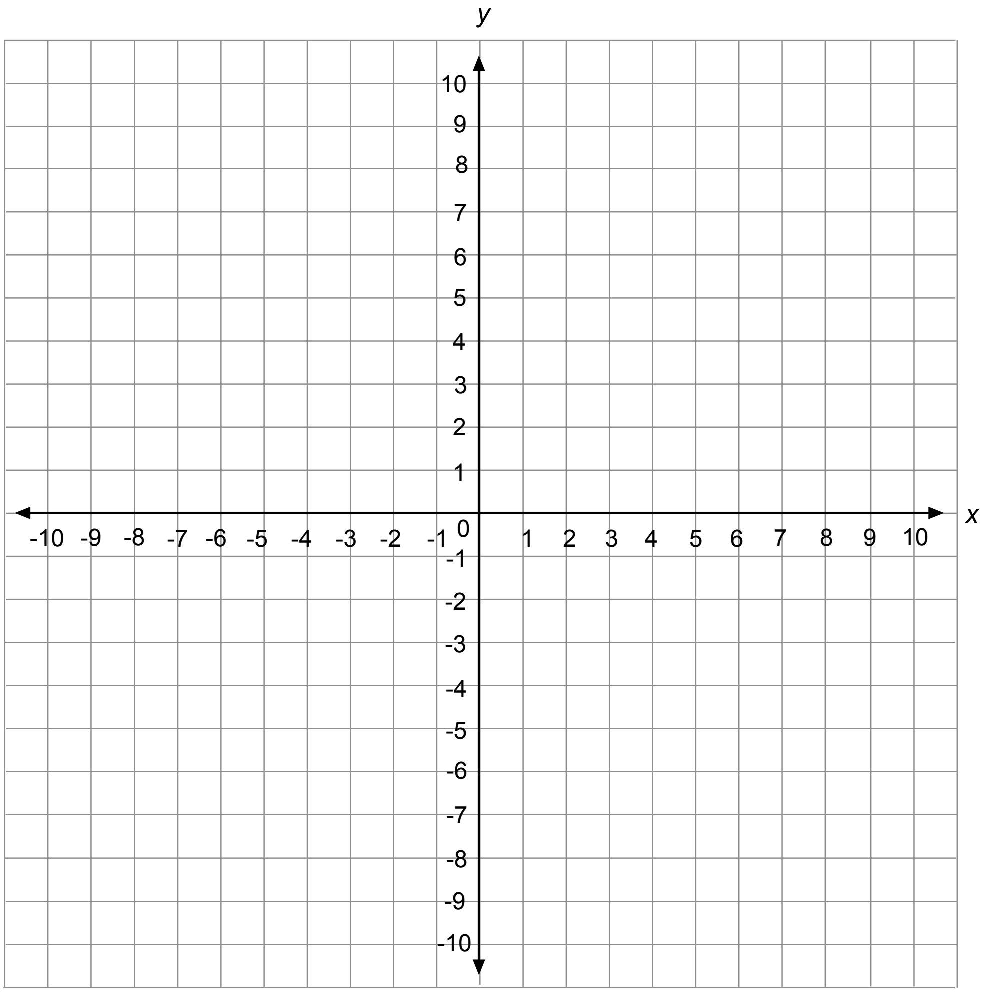
a. slope-intercept form = ____________________

b. y-intercept = __________

c. slope = __________

d. graph

Graph of $3x + 3y = 6$



4. $5x + y = 0$

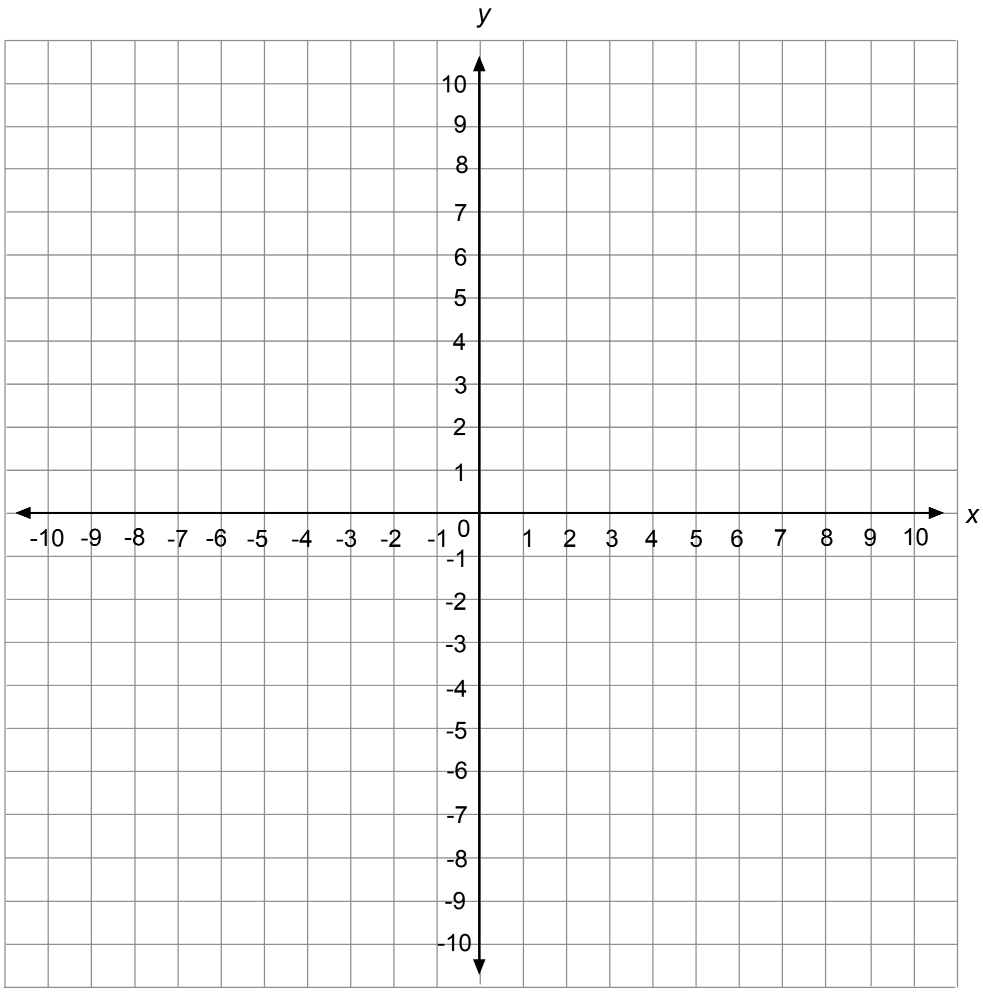
a. slope-intercept form = ______________________________

b. y-intercept = _______________

c. slope = _______________

d. graph

Graph of 5*x* + *y* = 0



5. $2x - y = -2$

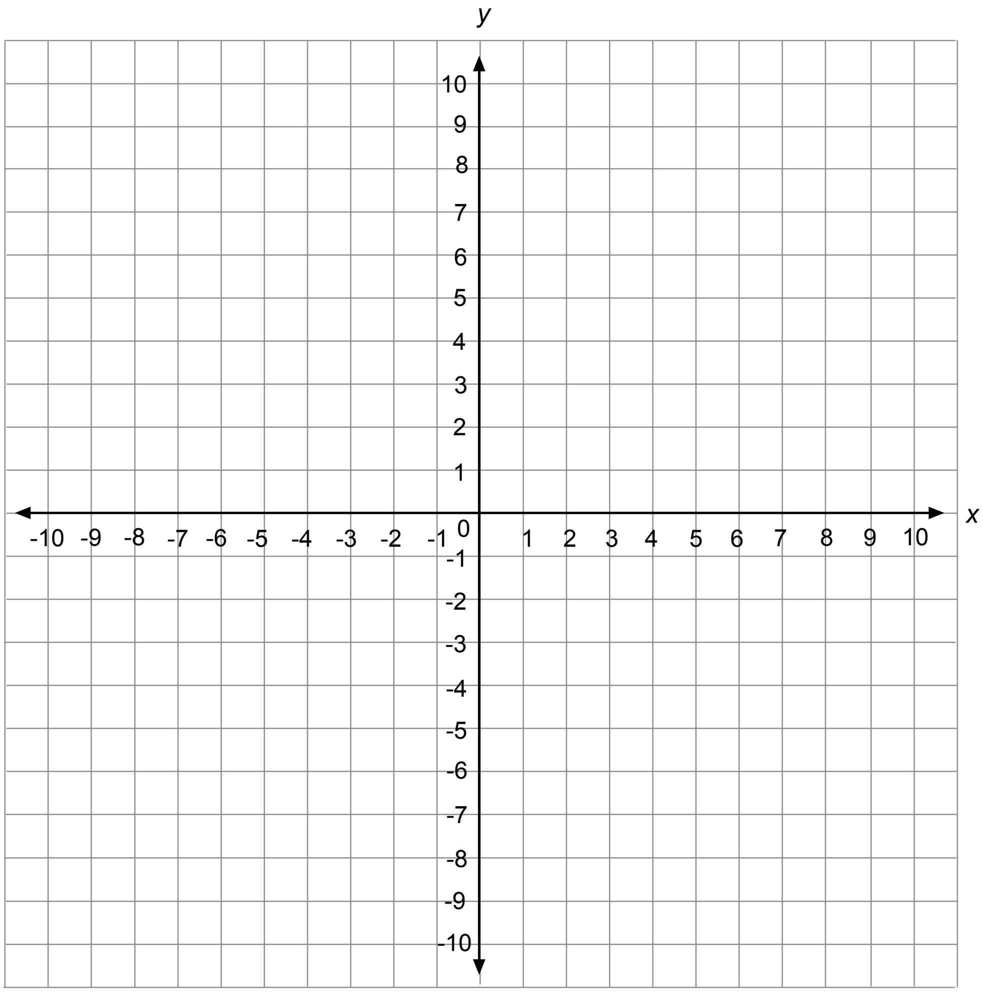
a. slope-intercept form = __________________________

b. y-intercept = _____________

c. slope = _____________

d. graph

Graph of $2x - y = -2$



6. $x - y = -8$

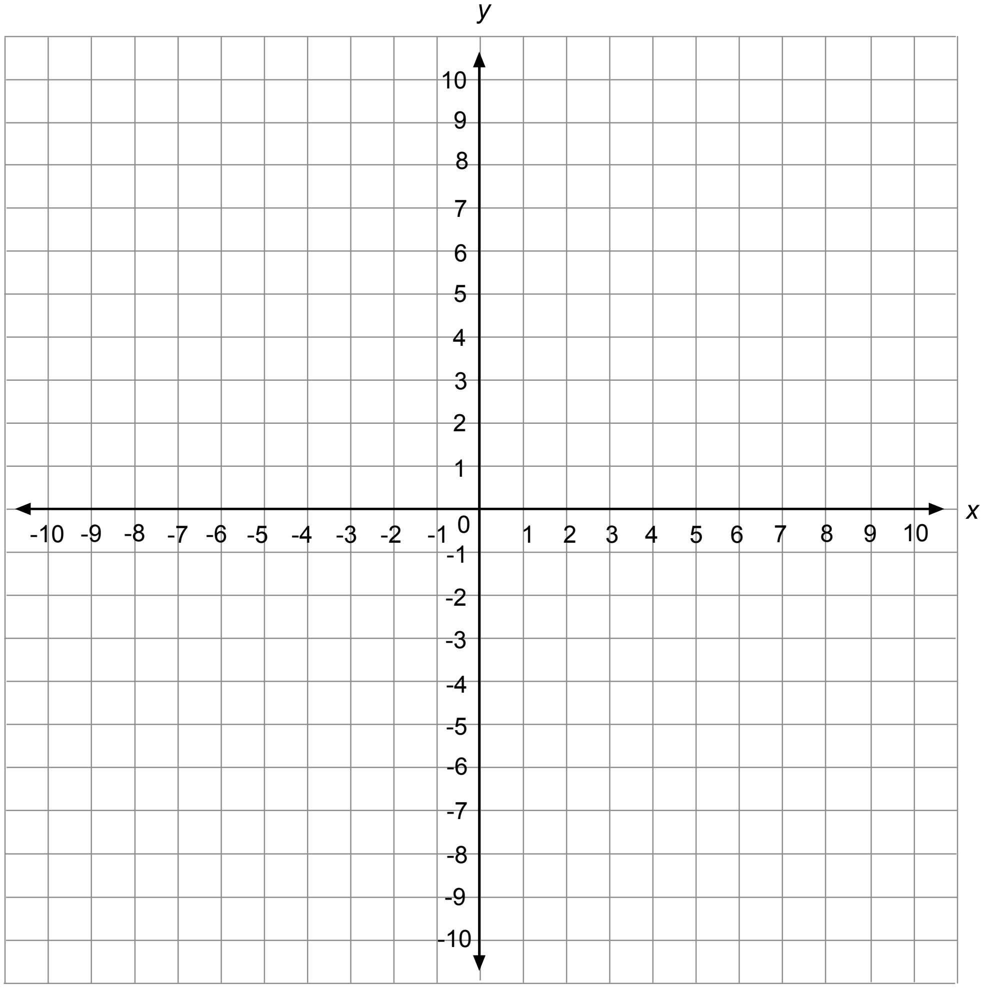
a. slope-intercept form = ______________________________

b. y-intercept = _______________

c. slope = _______________

d. graph

Graph of $x - y = -8$



Practice

Use the list below to write the correct term for each definition on the line provided.

denominator	**rise**	**slope**
linear equation	**run**	**slope-intercept form**
numerator		

________________________ 1. the vertical change on the graph between two points

________________________ 2. a form of a linear equation, $y = mx + b$, where m is the slope of the line and b is the y-intercept

________________________ 3. the ratio of change in the vertical axis (y-axis) to each unit change in the horizontal axis (x-axis) in the form $\frac{\text{rise}}{\text{run}}$; the constant, m, in the linear equation for the slope-intercept form $y = mx + b$

________________________ 4. the top number of a fraction, indicating the number of equal parts being considered

________________________ 5. the bottom number of a fraction, indicating the number of equal parts a whole was divided into

________________________ 6. the horizontal change on a graph between two points

________________________ 7. an equation whose graph in a coordinate plane is a straight line; an algebraic equation in which the variable quantity or quantities are raised to the zero or first power only and the graph is a straight line

Lesson Five Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.

- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.

- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Algebra Body of Knowledge

Standard 3: Linear Equations and Inequalities

- MA.912.A.3.10
  Write an equation of a line given any of the following information: two points on the line, its slope and one point on the line, or its graph. Also, find an equation of a new line parallel to a given line, or perpendicular to a given line, through a given point on the new line.

Geometry Body of Knowledge

Standard 1: Points, Lines, Angles, and Planes

- MA.912.G.1.4
  Use coordinate geometry to find slopes, parallel lines, perpendicular lines, and equations of lines.

Parallel and Perpendicular Lines

When two lines are on the same coordinate plane, there are two possibilities. Either the two lines are **parallel (||)** to each other or they **intersect** each other.

If two lines are *parallel* to each other, we can say that the lines are always the same distance apart and will never *intersect*. This happens when the two lines have the same *slant*. In other words, two **parallel lines** have *equal* slopes.

For example, the two lines, $y = \mathbf{5}x + 13$ and $y = \mathbf{5}x - 6$ are parallel because in each line, m has a value of **5**.

If two lines intersect, they cross each other at some point. You may not see that point where they cross on the particular picture, but remember that lines extend forever and their slopes may be such that they will eventually cross. If the two lines intersect at a **right angle** or at 90 **degrees (°)**, they are **perpendicular (⊥)**. Keep in mind that when this happens, their slopes will be negative **reciprocals** of each other.

A line whose equation is $y = \frac{3}{2}x - 5$ is *perpendicular* to a line whose equation is $y = -\frac{2}{3}x + 6$. Notice that their slopes are $\frac{3}{2}$ and $-\frac{2}{3}$.

Note: If you multiply the slopes of two **perpendicular lines**, the **product** will be -1, unless one of the lines was vertical.

Practice

Use the slope formula below to find the **slopes** *of* $\overleftrightarrow{AB}$ *and* $\overleftrightarrow{CD}$. *Then* **multiply the slopes** *to determine if they are* **parallel**, **perpendicular**, *or* **neither**. **Show all your work.** *Write the answer on the line provided.* **The first one has been done for you.**

slope formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Remember:

- If slopes are **equal**, the lines are **parallel**.
- If slopes are **negative reciprocals**, the lines are **perpendicular**.

parallel 1. *A* (3, 2), *B* (-5, 6), *C* (-4, 1), *D* (-2, 0)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \qquad m = y_2 - y_1 =$$

$$\frac{6 - 2}{-5 - 3} = \qquad \frac{0 - 1}{-2 - -4} =$$

$$\frac{6 + -2}{-5 + -3} = \qquad \frac{0 + -1}{-2 + {}^{+}4} =$$

$$\frac{4}{-8} = \qquad -\frac{1}{2}$$

$$-\frac{1}{2} =$$

The slopes are equal; therefore, the lines are *parallel*.

________________________ 2. *A* (5, 7), *B* (0, 4), *C* (2, -6), *D* (-3, 7)

________________________ 3. *A* (2, 4), *B* (-6, -6), *C* (3, -3), *D* (-1, -8)

________________________ 4. *A* (0, 5), *B* (3, 5), *C* (6, 7), *D* (6, 3)

Remember:

$\frac{0}{x}$ = a line that is horizontal with a zero (0) slope
$\frac{y}{0}$ = a line that is vertical with no slope

________________________ 5. *A* (3, 8), *B* (4, 5), *C* (0, 0), *D* (6, -4)

________________________ 6. *A* (4, 4), *B* (-4, -4), *C* (-4, 4), *D* (4, -4)

________________________ 7. *A* (-2, -2), *B* (2, 4), *C* (1, 6), *D* (-1, 3)

________________________ 8. *A* (8, -8), *B* (0, -6), *C* (3, 13), *D* (-3, -11)

Practice

Put equations in **slope-intercept form**. **Show all your work.** *Determine if the following* **lines** *are* **parallel**, **perpendicular**, *or* **neither**. *Write the answer on the line provided.*

slope-intercept form

$y = mx + b$

_______________________ 1. $3x + y = 7$
$y + 6 = 3x$

_______________________ 2. $x - y = -6$
$x + y = 6$

_______________________ 3. $x - 3y = -21$
$x + 3y = 21$

______________________ 4. $5x - y = 9$
$5x - y = 4$

______________________ 5. $3x - y = 4$
$4x - y = -3$

______________________ 6. $x = 6$
$y = -1$

______________________ 7. $2x + 3y = 5$
$3x - 2y = 7$

Practice

Use the list below to complete the following statements.

distance	**line segment**	**perpendicular**
horizontal	**midpoint**	**slope**
hypotenuse	**parallel**	**vertical**

1. The slant or ______________________ of a line is defined as $\frac{\text{rise}}{\text{run}}$.

2. A line that has no slope is called a ______________________ line.

3. The ______________________ between two points is the length of the segment that connects the two points.

4. The ______________________ is the segment in a right triangle that is opposite the right angle.

5. Lines in the same plane that do not intersect are called ______________________ lines.

6. A line that has zero slope is a ______________________ line.

7. The point located exactly halfway between two endpoints of a line segment is called the ______________________ .

8. If two lines intersect to form right angles, they are ______________________ lines.

9. The figure that contains two defined endpoints and all the points in between is called a ______________________ .

Practice

Match each definition with the correct term. Write the letter on the line provided.

_______ 1. the square of the hypotenuse (c) of a right triangle is equal to the sum of the square of the legs (a and b), as shown in the equation $c^2 = a^2 + b^2$

_______ 2. two lines, two line segments, or two planes that intersect to form a right angle

_______ 3. an angle whose measure is exactly 90°

_______ 4. two lines in the same plane that are a constant distance apart; lines with equal slopes

_______ 5. two numbers whose product is 1; also called *multiplicative inverses*

_______ 6. to meet or cross at one point

_______ 7. a way of expressing a relationship using variables or symbols that represent numbers

_______ 8. the result of multiplying numbers together

A. formula

B. intersect

C. parallel lines

D. perpendicular (⊥)

E. product

F. Pythagorean theorem

G. reciprocals

H. right angle

Lesson Six Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.

- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.

- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Algebra Body of Knowledge

Standard 1: Real and Complex Number Systems

- MA.912.A.1.8
  Use the zero product property of real numbers in a variety of contexts to identify solutions to equations.

Standard 3: Linear Equations and Inequalities

- MA.912.A.3.7
  Rewrite equations of a line into slope-intercept form and standard form.

- MA.912.A.3.10
  Write an equation of a line given any of the following information: two points on the line, its slope and one point on the line, or its graph. Also, find an equation of a new line parallel to a given line, or perpendicular to a given line, through a given point on the new line.

Geometry Body of Knowledge

Standard 1: Points, Lines, Angles, and Planes

- MA.912.G.1.4
  Use coordinate geometry to find slopes, parallel lines, perpendicular lines, and equations of lines.

Point-Slope Form

We know that if we have two points we can draw a line that connects them. But did you know we can also produce the equation of that line using those points?

To do this, we will use yet another format for the equation of a line. It is called the point-slope form. Notice that it looks a bit like the slope-intercept format, but it has a little extra.

$(y - y_1) = m(x - x_1)$ point-slope form

(x_1, y_1) is one of the coordinates given

m = slope

Let's see how this works.

Example 1

Find the equation of the line which passes through points (3, 5) and (-2, 1).

- Start with the following equation.

$$y - y_1 = m(x - x_1)$$

- Find the slope using the two points.

$$m = \frac{1-5}{-2-3} = \frac{-4}{-5} = \frac{4}{5} \qquad \longleftarrow \text{ the slope is } \frac{4}{5}$$

- Select one of the given points (3, 5).
- Replace x_1 and y_1 with the coordinates from the point you selected, and then replace m with the slope ($\frac{4}{5}$) that you found.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 5 &= \tfrac{4}{5}(x - 3) \end{aligned}$$

- Simplify.

$$\begin{aligned} y - 5 &= \tfrac{4}{5}(x - 3) && \\ y - 5 &= \tfrac{4}{5}x - \tfrac{12}{5} && \longleftarrow \text{ distribute } \tfrac{4}{5} \\ y &= \tfrac{4}{5}x - \tfrac{12}{5} + 5 && \longleftarrow \text{ add 5 to both sides} \\ y &= \tfrac{4}{5}x - \tfrac{12}{5} + \tfrac{25}{5} && \longleftarrow \text{ get a } \textbf{common denominator} \\ y &= \tfrac{4}{5}x + \tfrac{13}{5} && \longleftarrow \text{ simplify} \end{aligned}$$

This is the equation of the line in slope-intercept form: $y = mx + b$ with $m = \frac{4}{5}$, $b = \frac{13}{5}$.

We could also transform this equation to standard form of $ax + by = c$ using a bit of algebra.

$$\begin{aligned} y &= \tfrac{4}{5}x + \tfrac{13}{5} && \\ -\tfrac{4}{5}x + y &= \tfrac{13}{5} && \longleftarrow \text{ subtract } \tfrac{4}{5}x \text{ from both sides} \\ 4x - 5y &= -13 && \longleftarrow \text{ multiply both sides by -5} \end{aligned}$$

How about another example before you try this yourself?

Example 2

Find the equation of the line in both y-intercept and standard form that passes through the points (-4, 0) and (-2, 2).

- Start with the following equation.

$$y - y_1 = m(x - x_1)$$

- Find the slope.

$$m = \frac{2 - 0}{-2 - -4} = \frac{2}{2} = \frac{1}{1} = 1 \longleftarrow \text{the slope is 1}$$

- Select a point.

(-2, 2)

- Replace x_1, y_1, and m.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 2 &= 1(x - -2) \end{aligned}$$

- Simplify.

$$\begin{aligned} y - 2 &= 1(x - -2) \\ y - 2 &= 1(x + 2) \\ y - 2 &= x + 2 \\ y &= x + 4 \qquad \longleftarrow \text{equation in slope-intercept form} \end{aligned}$$

- Transform $y = x + 4$ into standard form.

$$\begin{aligned} -x + y &= 4 \qquad \longleftarrow \text{multiply both sides by -1} \\ x - y &= -4 \qquad \longleftarrow \text{equation in standard form} \end{aligned}$$

Look at some other situations when using the point-slope format is helpful.

Example 3

Write an equation in point-slope form of the line that passes through (2, -3) and has a slope of $-\frac{3}{8}$.

$$y - y_1 = m(x - x_1)$$

- We can skip finding the slope—it is already done for us!

$$m = -\frac{3}{8}$$

- There is no need to select a point because we only have one to choose.

(2, -3)

- Replace x_1, y_1, and m, and simplify.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - (-3) &= -\tfrac{3}{8}(x - 2) \\ y + 3 &= -\tfrac{3}{8}(x - 2) \end{aligned}$$

Ta-da! We are finished! We have written the equation in point-slope form.

Example 4

Write an equation in point-slope form for a horizontal line passing through the point (-4, 2).

$$y - y_1 = m(x - x_1)$$

- Slope = 0 (horizontal lines have zero slope)

$$m = 0$$

- Use the point (-4, 2), replace x_1, y_1, and m, and simplify.

$$\begin{aligned} y - y_1 &= m(x - x_1) \\ y - 2 &= 0(x - -4) \\ y - 2 &= 0(x + 4) \\ y - 2 &= 0x + 0 \\ y - 2 &= 0 \\ y &= 2 \end{aligned}$$

All *horizontal* lines have equations that look like y = "a number." That number will *always* be the y-coordinate from any point on the line.

Time for some practice…here we go!

Practice

Write an equation in **point-slope form** *for the* **line** *that passes through the* **given point with the given slope**.

1. $(7, 2), m = -\frac{3}{4}$

2. $(-1, -3), m = 8$

3. $(4, -5), m = -\frac{3}{8}$

4. $(2, 2), m = 0$

Put the following **equations of lines** *into* **slope-intercept form** *and* **standard form**.

slope-intercept form	**standard form**
$y = mx + b$	$ax + by = c$

5. $y - 2 = \frac{3}{2}(x - 8)$

   a. slope-intercept form = ____________________________

   b. standard form = ____________________________

6. $y + 3 = -5(x + 1)$

   a. slope-intercept form = ____________________________

   b. standard form = ____________________________

7. $y + 5 = 2(x - 4)$

   a. slope-intercept form = ______________________________

   b. standard form = ______________________________

8. $y - 4 = -6(x + 1)$

   a. slope-intercept form = ______________________________

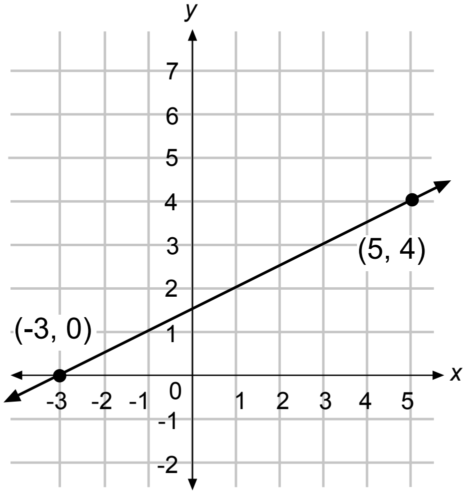
   b. standard form = ______________________________

Write an equation in **slope-intercept form** *for each* **line** *below.*

Hint: Use point-slope form and convert.

point-slope form
$(y - y_1) = m(x - x_1)$

9.



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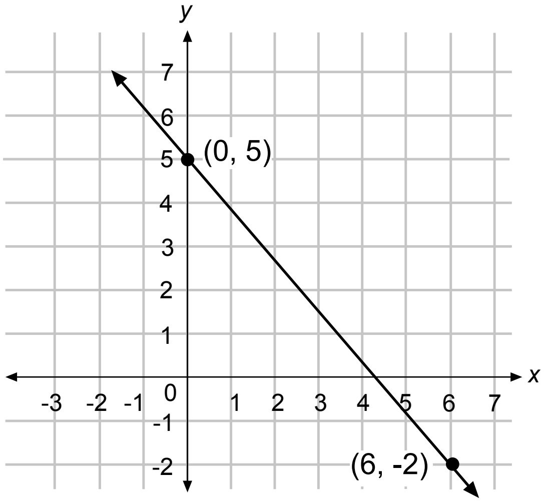

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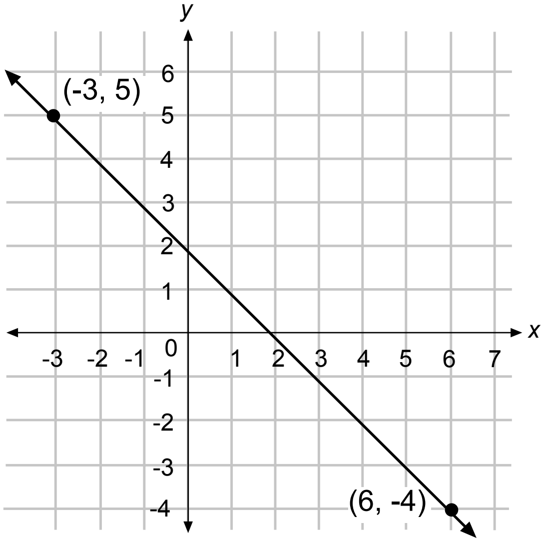
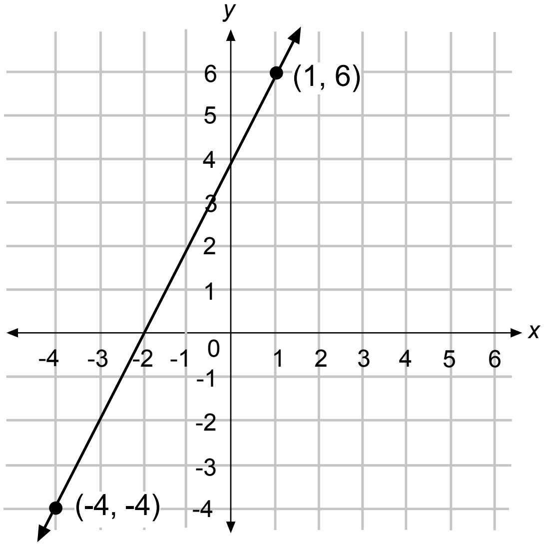

y
x
(-3, 5)
(6, -4)
6
5
4
3
2
1
0
-1
-2
-3
-4
-3
-2
-1
1
2
3
4
5
6
7

12.


y
x
(1, 6)
(-4, -4)
6
5
4
3
2
1
0
-1
-2
-3
-4
-4
-3
-2
-1
1
2
3
4
5
6

Write an equation in **slope-intercept form** *for the* **line** *which passes through each* **pair of points** *below.*

13. (-2, 4), (4, 5)

14. (1, 0), (-3, 8)

15. (0, 0), (7, -7)

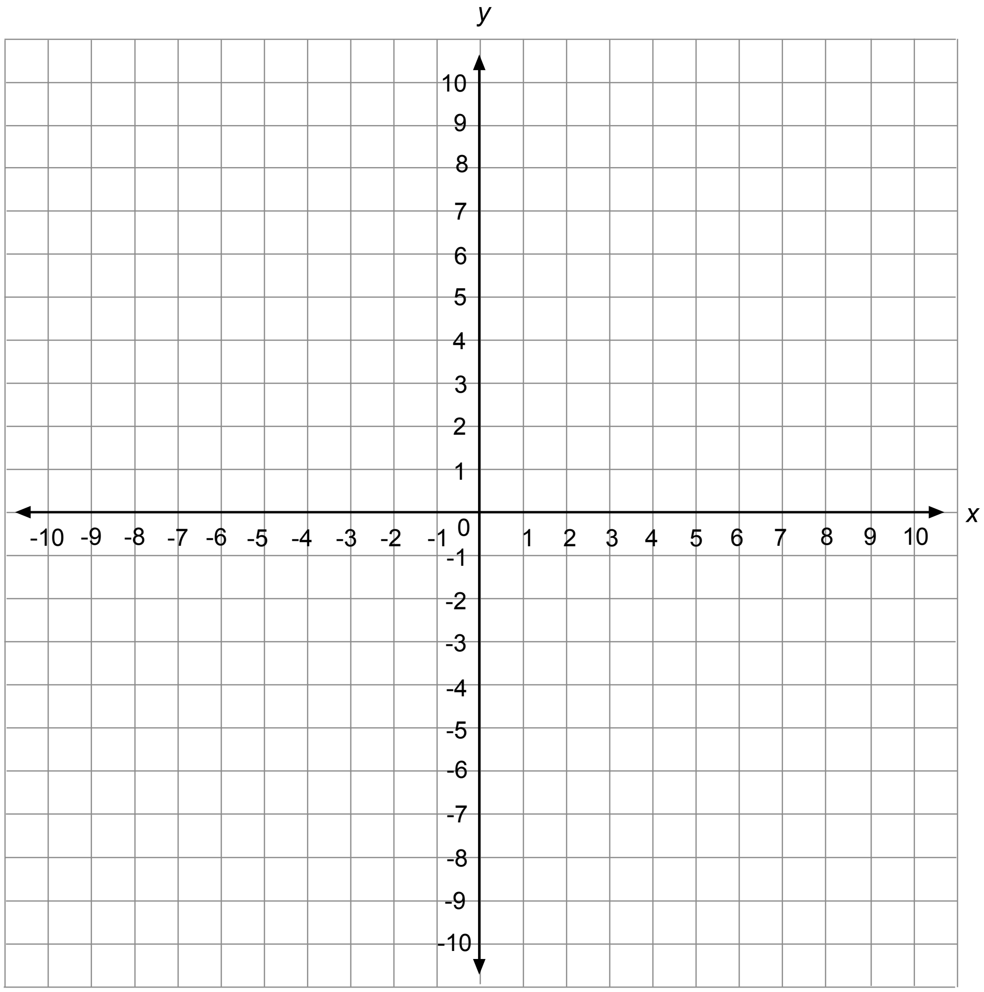
16. (4, -2), (-2, 8)

Unit Review

Solve the following.

1. Plot points (3, -2) and (-6, 4). Draw a triangle and use the Pythagorean theorem below to find the distance between the two points.

Pythagorean theorem
$a^2 + b^2 = c^2$



2. Use the distance formula below to find the distance between (-2, 4) and (7, -3).

distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

3. Use either the method above or the Pythagorean theorem below to find the distance between (5, 1) and (-1, 9).

Pythagorean theorem

$$a^2 + b^2 = c^2$$

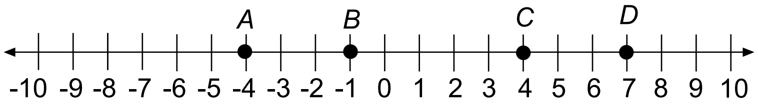
4. On the number line below, find the **midpoint** between *A* and *B*. Use either of the methods below.

   - **Use the number line and count in from both endpoints** of a line segment until you reach the middle to determine the midpoint.

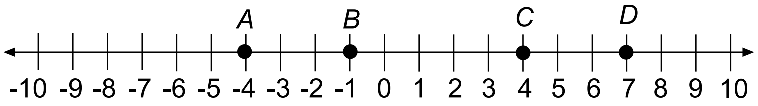
   - **Use the Method One midpoint formula** and add the two endpoints together, then divide by two. **Show all your work.**

Method One midpoint formula

$$\frac{a+b}{2}$$



5. On the number line below, find the **midpoint** of $\overline{BD}$. Use either method above.

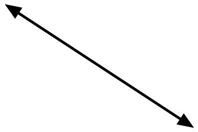


Use the list below to correctly describe the following lines. Write the answer on the line provided.

falling
horizontal
rising
vertical

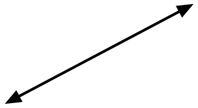
________________________ 6.

________________________ 7.



________________________ 8.

________________________ 9.



Use the **slope formula** *below to find the* **slopes** *through each* **pair of points**.

slope formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

10. (3, -8), (5, 7)

11. (-2, 0), (6, -3)

Use the **slope-intercept form** *below to find the* **slope** *for each* **line**.

slope-intercept form

$$y = mx + b$$

12. $y = \frac{1}{2}x - 7$

13. $y = -2x + 6$

Use the **slope formula** *below to find the* **slopes** *of* $\overleftrightarrow{AB}$ *and* $\overleftrightarrow{CD}$*. Then* **multiply the slopes** *to determine if they are* **parallel***,* **perpendicular***, or* **neither**. **Show all your work.** *Write the answer on the line provided. .*

slope formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

________________________ 14. *A* (2, -5), *B* (4, 5), *C* (-3, 8), *D* (2, 7)

________________________ 15. *A* (-4, 0), *B* (6, 1), *C* (4, 3), *D* (-6, 2)

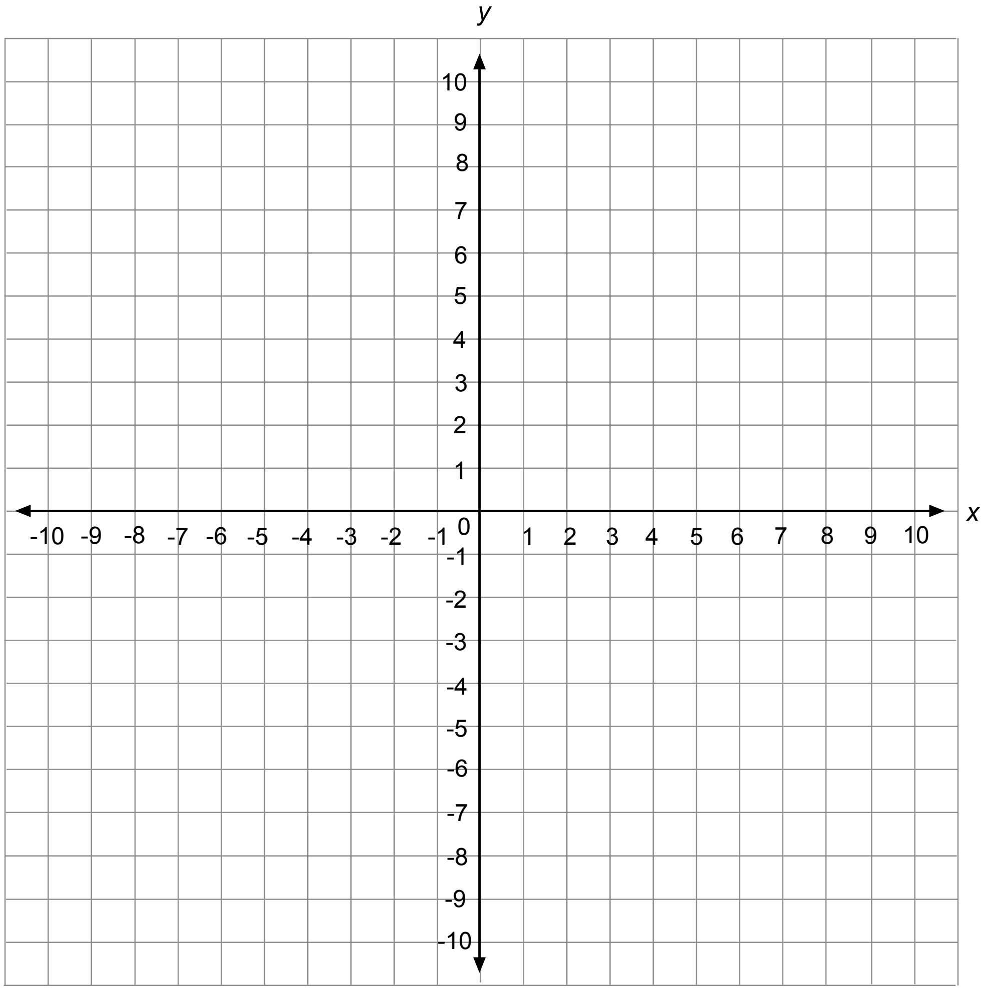
Use the equation $y = -\frac{3}{2}x + 4$ *to answer each of the following.*

16. Give the *y*-intercept. ______________

17. Give the slope. ______________

18. Graph the line.

Graph of $y = -\frac{3}{2}x + 4$



19. Write the equation in standard form.

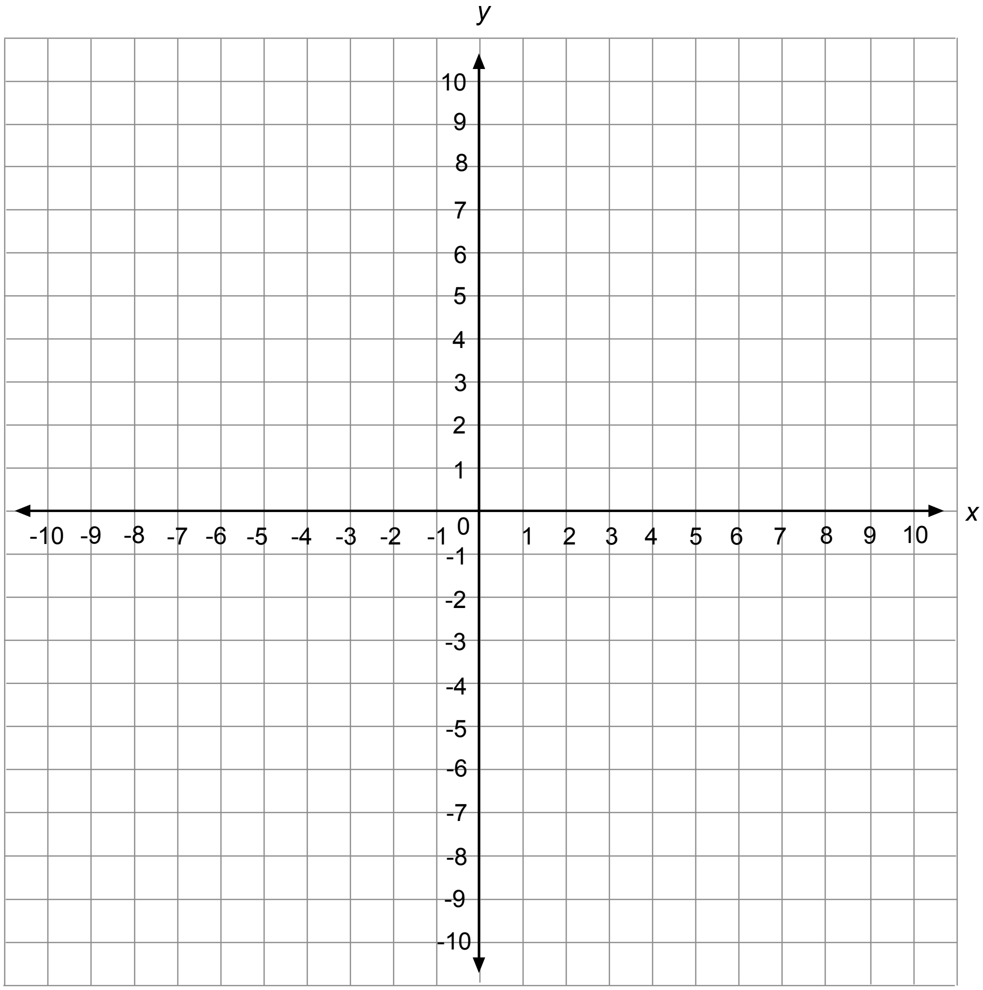
Use the equation 2x – 4y = -12 *to answer the following.*

20. Find the y-intercept. ______________

21. Find the x-intercept. ______________

22. Graph the line.

Graph of 2*x* – 4*y* = -12



Express these lines in **slope-intercept form**.

23. $2x - 3y = 4$

24. $2x + y = 16$

Put equations in **slope-intercept form**. **Show all your work.** *Determine if the following* **lines** *are* **parallel**, **perpendicular**, *or* **neither**. *Write the answer on the line provided.*

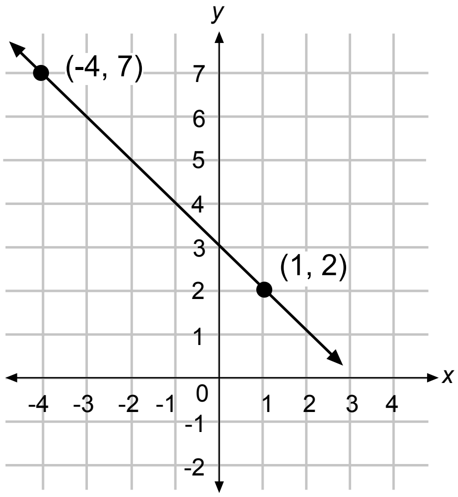
________________________ 25. $x - 2y = 12$
$2x - y = -6$

________________________ 26. $5x - 8y = -8$
$8x + 5y = -15$

27. Write an equation in point-slope form for the line that passes through point (-2, 5) with a slope of $\frac{2}{3}$.

28. Put $y - 4 = \frac{3}{8}(x + 2)$ into slope-intercept form.

29. Write an equation in slope-intercept form for the line given.



30. Write an equation in slope-intercept form for the line which passes through points (-3, 8) and (5, 7).

Unit 9: Having Fun with Functions

Students will learn and use the terminology and symbolism associated with functions.

Unit Focus

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Algebra Body of Knowledge

Standard 1: Real and Complex Number Systems

- MA.912.A.1.8
  Use the zero product property of real numbers in a variety of contexts to identify solutions to equations.

Standard 2: Relations and Functions

- MA.912.A.2.3
  Describe the concept of a function, use function notation, determine whether a given relation is a function, and link equations to functions.

- MA.912.A.2.4
  Determine the domain and range of a relation.

- MA.912.A.2.13
  Solve real-world problems involving relations and functions.

Standard 3: Linear Equations and Inequalities

- MA.912.A.3.11
  Write an equation of a line that models a data set and use the equation or the graph to make predictions. Describe the slope of the line in terms of the data, recognizing that the slope is the rate of change.

Standard 7: Quadratic Equations

- MA.912.A.7.1
  Graph quadratic equations with and without graphing technology.

- MA.912.A.7.10
  Use graphing technology to find approximate solutions of quadratic equations.

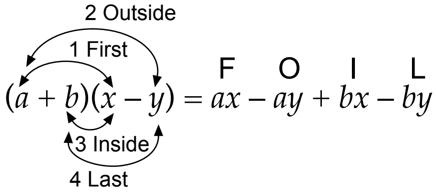
Standard 10: Mathematical Reasoning and Problem Solving

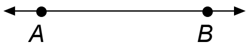
- MA.912.A.10.3
  Decide whether a given statement is always, sometimes, or never true (statements involving linear or quadratic expressions, equations, or inequalities, rational or radical expressions, or logarithmic or exponential functions).

Vocabulary

Use the vocabulary words and definitions below as a reference for this unit.

axis of symmetry	vertical line passing through the maximum or minimum point of a parabola
coefficient	the number that multiplies the variable(s) in an algebraic expression *Example*: In $4xy$, the coefficient of xy is 4. If no number is specified, the coefficient is 1.
coordinates	numbers that correspond to points on a coordinate plane in the form (x, y), or a number that corresponds to a point on a number line
data	information in the form of numbers gathered for statistical purposes
domain	set of x-values of a relation
element	one of the objects in a set
equation	a mathematical sentence stating that the two expressions have the same value *Example*: $2x = 10$
estimation	the use of rounding and/or other strategies to determine a reasonably accurate approximation, without calculating an exact answer *Examples*: clustering, front-end estimating, grouping, etc.

expression	a mathematical phrase or part of a number sentence that combines numbers, operation signs, and sometimes variables *Examples*: $4r^2$; $3x + 2y$; $\sqrt{25}$ An expression does *not* contain equal (=) or inequality ($<$, $>$, $\leq$, $\geq$, or $\neq$) signs.
factor	a number or expression that divides evenly into another number; one of the numbers multiplied to get a product *Examples*: 1, 2, 4, 5, 10, and 20 are factors of 20 and $(x + 1)$ is one of the factors of $(x^2 - 1)$.
factoring	expressing a polynomial expression as the product of monomials and polynomials *Example*: $x^2 - 5x + 4 = 0$ $(x - 4)(x - 1) = 0$
FOIL method	a pattern used to multiply two binomials; multiply the first, outside, inside, and last terms: **F** **F**irst terms **O** **O**utside terms **I** **I**nside terms **L** **L**ast terms. *Example*: 
function notation	a way to name a function that is defined by an equation Example: In function notation, the equation $x = 5x + 4$ is written as $f(x) = 5x + 4$.
function (of x)	a relation in which each value of x is paired with a unique value of y

horizontal	parallel to or in the same plane of the horizon 
intersect	to meet or cross at one point
line (↔)	a collection of an infinite number of points forming a straight path extending in opposite directions having unlimited length and no width 
linear function	an equation whose graph is a nonvertical line
maximum	the highest point on the vertex of a parabola, which opens downward
mean (or average)	the arithmetic average of a set of numbers; a measure of central tendency
minimum	the lowest point on the vertex of a parabola, which opens upward
ordered pair	the location of a single point on a rectangular coordinate system where the first and second values represent the position relative to the x-axis and y-axis, respectively *Examples*: (x, y) *or* (3, -4)
origin	the point of intersection of the x- and y-axes in a rectangular coordinate system, where the x-coordinate and y-coordinate are both zero (0)
parabola	the graph of a quadratic equation
point	a specific location in space that has no discernable length or width

quadratic equation	an equation in the form of $ax^2 + bx + c = 0$
quadratic function	an equation in the form $y = ax^2 + bx + c$, where $a \neq 0$
range	set of y-values of a relation
relation	a set of ordered pairs (x, y)
roots	the solutions to a quadratic equation
rounded number	a number approximated to a specified place *Example*: A commonly used rule to round a number is as follows. • If the digit in the first place after the specified place is 5 or more, *round up* by adding 1 to the digit in the specified place ($\overset{\curvearrowleft}{\underline{4}6}1$ rounded to the nearest hundred is 500). • If the digit in the first place after the specified place is less than 5, *round down* by *not* changing the digit in the specified place ($\overset{\curvearrowleft}{\underline{4}4}1$ rounded to the nearest hundred is 400).
set	a collection of distinct objects or numbers
slope	the ratio of change in the vertical axis (y-axis) to each unit change in the horizontal axis (x-axis) in the form $\frac{\text{rise}}{\text{run}}$ or $\frac{\Delta y}{\Delta x}$; the constant, m, in the linear equation for the slope-intercept form $y = mx + b$

solution	any value for a variable that makes an equation or inequality a true statement *Example*: In $y = 8 + 9$ $y = 17$ 17 is the solution.
solve	to find all numbers that make an equation or inequality true
value (of a variable)	any of the numbers represented by the variable
variable	any symbol, usually a letter, which could represent a number
vertex	the maximum or minimum point of a parabola
vertical	at right angles to the horizon; straight up and down 
vertical line test	if any vertical line passes through no more than one point of the graph of a relation, then the relation is a function
***x*-axis**	the horizontal number line on a rectangular coordinate system
***x*-intercept**	the value of x at the point where a line or graph intersects the x-axis; the value of y is zero (0) at this point
***y*-axis**	the vertical number line on a rectangular coordinate system

***y*-intercept**	the value of y at the point where a line or graph intersects the y-axis; the value of x is zero (0) at this point
zero product property	for all numbers a and b, if $ab = 0$, then $a = 0$ and/or $b = 0$
zeros	the points where a graph crosses the x-axis; the roots, or x-intercepts, of a quadratic function

Unit 9: Having Fun with Functions

Introduction

We will explore a number of relations through the use of tables and graphs. We will create tables and graphs for specific problems. We will also link equations to functions when given a function.

Lesson One Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.
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  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Algebra Body of Knowledge

Standard 2: Relations and Functions

- MA.912.A.2.3
  Describe the concept of a function, use function notation, determine whether a given relation is a function, and link equations to functions.

- MA.912.A.2.4
  Determine the domain and range of a relation.

Functions

In the unit on Venn diagrams we learned that a **set** of **ordered pairs**, such as {(2, 4), (3, 8), (5, 7), (-2, 1)}, is called a **relation**. Each **element** in a *relation* has a **value**—an x-value and a y-value (x, y). The **ordered pairs** are called **coordinates** (x, y) of a point on a graph.

The *set* containing all of the x-values is called the **domain**, while the set of all y-values is called the **range**.

From the example {(2, 4), (3, 8), (5, 7), (-2, 1)} the *domain* would be {2, 3, 5, -2} and the *range* would be {4, 8, 7, 1}.

A *relation* in which no x-value is repeated is called a **function**. Another way to say that is each element of the domain is paired with only one element of the range.

Set of Ordered Pairs—Relation

{(2, 4), (3, 8), (5, 7), (-2, 1)}

x-values, or first numbers of the ordered pairs—domain = {2, 3, 5, -2}

(2, 4)
(3, 8)
(5, 7)
(-2, 1)

This relation is a function because of the following.

- no *x*-value is repeated
- each element of the domain, or *x*-values, can be paired with only one element of the range, or *y*-values

y-values, or second numbers of the ordered pairs—range = {4, 8, 7, 1}

Note: Usually *values* are listed in numerical order. However, for giving the domain (x-values) and the range (y-values) for relations, numerical order is *not* required. If a value in a domain or in a range is repeated, list the value *one* time.

Practice

Decide if the **relations** *below are* **functions**. *Write* **yes** *if it is a function, write* **no** *if it is* not *a function.*

____________ 1. {(2, 3), (5, 6), (4, 9), (3, 8)}

____________ 2. {(3, 6), (4, 7), (3, -9), (8, 2)}

____________ 3. {(4, 2), (2, 4), (3, 6), (6, 3)}

____________ 4. {(4, -1), (5, 8), (4, 6), (3, 0)}

____________ 5. {(10, 4), (8, -6), (0, 0), (10, 3)}

____________ 6. {(6, 3), (6, 2), (6, 0), (6, -2)}

____________ 7. {(4, 1), (5, 1), (6, 1)}

____________ 8. {(3, 4), (4, 5), (5, 6), (6, 7), (7, 8)}

Practice

Give the **domain** *and the* **range** *for each* **relation**.

Remember: The domains and ranges do *not* have to be listed in numerical order. If a value in a domain or in a range is repeated, list the value *one* time.

1. {(2, 3), (5, 6), (4, 9), (3, 8)}

   a. domain = ______________________

   b. range = ______________________

2. {(3, 6), (4, 7), (3, -9), (8, 2)}

   a. domain = ______________________

   b. range = ______________________

3. {(4, 2), (2, 4), (3, 6), (6, 3)}

   a. domain = ______________________

   b. range = ______________________

4. {(4, -1), (5, 8), (4, 6), (3, 0)}

   a. domain = ______________________

   b. range = ______________________

5. {(10, 4), (8, -6), (0, 0), (10, 3)}

   a. domain = ____________________

   b. range = ____________________

6. {(6, 3), (6, 2), (6, 0), (6, -2)}

   a. domain = ____________________

   b. range = ____________________

7. {(4, 1), (5, 1), (6, 1)}

   a. domain = ____________________

   b. range = ____________________

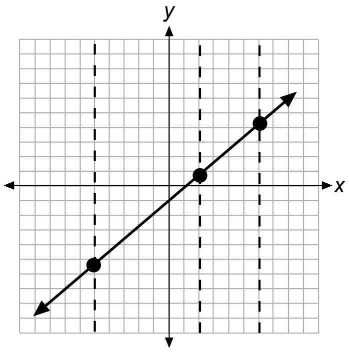
8. {(3, 4), (4, 5), (5, 6), (6, 7), (7, 8)}

   a. domain = ____________________

   b. range = ____________________

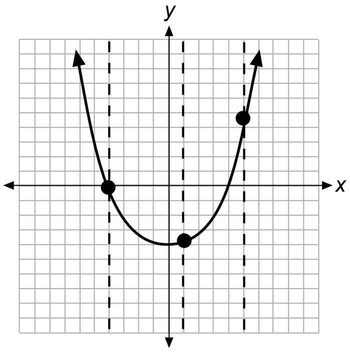
Graphs of Functions

Using the **vertical line test**, it is possible to tell from a graph whether a relation is a *function* or not. If any **vertical** line (line that is straight up and down) can be drawn that touches the graph at *no more than* one **point** of the graph, then the relation is a function. However, if the *vertical* line touches the graph at *more than* one *point*, the relation is *not* a function.



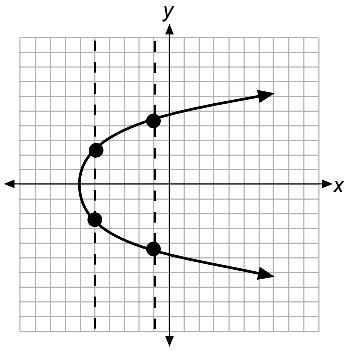
a function

The three *vertical test lines* touch the graph at only one point each.



a function

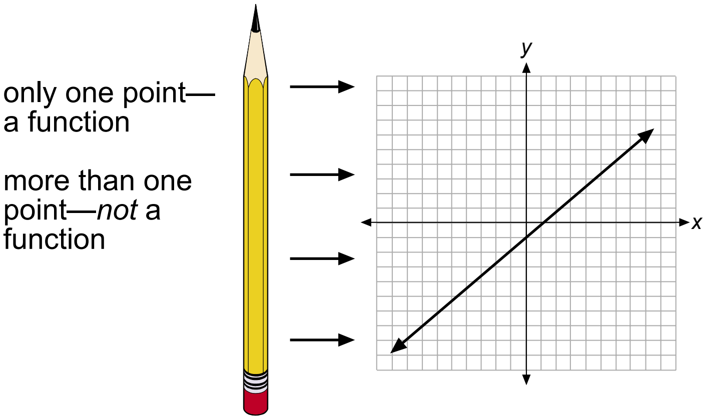
The three *vertical test lines* touch the graph at only one point each.



not a function

The two *vertical test lines* touch the graph at more than one point each.

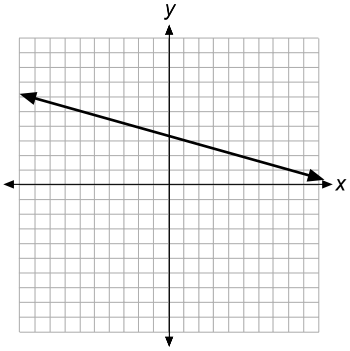
Tip: A vertical test line can use any straight-edged object, such as a pencil or pen, to perform the test. Place your pencil next to the graph. Line the pencil up vertically with the graph and move it slowly across the graph.



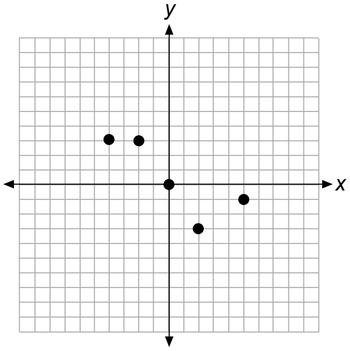
Practice

Determine if these **graphs** *represent* **functions**. *Write* **yes** *if it is a function. Write* **no** *if it is* not *a function.*

_____________ 1.



_____________ 2.



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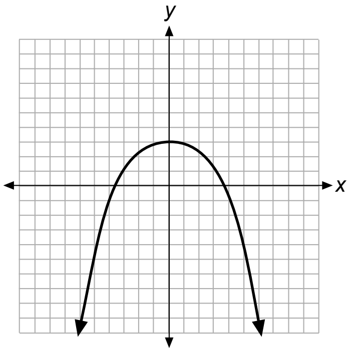

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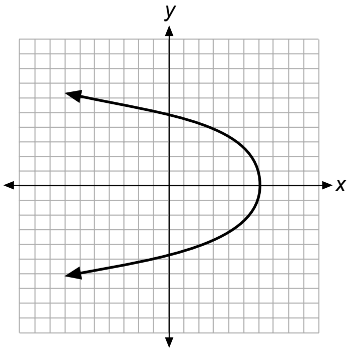

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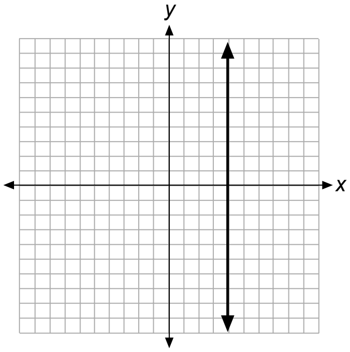

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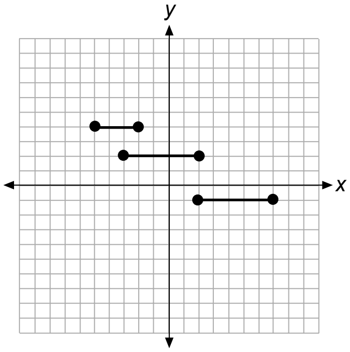

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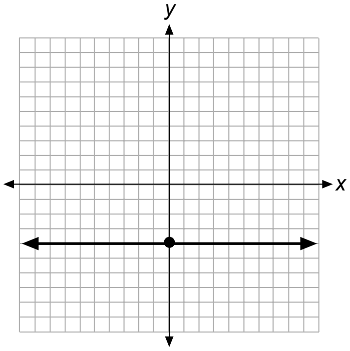

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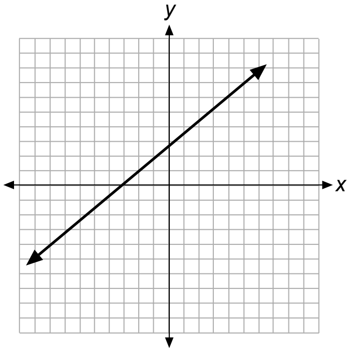


Practice

Use the list below to complete the following statements.

domain	**range**
function	**relation**
ordered pair	

1. (x, y) represents a(n) ____________________ .

2. A set of ordered pairs is called a(n) ____________________ .

3. A relation in which no x-value is repeated is called a(n)

   ____________________ .

4. The set of x-values from a relation is the ____________________ .

5. The set containing the y-values from a set of ordered pairs is the

   ____________________ .

Lesson Two Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.

- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.

- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Algebra Body of Knowledge

Standard 2: Relations and Functions

- MA.912.A.2.3
  Describe the concept of a function, use function notation, determine whether a given relation is a function, and link equations to functions.

The Function of *X*

Functions are so important that they have their own notation called a **function notation**. A *function notation* is a way to name a function defined by an **equation**. An *equation* is a mathematical sentence stating that the two **expressions** have the same value, connected by an equal sign (=). Think of a function as a math machine that will work problems the way you instruct it.

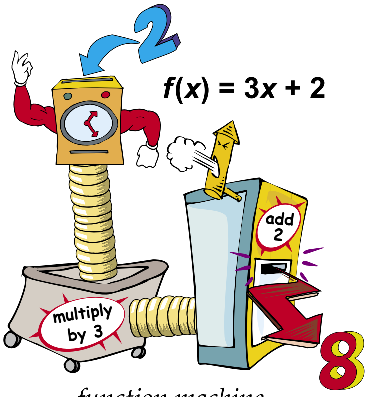
Function Machine

$f(x) = 3x + 2$

Notice the notation on the function machine above—$f(x) = 3x + 2$. The $f(x)$ is read "the function of x." We sometimes shorten that and read the entire sentence as f of x equals $3x + 2$.

The machine works when you put in numbers from a domain (set of x-values). So if our domain is {2, 4, 5, 9} and we use the function machine, we get the following.

2	$f(x) = 3x + 2$	8	because $3(2) + 2 = 8$
4	$f(x) = 3x + 2$	14	$3(4) + 2 = 14$
5	$f(x) = 3x + 2$	17	$3(5) + 2 = 17$
9	$f(x) = 3x + 2$	29	$3(9) + 2 = 29$



function machine

So now we see our range (y-values) is {8, 14, 17, 29}.

Together the domain and range give us the relation.

{(2, 8), (4, 14), (5, 17), (9, 29)}

This relation is a function because no y-value is repeated.

Although $f(x)$ is most commonly used, it is not unusual to see a function expressed as $g(x)$ or $h(x)$ and occasionally other letters as well. Did you notice we work these the same as if the problem had read $y = 3x + 2$?

Let's practice a bit, shall we?

Practice

Use the **domain** *below to give the* **range** *for the following* **functions**.

{-4, -2, 0, 1, 3}

Remember: The range does *not* have to be listed in numerical order. If a value in a range is repeated, list the value *one* time.

1. $f(x) = 2x - 9$

2. $f(x) = 5x + 4$

3. $g(x) = -2x - 3$

Use the **domain** *below to give the* **range** *for the following* **functions**.

{-4, -2, 0, 1, 3}

4. $f(x) = x^2$

5. $f(x) = 3x^2$

6. $h(x) = x^2 + 2$

Use the **domain** *below to give the* **range** *for the following* **functions**.

{-4, -2, 0, 1, 3}

7. $f(x) = (5x)^2$

8. $g(x) = (x + 3)^2$

Lesson Three Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Algebra Body of Knowledge

Standard 3: Linear Equations and Inequalities

- MA.912.A.3.11
  Write an equation of a line that models a data set and use the equation or the graph to make predictions. Describe the slope of the line in terms of the data, recognizing that the slope is the rate of change.

Standard 10: Mathematical Reasoning and Problem Solving

- MA.912.A.10.3
  Decide whether a given statement is always, sometimes, or never true (statements involving linear or quadratic expressions, equations, or inequalities, rational or radical expressions, or logarithmic or exponential functions).

Graphing Functions

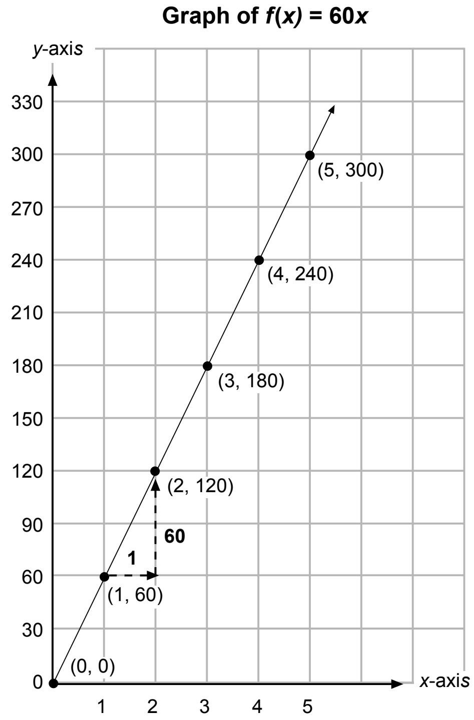
We graph functions in the same way we do equations. We can still identify **slopes** and ***y*-intercepts** from functions whose graph is a **line**. Remember the slope-intercept form $y = mx + b$? Well, if a function is expressed as $f(x) = mx + b$, it is a **linear function**. A *linear function* is an equation whose graph is a nonvertical *line*.

Sometimes a graph will pass through the **origin**. That happens when $f(0) = 0$ or when the point (0, 0) is in the relation.

As we know, the set

$$\{(0, 0), (1, 60), (2, 120), (3, 180), (4, 240), (5, 300)\}$$

can be called a relation (which is any set of ordered pairs). These ordered pairs could be graphed by hand on a coordinate grid or on a graphing calculator. A function is a relation in which each value of x is paired with a unique value of y. This relation is also a function because its graph is a *nonvertical* line.



equation: $y = 60x$

slope: 60 or $\frac{60}{1}$

Think about This!

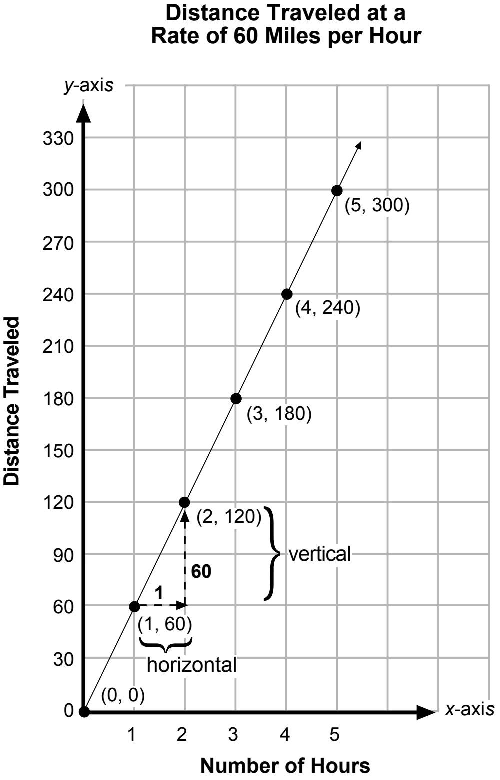
- As the first *coordinate* increases by 1, the second coordinate increases by 60.

60
(1, 60), (2, 120)
1

- If these points were plotted, they would lie in a line.
- An equation for the line would be $y = 60x$.
- The line will pass through the origin so the ***x*-intercept** is (0, 0) and the *y*-intercept is (0, 0).

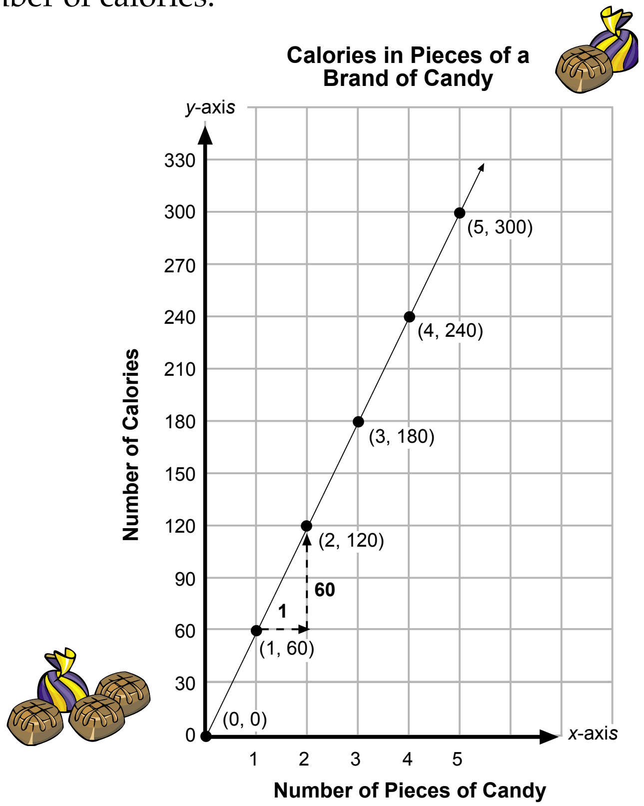
Remember: The *x*-intercept is the value of *x* on a graph when *y* is zero. The line passes through the ***x*-axis** at this point. The *y*-intercept is the value of *y* on a graph when *x* is zero. The line passes through the ***y*-axis** at this point.

- The slope of the line will be 60 or $\frac{60}{1}$ because for each increase of 1 in *x*, there is an increase of 60 in *y*. From any given point on the line, a **horizontal** (⟷) movement of 1 unit followed by a vertical (↕) movement of 60 units will produce another point on the line.
- If the ordered pairs are describing the distance traveled at a rate of 60 miles per hour, then *x* could represent the number of hours and *y* would represent the distance traveled.



From the function $f(x) = 60x$ or its graph, we can predict how far we could travel in 8 hours at 60 mph. If $f(x) = 60x$ and $x = 8$, $f(8) = 60(8)$. We could travel 480 miles in 8 hours.

- If the ordered pairs are describing the number of calories in a certain brand of candy, then x could represent the number of pieces of candy and y would represent the number of calories.



What function could be written to describe the graph above?

The function would be

$$f(x) = 60x$$

because the relationship between x and y in each ordered pair indicates that x times $60 = y$.

Practice

Complete the following for the **set of ordered pairs** *below.*

{(0, 0), (1, 1), (2, 2), (3, 3), (4, 4), (5, 5)}

1. As the first coordinate increases by 1, the second coordinate increases by ____________ .

2. If these points are plotted, they ________________________ (always, sometimes, never) lie in a line.

3. This line ________________________ (always, sometimes, never) passes through the origin.

4. The slope of the line will be ____________ because for each increase of 1 in x, there is an increase of ____________ in y. From any given point on the line, a *horizontal* (↔) movement of 1 unit followed by a vertical (↕) movement of ____________ unit(s) will produce another point on the line.

5. The function for the line would be $f(x)$ = ________________________ .

6. Create a situation the set of order pairs might describe.

____

____

Practice

Graph each function **two** *times. Use a* **table of values** *in one of your graphs and the* **slope-intercept method** *in the other. Use* **x-values** *of -2, 0, and 2 in your* **table**.

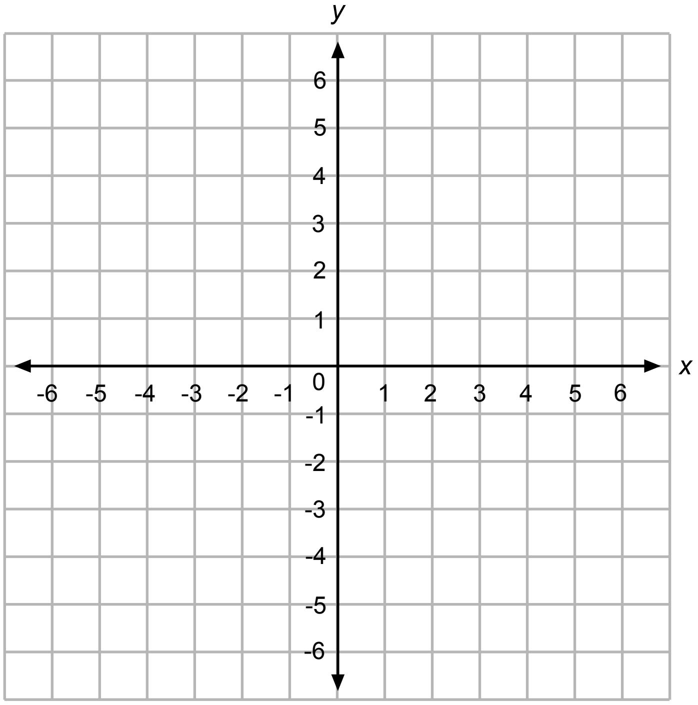
1. $f(x) = 3x$

   a. Table of Values Method

Table of Values

$f(x) = 3x$	
x	$f(x)$
-2	
0	
2	

b. **Graph of $f(x) = 3x$**



c. Slope-Intercept Method

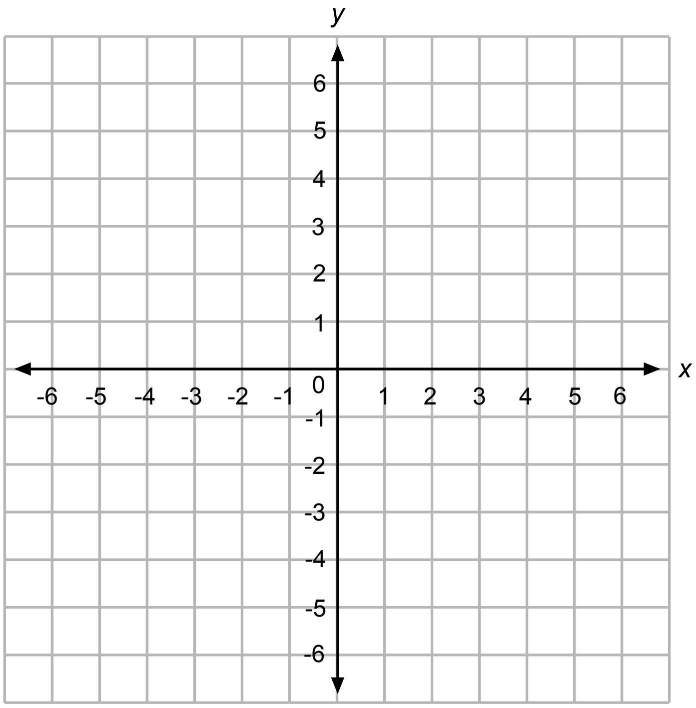
$f(x) = mx + b$

$f(x) = 3x$

Remember: In the function $f(x) = 3x$, the **variable** b, which is the y-intercept, is zero. A *variable* is any symbol, usually a letter, which could represent a number.

Slope is ______________ or $\frac{\quad}{1}$

d. **Graph of $f(x) = 3x$**



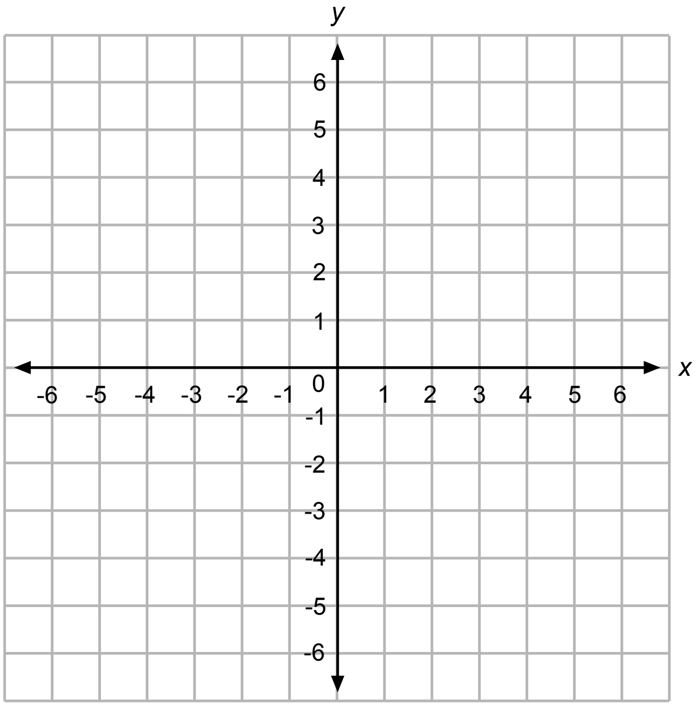
2. $f(x) = -2x + 1$

a. Table of Values Method

Table of Values

$f(x) = -2x + 1$	
x	$f(x)$
-2	
0	
2	

b. **Graph of $f(x) = -2x + 1$**



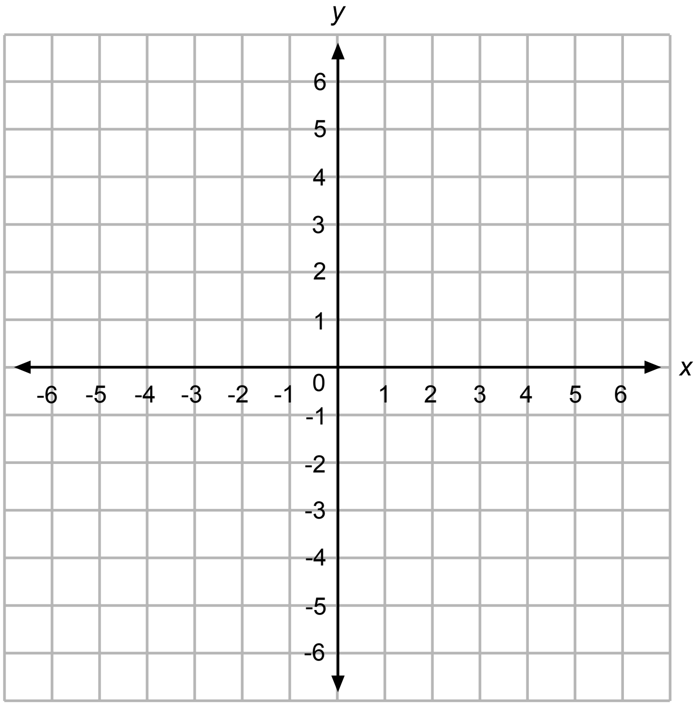
c. Slope-Intercept Method

$f(x) = mx + b$

$f(x) = -2x + 1$

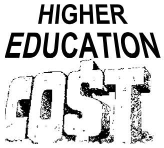
Slope is ______________ or $\frac{\quad}{1}$

d. **Graph of $f(x) = -2x + 1$**



Linear Relations in the Real World

As you look ahead and consider the cost of higher education, you will find that *tuition costs* tend to represent a *linear relationship*. Universities tend to have a *fixed price for each semester hour of credit*. There is often a difference in the fixed price for a semester hour of undergraduate credit and a semester hour of graduate credit. Special areas of study may have increased costs. The following set of practices deals with costs involved in higher education. You will likely find technology, such as computer programs and some calculators support the making of tables and graphs, which are often used when considering **data** to be displayed when making comparisons.



Practice

Answer the following.

1. The function used to determine the tuition for each semester hour of undergraduate credit for Florida residents at Florida State University was $f(x) = 84.58x + 4.9x$ for all main-campus students. If a student was enrolled in a course at an extension site, the equation was $g(x) = 84.58x$. Most students take 12 to 15 hours each semester.

   Complete the table below.

   Tuition for Florida State University—Florida Residents

Number of Semester Hours x	Cost for Main-Campus Students (Florida Residents) $f(x) = 84.58x + 4.9x$	Cost for Extension-Site Students (Florida Residents) $g(x) = 84.58x$
12		
13		
14		
15		

   If these points were plotted on a coordinate grid, they would appear linear. If we make a graph of $g(x) = 84.58x$ when x can be any number, we tend to show the line passing through the points.

   The additional charge of \$4.90 per semester hour for main-campus students used to improve the overall campus infrastructure for all students is called the transportation access fee.

2. The function used to determine the tuition for each semester hour of undergraduate credit for non-Florida residents at Florida State University was $f(x) = 402.71x + 4.9x$ for all main-campus students. If a student was enrolled in a course at an extension site, the equation was $g(x) = 402.71x$. Most students take 12 to 15 hours each semester.

Complete the table below.

Tuition for Florida State University—Non-Florida Residents

Number of Semester Hours x	Cost for Main-Campus Students (Non-Florida Residents) $f(x) = 402.71x + 4.9x$	Cost for Extension-Site Students (Non-Florida Residents) $g(x) = 402.71x$
12		
13		
14		
15		

3. The graph below shows the three functions related to tuition costs for Florida residents who are undergraduate students, graduate students, and law students. The total cost per semester hour was rounded to the nearest $10 and included the transportation fee. The points were connected to emphasize the slope of each line. (The slope directly relates to *per hour costs*.)

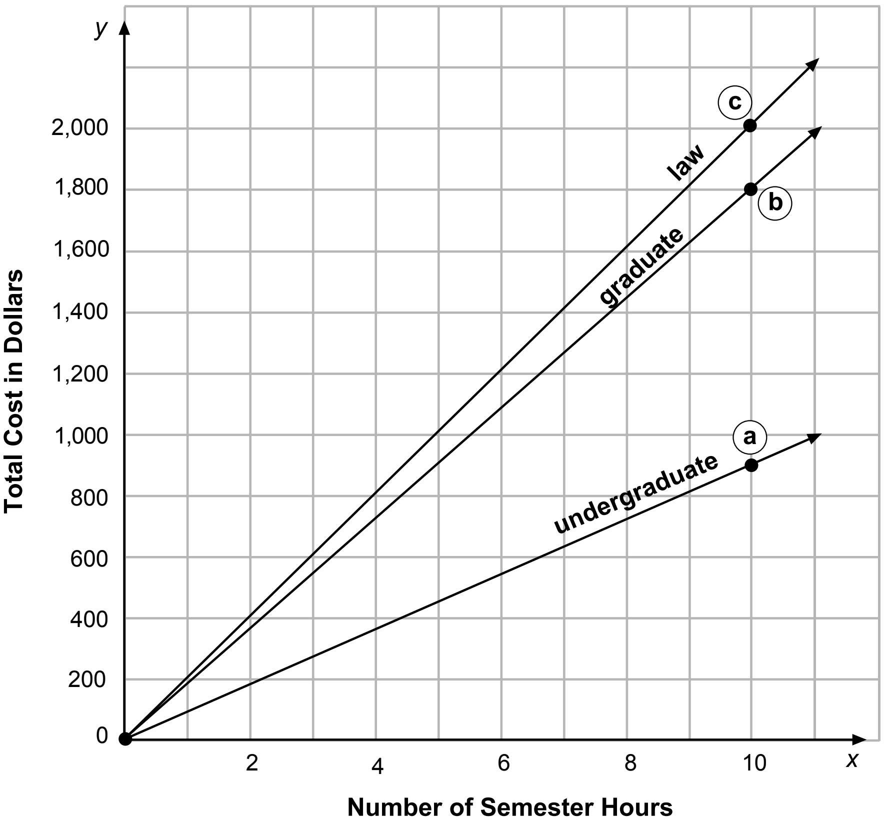
   Use the graph to **estimate** the cost per semester hour for each of the three types for Florida residents.

   a. undergraduate students: approximately ____________ per semester hour

   b. graduate students: approximately ____________ per semester hour

   c. law students: approximately ____________ per semester hour

Tuition for Florida State University—Florida Residents

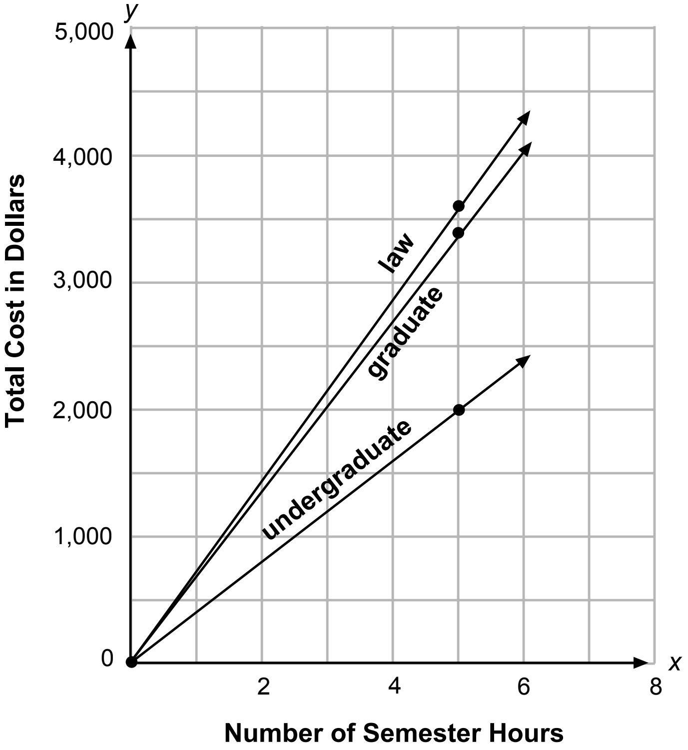


4. The cost per semester hour including the transportation access fee for each of the three types for non-Florida residents was as follows.

    - undergraduate school: $402.71 + 4.90 per semester hour
    - graduate school: $670.92 + 4.90 per semester hour
    - law school: $712.59 + 4.90 per semester hour

    A graph is provided below for this data.

Tuition for Florida State University—Non-Florida Residents



Write **three** statements comparing tuition for **each type** of student, based on being Florida residents *or* non-Florida residents. Explain whether you prefer to use equation, table, or graph models when writing such comparison statements.

Statement 1: ______________________________

____

____

Statement 2: ______________________________

____

____

Statement 3: ______________________________

____

____

Preference: ________________

Explanation: ______________________________

____

____

5. The following functions would allow you to compute the tuition for a semester hour of credit in an undergraduate course at the University of Florida for Florida residents and non-Florida residents.

$f(x) = 92.68x$ where $f(x)$ is the total tuition for a Florida resident for x number of hours of undergraduate-level courses.

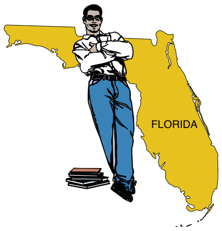
$g(x) = 460.28x$ where $g(x)$ is the total tuition for a non-Florida resident for x number of hours of undergraduate-level courses.

Complete the table below.

Tuition for University of Florida—Florida Residents and Non-Florida Residents

Number of Semester Hours x	**Cost for Florida Resident in Undergraduate Courses** $f(x) = 92.68x$	**Cost for Non-Florida Resident in Undergraduate Courses** $g(x) = 460.28x$
6		
9		
12		
15		

6. Complete the following.



   a. A Florida resident paid $2,873.64 in tuition for 14 hours of graduate-level courses. What was the cost per credit hour?

      Answer: ______________________

   b. A non-Florida resident received $27.96 in change from his payment of $10,000 for 12 credit hours in law courses. What was the cost per credit hour? **Round to the nearest dollar.**

      Answer: ______________________

c. A non-Florida resident has a budget of $25,000 for tuition for two semesters and is taking graduate-level courses with a cost per credit hour of $774.53. What is the greatest number of hours she can take and not exceed her budget?

   Answer: ____________________

More about the Slope of a Line

You are a member of a private club that offers valet parking for its members. The club charges you \$3.00 to have the parking attendant park and retrieve your car and \$2 per hour for parking. A set of ordered pairs for this situation would include the following.

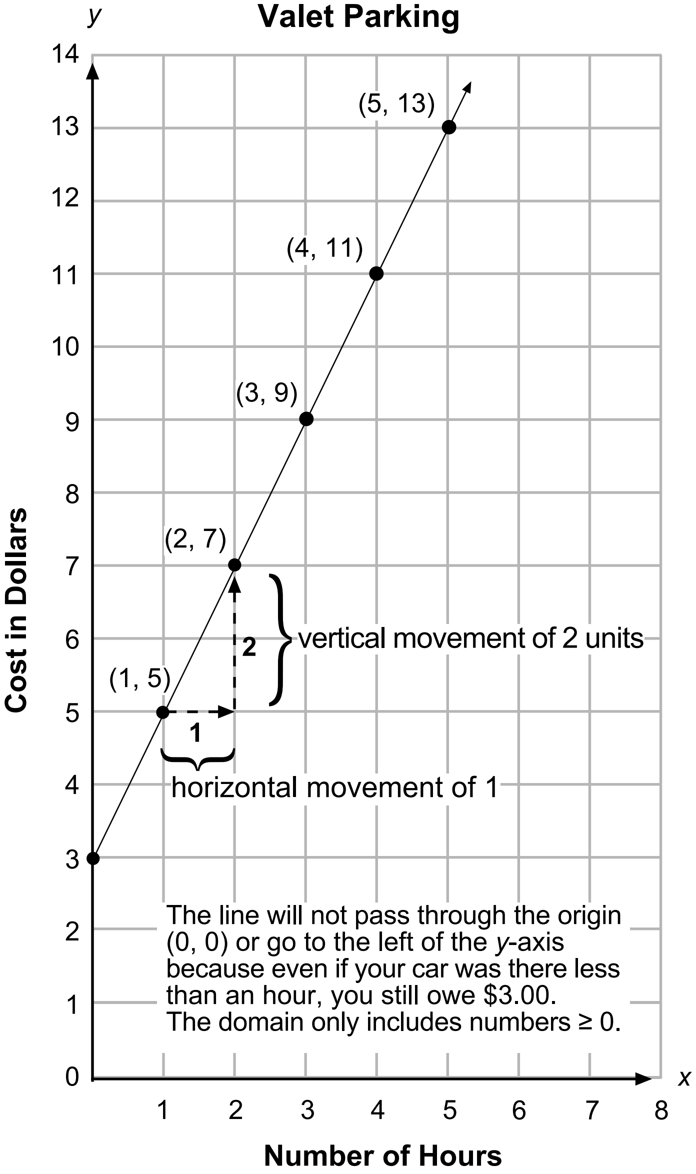
$\{(1, 5), (2, 7), (3, 9), (4, 11), (5, 13)\}$

If x represents the number of hours your car is parked, then y would represent the cost.

The equation for this line would be $y = 2x + 3$.

In function notation, $f(x) = 2x + 3$.

valet parking for members



The line will not pass through the origin (0, 0) or go to the left of the *y*-axis because even if your car was there less than an hour, you still owe \$3.00. The domain only includes numbers ≥ 0.

Practice

Complete the following for the **set of ordered pairs** *below.*

{(0, 10), (1, 13), (2, 16), (3, 19), (4, 22), (5, 25)}

1. As the first coordinate increases by 1, the second coordinate increases by ____________ .

2. If these points were plotted, they ____________ would (always, sometimes, never) lie in a line.

3. The line ____________ (will, will not) pass through the origin.

4. The slope of the line will be ____________ or $\frac{}{1}$ because for each increase of 1 in x, there is an increase of ____________ in y. From any given point on the line, a horizontal movement of 1 unit, followed by a vertical movement of ____________ unit(s), will produce another point on the line.

5. The equation for the line would be $y =$ ________________________ and in function notation it would be $f(x) =$ ________________________ .

6. Create a situation the set of order pairs might describe.

Situation: ____

____

____

____

Practice

Answer the following.

1. The cost for Florida residents per credit hour at Tallahassee Community College was $53 per credit hour, plus a $10 student service fee and $10 for the math lab.

   a. Write a function that would permit tuition and fees to be calculated for x number of hours.

   Function: ______________________

   b. If a graph were made for your function, the slope would be

   ___________ and the y-intercept would be located at

   (0, _____). You know that an active student takes one or more

   credit hours. Therefore, the y-intercept has meaning for the

   general equation but not for its specific application when used

   for tuition and fee costs.

2. Tuition at the University of Miami was $1,074 per credit hour for undergraduate students taking 1-11 hours. In addition, the university charged $299.50 for a combination of four different student fees: activity, athletic, wellness center, and university.

   a. Write a function that would permit tuition and fees to be calculated for x number of hours where x can be from 1 through 11.

   Function: ______________________

b. Use your function to determine the cost of tuition and fees for a student taking 9 hours.

Answer: ____________________

3. The University of Miami charged a flat rate of $12,919 for 12-20 credit hours plus $226.50 for a combination of three fees for full-time students.

a. Write a function that would represent the total cost of tuition and fees for a student taking 15 credit hours.

Function: ____________________

b. Use your function to determine the total cost for 12 hours.

Answer: ____________________

c. Based on your answer for **b**, what is the **mean (or average)** cost per credit hour when 12 credit hours are taken? **Round to the nearest hundredth.**

Answer: ____________________

d. Use your function to determine the total cost for 20 hours.

Answer: ____________________

Practice

Answer the following.

1. Harvard charged a flat rate of $26,066 for tuition, plus fees of $1,142 for health services, $1,852 for student services, and $35 for the undergraduate council. Determine the *mean* cost per credit hour for a student taking 15 credit hours. **Round to the nearest hundredth.**

   Answer: ____________________

2. The cost per credit hour at the undergraduate level at Florida A & M University was $90.09, plus fees of $45 for transportation and access, $59 for health if taking 6 or more hours, and a materials and supply fee ranging from $15 to $60. Assuming the materials and supply fee was $37.50, write a function that would allow you to calculate cost for x number of hours where x represents 6 or more.

   Function: ____________________

3. Consider the following.

Public Undergraduate Tuition Rates and Fees

Institution of Higher Learning (Public)	Cost of Tuition and Fees for 15 Undergraduate Credit Hours for Florida Resident
Florida A & M University	$1,492.85
Florida State University	$1,342.20
Tallahassee Community College	$815.00
University of Florida	$1,390.20

Private Undergraduate Tuition Rates and Fees

Institution of Higher Learning (Private)	Cost of Tuition and Fees for 15 Undergraduate Credit Hours
Harvard University	$29,095.00
University of Miami	$13,145.50

a. Use the figures in the table above and on the previous page. Explain why you could not divide each of the costs by 15 and then multiply by 12 to get the costs for 12 credit hours for each institution.

Explanation: ____

____

____

____

b. If you were considering these schools in your future, would you find information pertaining to tuition and fees more helpful if this were modeled by equations, tables, or graphs? Explain the basis for your choice.

Explanation: ____

____

____

____

Lesson Four Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.

- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.

- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Algebra Body of Knowledge

Standard 1: Real and Complex Number Systems

- MA.912.A.1.8
  Use the zero product property of real numbers in a variety of contexts to identify solutions to equations.

Standard 7: Quadratic Equations

- MA.912.A.7.1
  Graph quadratic equations with and without graphing technology.

- MA.912.A.7.10
  Use graphing technology to find approximate solutions of quadratic equations.

Graphing Quadratics

Any function whose equation is in the format $f(x) = ax^2 + bx + c$ (when $a \neq 0$) is called a **quadratic function**. The presence of the ax^2 term is a big hint that this is a quadratic expression. You'll also remember that the ax^2 term is a hint that **factoring** is involved for solving x.

Graphs of *quadratic functions* are called **parabolas** and have a shape that looks like an airplane wing.

Let's look at two examples.

Example 1

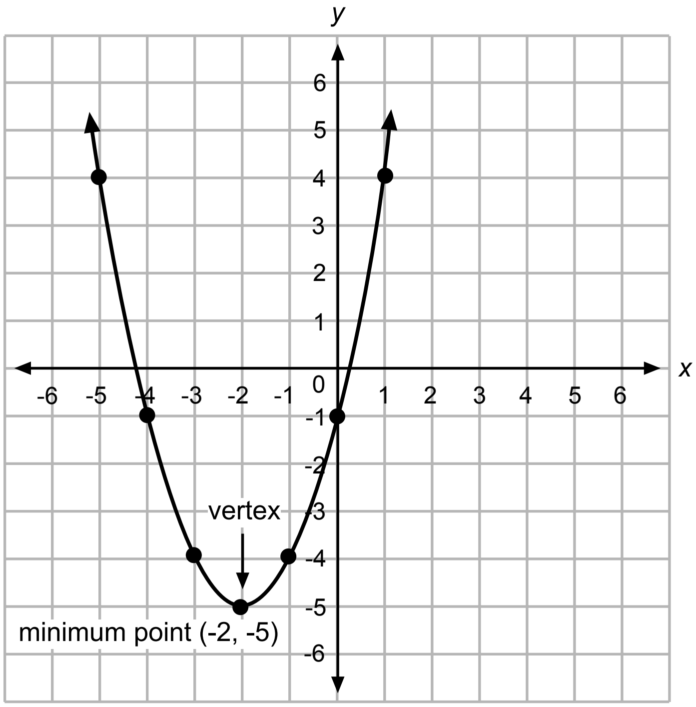
$$f(x) = x^2 + 4x - 1$$

We will use a table of values to graph this function.

Table of Values

$f(x) = x^2 + 4x - 1$	
x	$f(x)$
-5	4
-4	-1
-3	-4
-2	-5
-1	-4
0	-1
1	4

Graph of $f(x) = x^2 + 4x - 1$



We plot the points, and knowing that the graph will look like an airplane wing, we connect the dots with a smooth curve. A **coefficient** is the number part in front of an algebraic term. The *coefficient* in front of x^2 in the function $f(x) = x^2 + 4x - 1$ is understood to be a +1.

Because the x value of the coefficient is positive, the *parabola* will open upward and will have a *lowest point*, or **vertex**, called the **minimum** point.

To tell exactly where that *minimum* point will be on our graph, we use information from the equation. Remembering that $f(x) = ax^2 + bx + c$, we use $x = \frac{-b}{2a}$ to tell us the x-value of the lowest point.

So from our function $f(x) = x^2 + 4x - 1$, where $a = 1$, $b = 4$, $c = -1$, we get the following.

$$x = \frac{-b}{2a}$$
$$x = \frac{-4}{2(1)}$$
$$x = \frac{-4}{2}$$
$$\boxed{x = -2}$$

So, the minimum point occurs when $x = -2$.

Using the function again,

Remember: $f(x) = ax^2 + bx + c$
$f(x) = x^2 + 4x - 1$, where
$a = 1, b = 4, c = -1$

$$f(-2) = (-2)^2 + 4(-2) - 1$$
$$f(-2) = 4 + -8 - 1$$
$$f(-2) = -5$$

Therefore, the minimum point is (-2, -5).

Another thing we can tell from the equation $x = -2$ in the box above is the **axis of symmetry**. Recall that the graph of $x = -2$ is a vertical line through -2 on the *x-axis*. This is the line that divides the parabola exactly in half. If you fold the graph along the *axis of symmetry*, each half of the parabola will match the other side exactly.

Note that the graph is a function because it passes the *vertical line test*. Any vertical line you draw will only **intersect** the graph (parabola) at one point.

Let's look at another example.

Example 2

$$f(x) = -x^2 + 2x - 3$$

Notice that the coefficient of x^2 is -1.

Because the value of the coefficient of x is negative, the parabola will open downward and have a highest point, or *vertex*, called a **maximum** point.

Find the axis of symmetry.

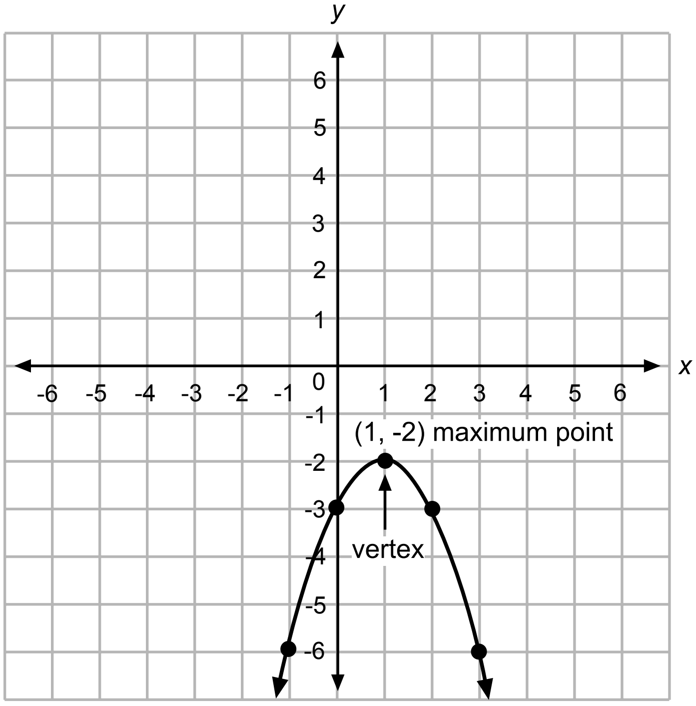
$$x = \frac{-b}{2a}$$
$$x = \frac{-2}{-2}$$
$$\boxed{x = 1} \longleftarrow \text{axis of symmetry}$$

Our *maximum* point occurs when $x = 1$. Let's make a table of values (be sure to include 1 as a value for x).

Table of Values

$f(x) = -x^2 + 2x - 3$	
x	$f(x)$
-1	-6
0	-3
1	-2
2	-3
3	-6

Graph of $f(x) = -x^2 + 2x - 3$



Graph the ordered pairs and connect them with a smooth curve. Note that the vertex of the parabola has a maximum point at (1, -2) and the line of symmetry is at $x = 1$.

Refer to the examples above as you try the following.

Practice

For each **function** *do the following.*

- Find the equation for the axis of symmetry.
- Find the coordinates of the vertex of the graph.
- Tell whether the vertex is a maximum or minimum vertex.
- Graph the function.

1. $f(x) = x^2 + 2x - 3$

   a. axis of symmetry = ______________________

   b. coordinates of vertex = ______________________

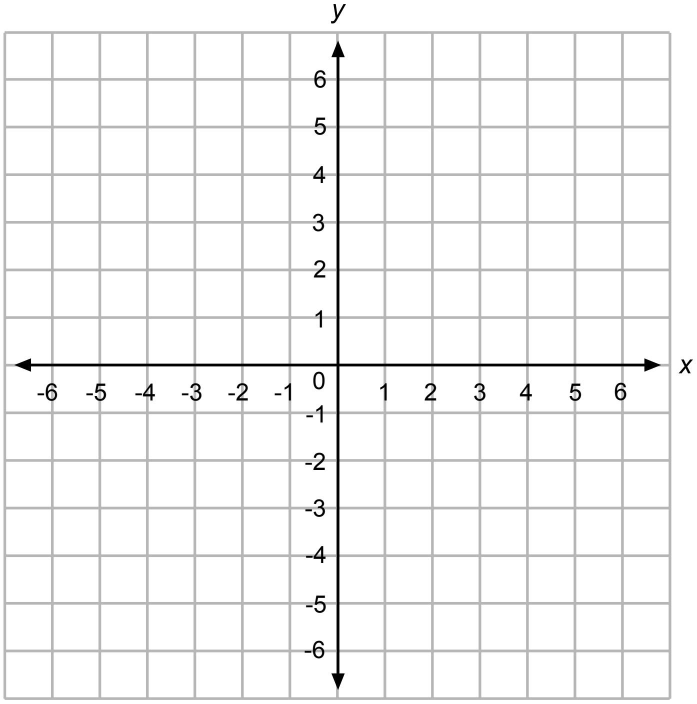
   c. maximum or minimum = ______________________

   d. graph

Table of Values

$f(x) = x^2 + 2x - 3$	
x	$f(x)$

Graph of $f(x) = x^2 + 2x - 3$



2. $f(x) = x^2 - 2x - 3$

a. axis of symmetry = ________________________

b. coordinates of vertex = ________________________

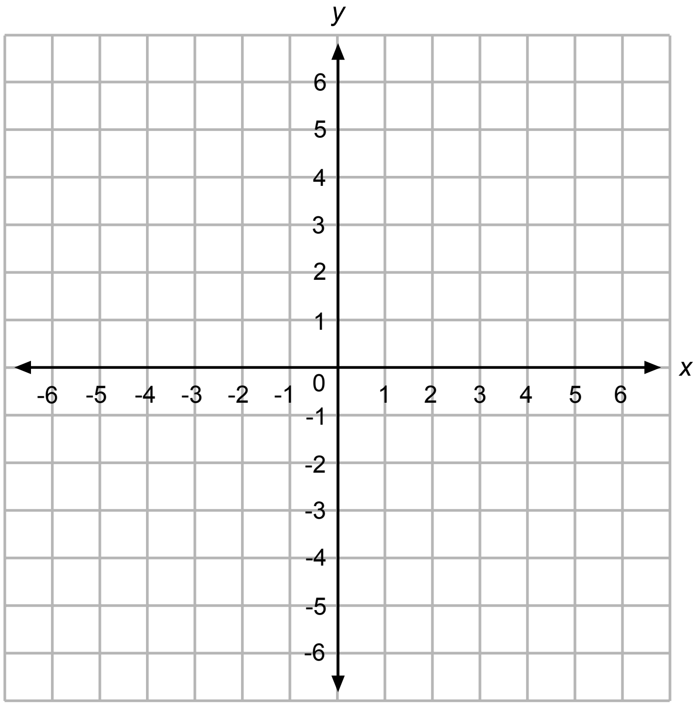
c. maximum or minimum = ________________________

d. graph

Table of Values

$f(x) = x^2 - 2x - 3$	
x	$f(x)$

Graph of $f(x) = x^2 - 2x - 3$



3. $f(x) = -x^2 + 1$

a. axis of symmetry = ________________________

b. coordinates of vertex = ________________________

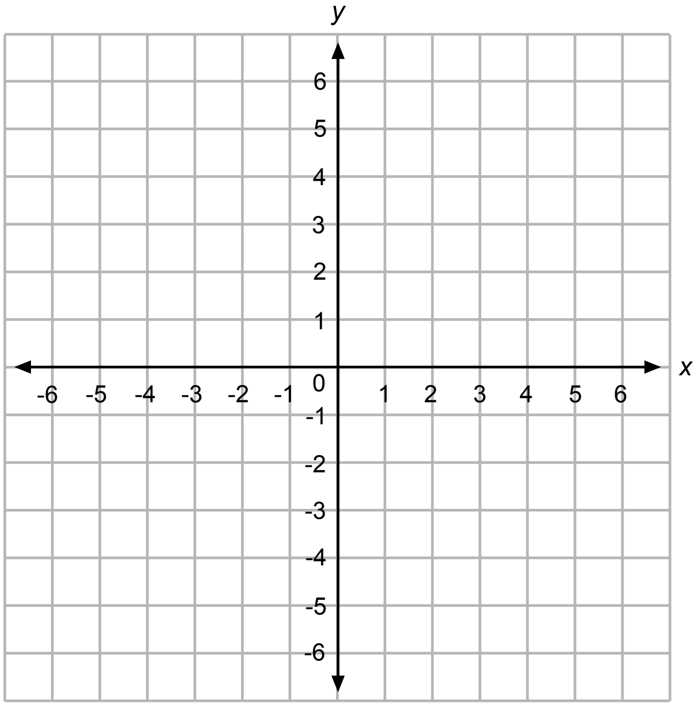
c. maximum or minimum = ________________________

d. graph

Table of Values

$f(x) = -x^2 + 1$	
x	$f(x)$

Graph of $f(x) = -x^2 + 1$



4. $f(x) = x^2 + 2x$

   a. axis of symmetry = ____________________

   b. coordinates of vertex = ____________________

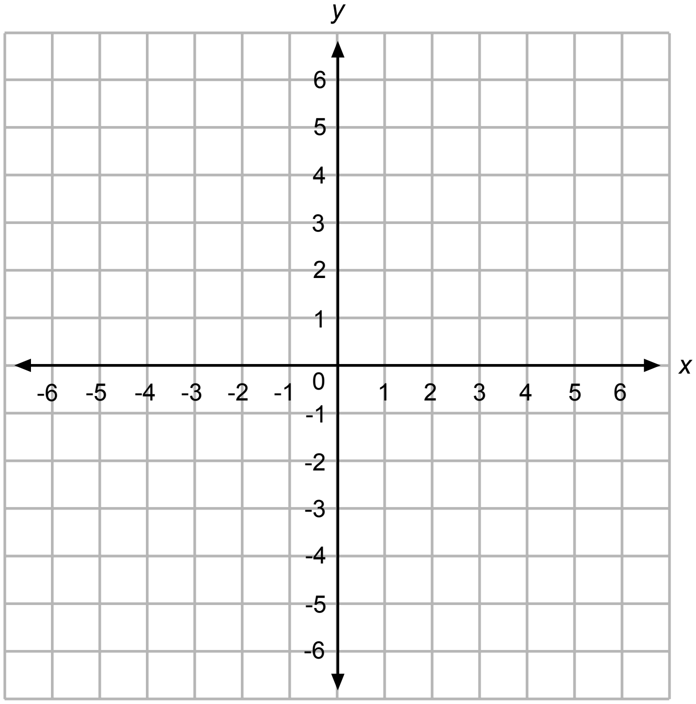
   c. maximum or minimum = ____________________

   d. graph

Table of Values

$f(x) = x^2 + 2x$	
x	$f(x)$

Graph of $f(x) = x^2 + 2x$



5. $g(x) = -x^2 - 4x + 4$

a. axis of symmetry = ______________________

b. coordinates of vertex = ______________________

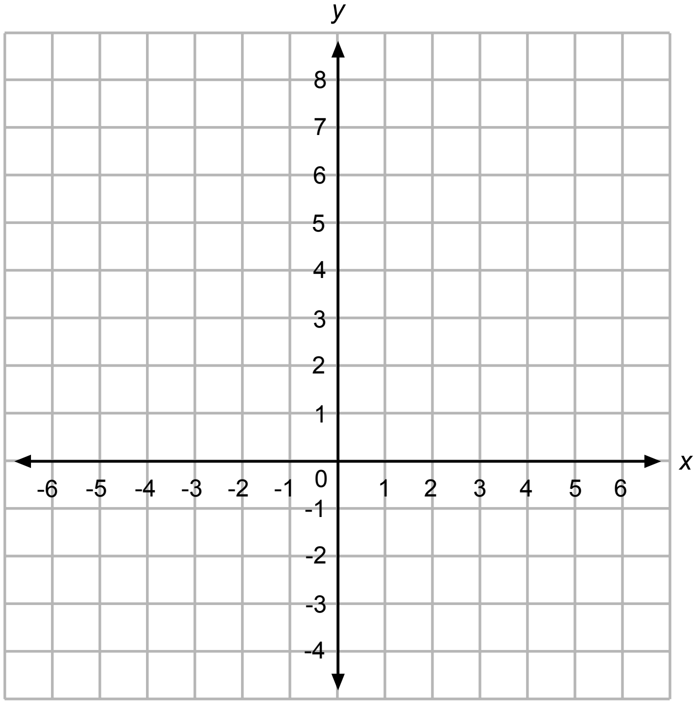
c. maximum or minimum = ______________________

d. graph

Table of Values

$g(x) = -x^2 - 4x + 4$	
x	$g(x)$

Graph of $g(x) = -x^2 - 4x + 4$



6. $f(x) = 3x^2 + 6x - 2$

a. axis of symmetry = ______________________

b. coordinates of vertex = ______________________

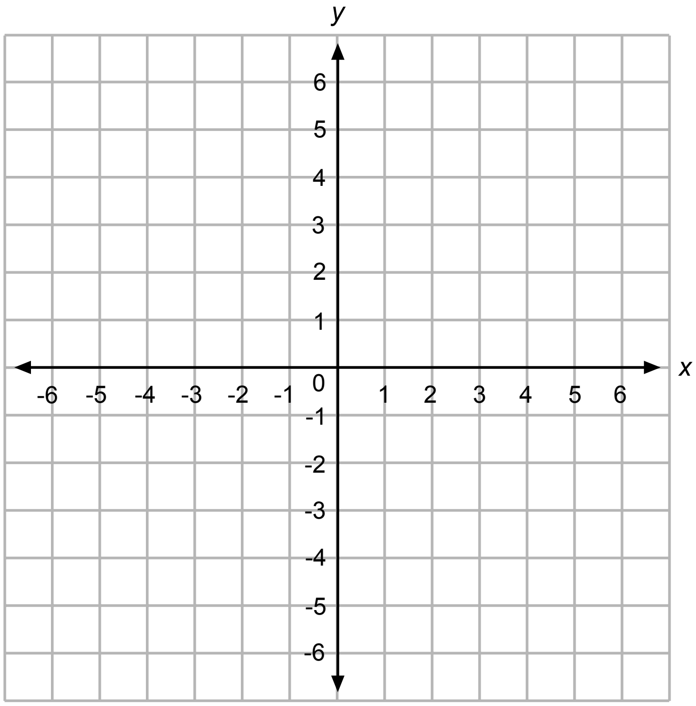
c. maximum or minimum = ______________________

d. graph

Table of Values

$f(x) = 3x^2 + 6x - 2$	
x	$f(x)$

Graph of $f(x) = 3x^2 + 6x - 2$



7. $g(x) = -x^2 + 2x$

a. axis of symmetry = ______________________

b. coordinates of vertex = ______________________

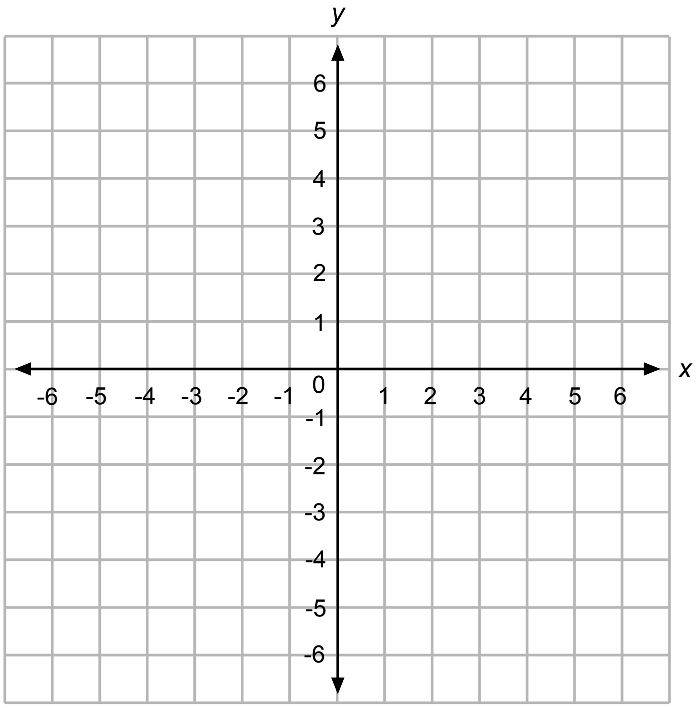
c. maximum or minimum = ______________________

d. graph

Table of Values

$g(x) = -x^2 + 2x$	
x	**$g(x)$**

Graph of $g(x) = -x^2 + 2x$



8. $g(x) = (x - 1)^2$

Note: You must first use the **FOIL method**.

Remember:
F First terms
O Outside terms
I Inside terms
L Last terms.

a. axis of symmetry = ________________________

b. coordinates of vertex = ________________________

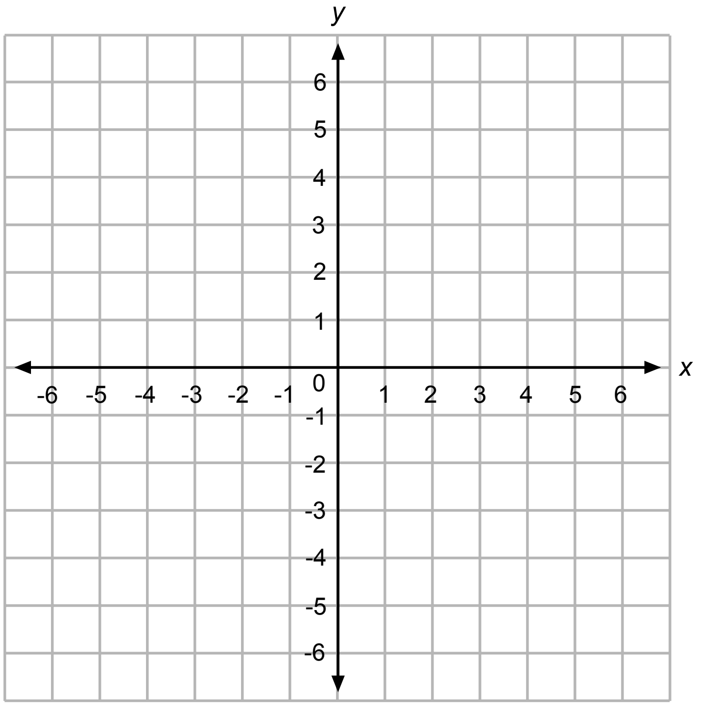
c. maximum or minimum = ________________________

d. graph

Table of Values

$g(x) = (x - 1)^2$	
x	***g*(*x*)**

Graph of $g(x) = (x - 1)^2$



Solving Quadratic Equations

The **solutions** to **quadratic equations** are called the **roots** of the equations. In factoring *quadratic equations*, set each **factor** equal to 0 to **solve** for values of x. Those values of x are the *roots* of the equation.

Example

Solve by factoring

$x^2 + 10x + 9 = 0$ ← factor

$(x + 1)(x + 9) = 0$ ←

Set each factor equal to 0

$x + 1 = 0$ ← **zero product property**

$x = -1$ ← add -1 to each side

$x + 9 = 0$ ← zero product property

$x = -9$ ← add -9 to each side

-1 and -9 are roots

We can also find these roots by graphing the related function $f(x) = x^2 + 10x + 9$ and finding the x-intercepts. The x-intercepts are the points where the graph crosses the x-axis, which are also known as the **zeros** of the function.

Let's see how this works.

$$f(x) = x^2 + 10x + 9$$

The equation for the axis of symmetry is as follows.

$$x = \frac{-10}{2(1)}$$

$$\boxed{x = -5}$$

$$f(-5) = (-5)^2 + 10(-5) + 9$$
$$f(-5) = -16$$

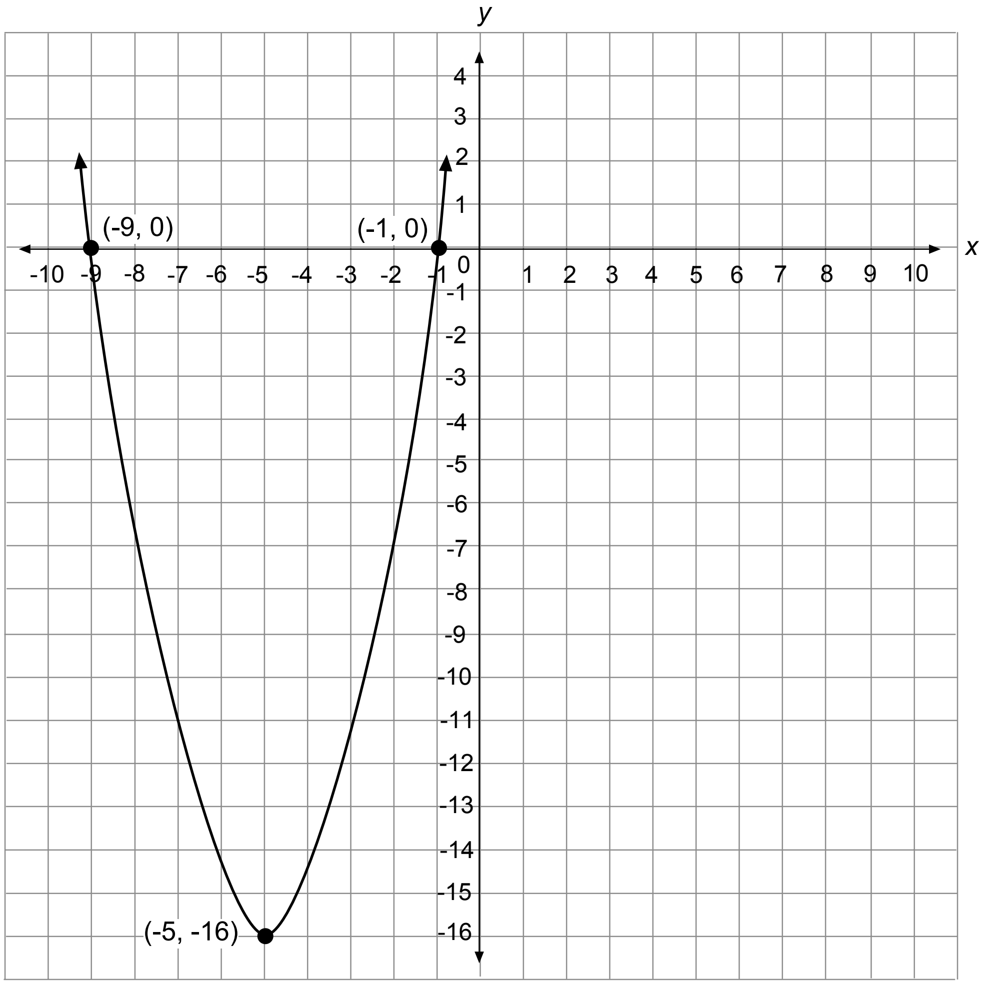
The vertex is at (-5, -16).

Find the x-intercepts by letting $f(x) = 0$.

$$0 = x^2 + 10x + 9$$

The x-intercepts are at (-9, -1). Thus the solutions are -9 and -1.

Graph of $f(x) = x^2 + 10x + 9$



You may find the solutions more efficiently by using your graphing calculator. When the x-intercepts are *not* integers, use your calculator to estimate them to the nearest integer.

Practice

Solve the following by **graphing**. **Show each step indicated.**

Check your work
using your graphing
calculator.

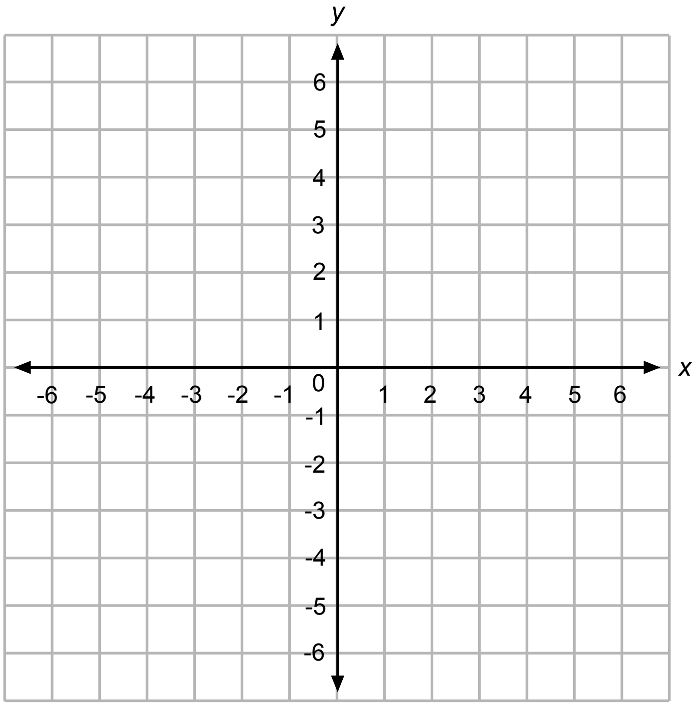
1. $x^2 + 3x + 2 = 0$

   a. axis of symmetry = ____________________

   b. x-intercepts = ____________________

   c. graph

Graph of $f(x) = x^2 + 3x + 2$



   d. solutions = ____________________

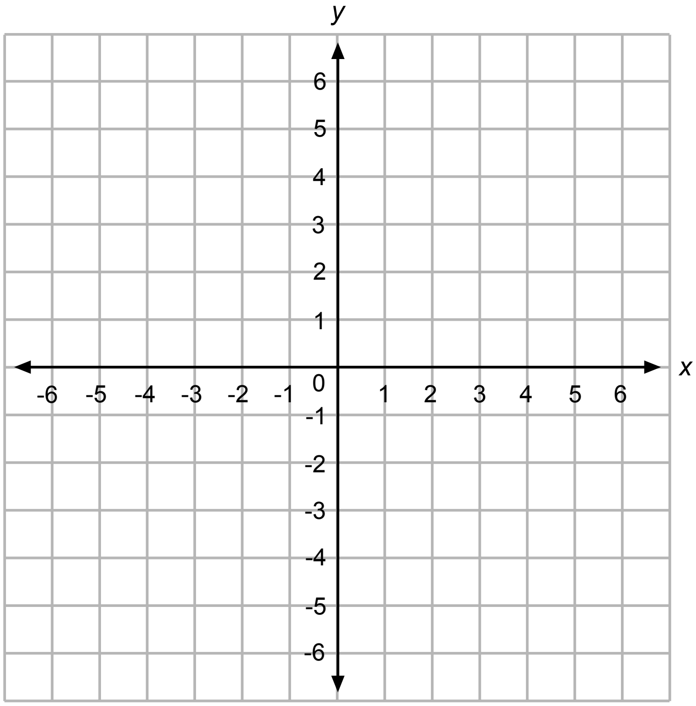
2. $x^2 - 3x + 2 = 0$

a. axis of symmetry = ____________________

b. x-intercepts = ____________________

c. graph

Graph of $f(x) = x^2 - 3x + 2$



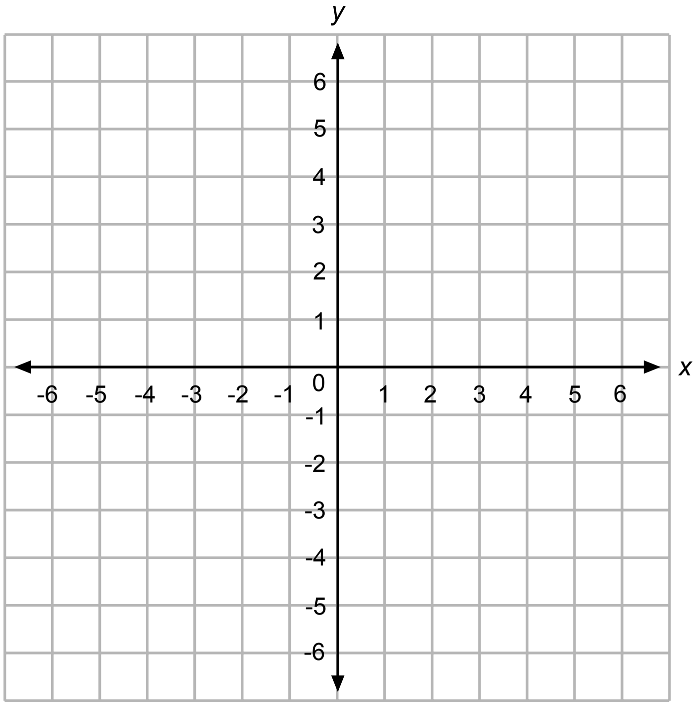
d. solutions = ____________________

3. $-x^2 - 3x - 2 = 0$

   a. axis of symmetry = ______________________

   b. x-intercepts = ______________________

   c. graph



Graph of $f(x) = -x^2 - 3x - 2$

   d. solutions = ______________________

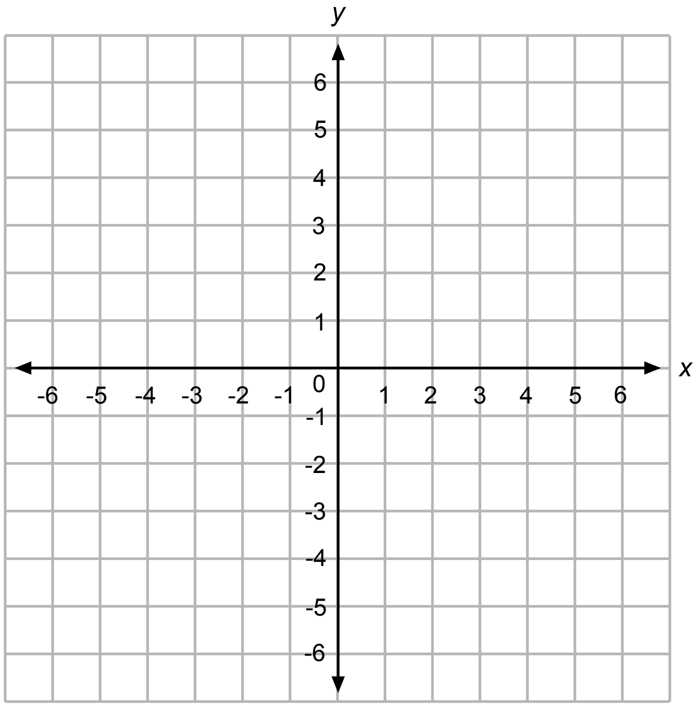
4. $x^2 + 9x + 14 = 0$

a. axis of symmetry = ____________________________

b. x-intercepts = ____________________________

c. graph

Graph of $f(x) = x^2 + 9x + 14$



d. solutions = ____________________________

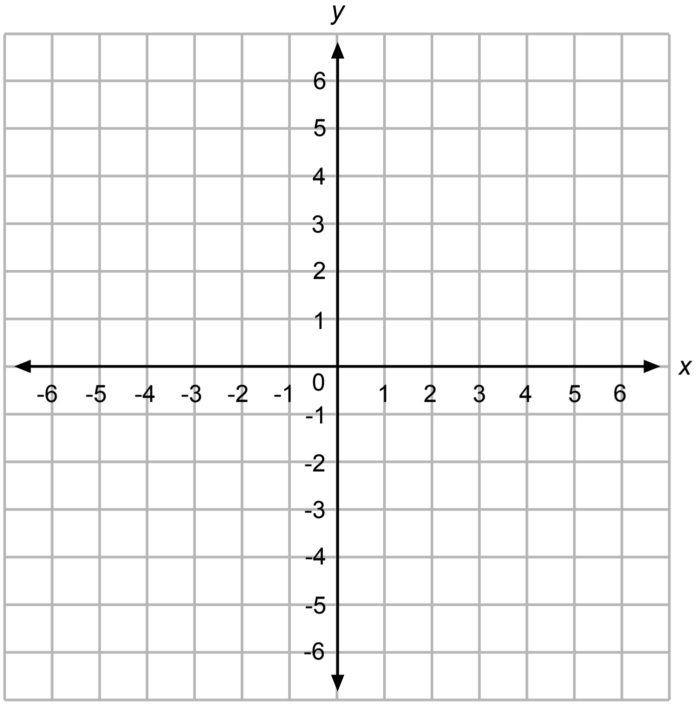
5. $-x^2 + 9x - 14 = 0$

a. axis of symmetry = ______________________________

b. x-intercepts = ______________________________

c. graph

Graph of $f(x) = -x^2 + 9x - 14$



d. solutions = ______________________________

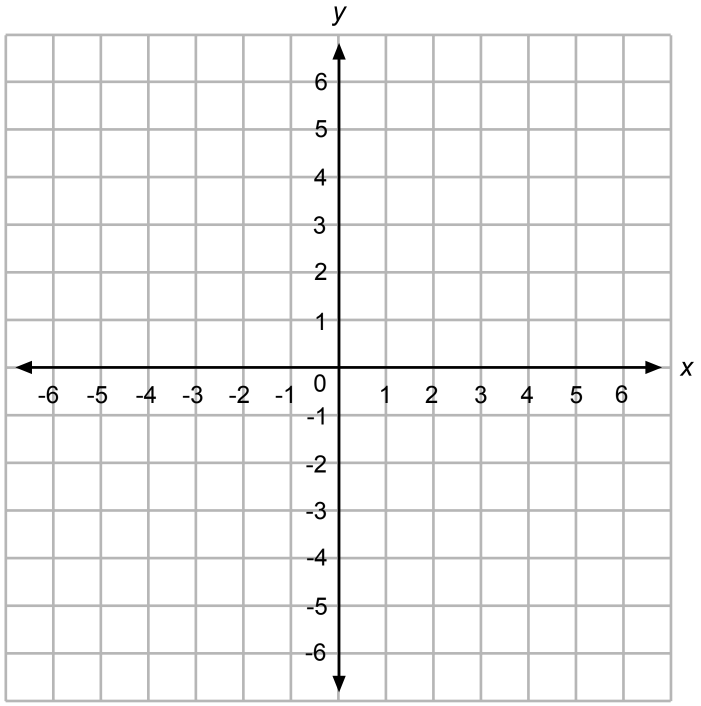
6. $2x^2 + 3x + 1 = 0$

a. axis of symmetry = ____________________________

b. x-intercepts = ____________________________

c. graph

Graph of $f(x) = 2x^2 + 3x + 1$



d. solutions = ____________________________

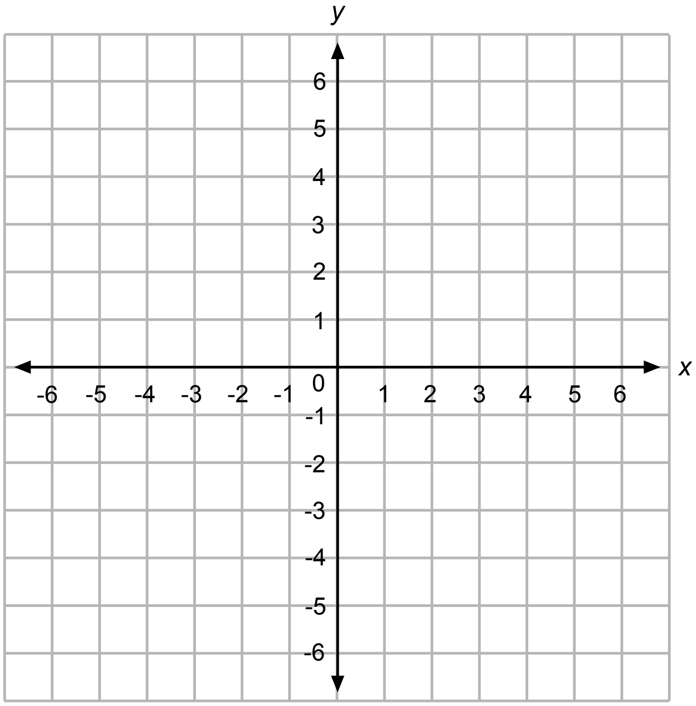
7. $-2x^2 - 5x - 3 = 0$

a. axis of symmetry = __________________________

b. x-intercepts = __________________________

c. graph

Graph of $f(x) = -2x^2 - 5x - 3$



d. solutions = __________________________

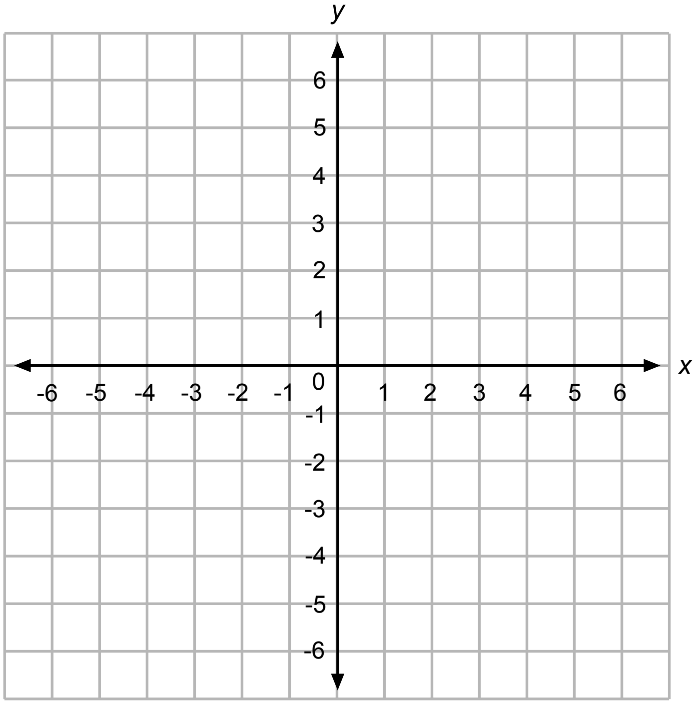
8. $2x^2 - 5x = 0$

a. axis of symmetry = ____________________________

b. x-intercepts = ____________________________

c. graph

Graph of $f(x) = 2x^2 - 5x$



d. solutions = ____________________________

9. $x^2 + 6x + 6 = 0$

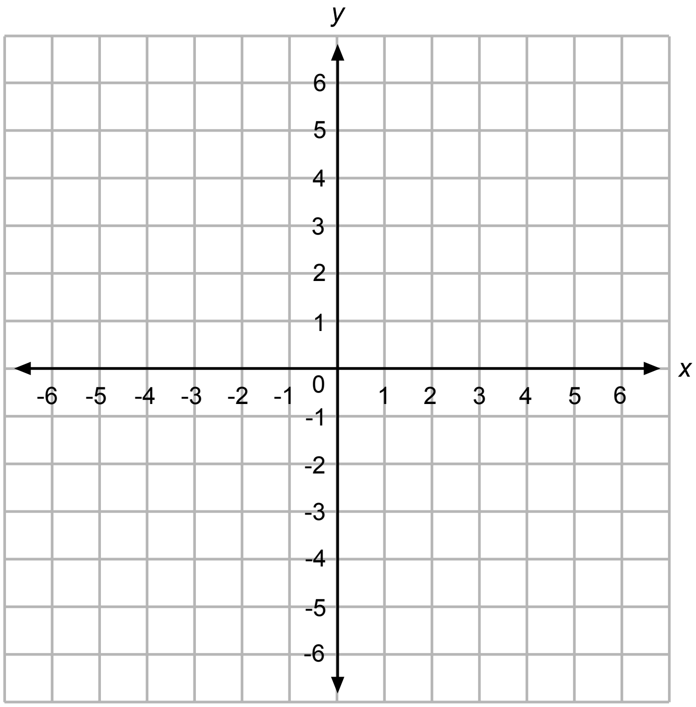
a. axis of symmetry = ____________________

b. x-intercepts = ____________________

Hint: Use your calculator to estimate x-intercepts.

c. graph

Graph of $f(x) = x^2 + 6x + 6$



d. solutions = ____________________

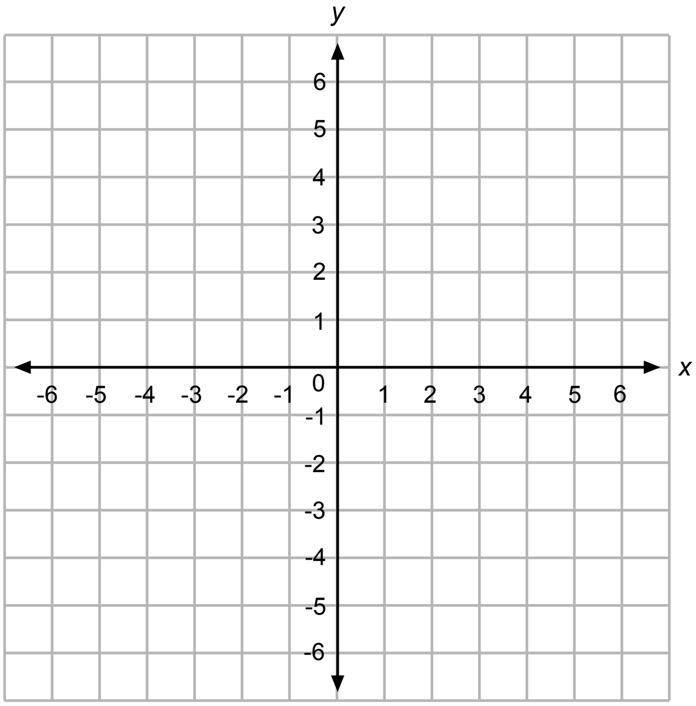
10. $x^2 - x - 4 = 0$

a. axis of symmetry = ______________________________

b. x-intercepts = ______________________________

Hint: Use your calculator to estimate x-intercepts.

c. graph



Graph of $f(x) = x^2 - x - 4$

d. solutions = ______________________________

Practice

Match each definition with the correct term. Write the letter on the line provided.

_______ 1. the lowest point on the vertex of a parabola, which opens upward

_______ 2. set of y-values of a relation

_______ 3. the value of x at the point where a line or graph intersects the x-axis; the value of y is zero (0) at this point

_______ 4. set of x-values of a relation

_______ 5. a set of ordered pairs (x, y)

_______ 6. vertical line passing through the maximum or minimum point of a parabola

_______ 7. the highest point on the vertex of a parabola, which opens downward

_______ 8. a relation in which each value of x is paired with a unique value of y

_______ 9. the maximum or minimum point of a parabola

A. axis of symmetry

B. domain

C. function (of x)

D. maximum

E. minimum

F. range

G. relation

H. vertex

I. x-intercept

Unit Review

Use the **set** *below to answer the following.*

{(2, 6), (3, -1), (7, 2), (-3, 5), (0, -2)}

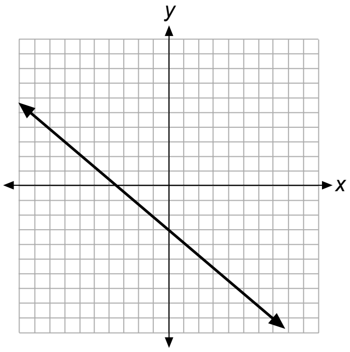
1. Is the set a relation? ______________
2. Give the domain. ____________________________

Remember: The domains and ranges do *not* have to be listed in numerical order. If a value in a domain or in a range is repeated, list the value *one* time.

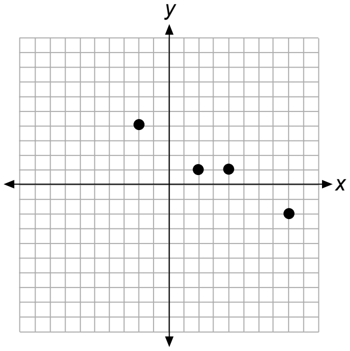
3. Give the range. ____________________________
4. Is the set a function? ______________

Determine if the **graphs** *represent* **functions**.

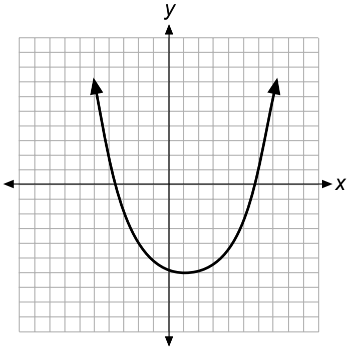
______________ 5.



______________ 6.



______________ 7.



Use the **domain** *below to give the* **range** *for the following* **functions***.*

{-2, -1, 0, 1, 2}

Remember: The domains and ranges do *not* have to be listed in numerical order. If a value in a domain or in a range is repeated, list the value *one* time.

8. $f(x) = 2x + 5$

9. $g(x) = x^2 + 2$

10. $h(x) = -3x + 2$

Use the **relation** *below to answer the following.*

{(0, 0), (1, 2), (2, 4), (3, 6), (4, 8)}

11. If these points are plotted, they will ________________________
    (always, sometimes, never) lie in a line.

12. The function for the line will be $f(x) =$ ________________________ .

13. Create a situation the relation might describe.

____

____

____

For each **function**, *fill in the* **table of values** *and then* **graph the function**.

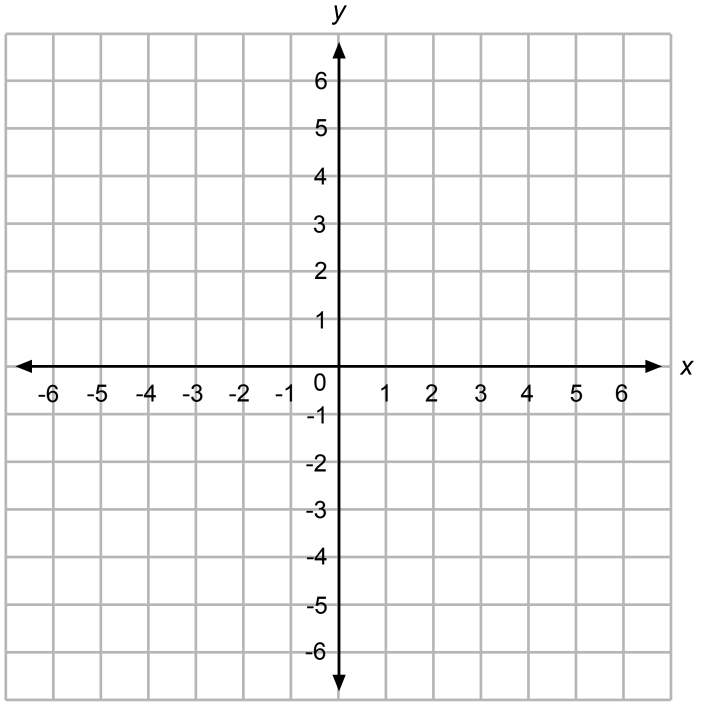
14. $f(x) = 2x - 2$

a. **Table of Values**

$f(x) = 2x - 2$	
x	**$f(x)$**
-2	
0	
2	

b. graph

Graph of $f(x) = 2x - 2$



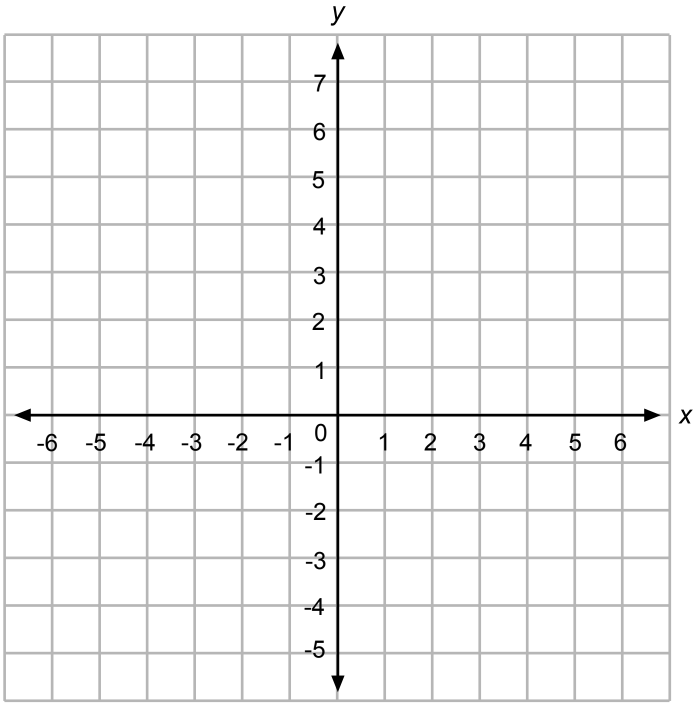
15. $f(x) = -3x + 1$

a. **Table of Values**

$f(x) = -3x + 1$	
x	$f(x)$
-2	
0	
2	

b. graph

Graph of $f(x) = -3x + 1$



16. In this equation, $f(x) = 92.68x$ represents the total tuition for a Florida resident at the University of Florida. The x in the equation represents the number of hours of courses an undergraduate student takes.

Calculate the tuition for a student who takes 14 hours.

Answer: ____________

17. The University of Miami charges $1,000 for each credit hour plus $226.50 for various fees for full time students.

a. Write a function to express the total cost for a semester.

$f(x) =$ ____

b. Calculate the total cost for a student taking 15 hours.

Answer: ____________

For each **function** *do the following.*

- Find the equation for the axis of symmetry.
- Find the coordinates of the vertex of the graph.
- Tell whether the vertex is a maximum or minimum.
- Graph the function.

18. $f(x) = x^2 + 2x - 3$

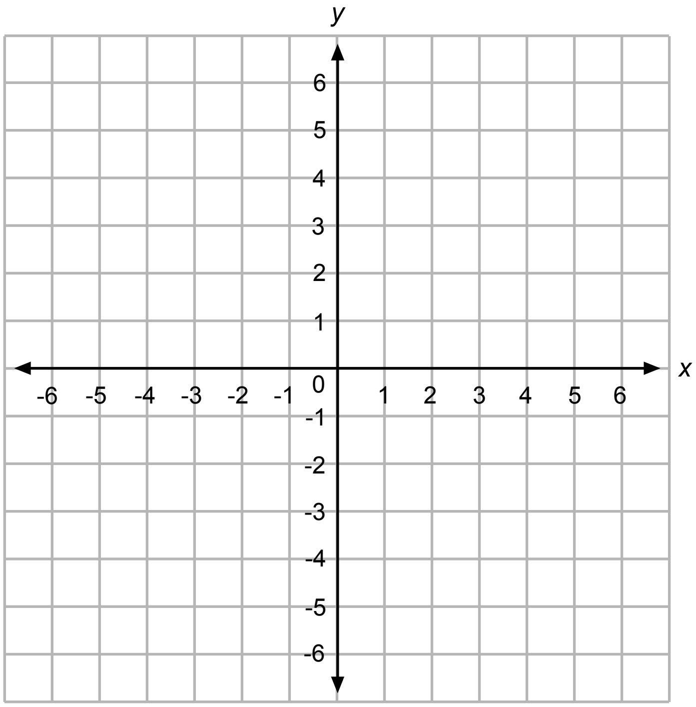
a. axis of symmetry = ____________________

b. coordinates of vertex = ____________________

c. maximum or minimum = ____________________

d. graph

Graph of $f(x) = x^2 + 2x - 3$



19. $g(x) = -x^2 + x$

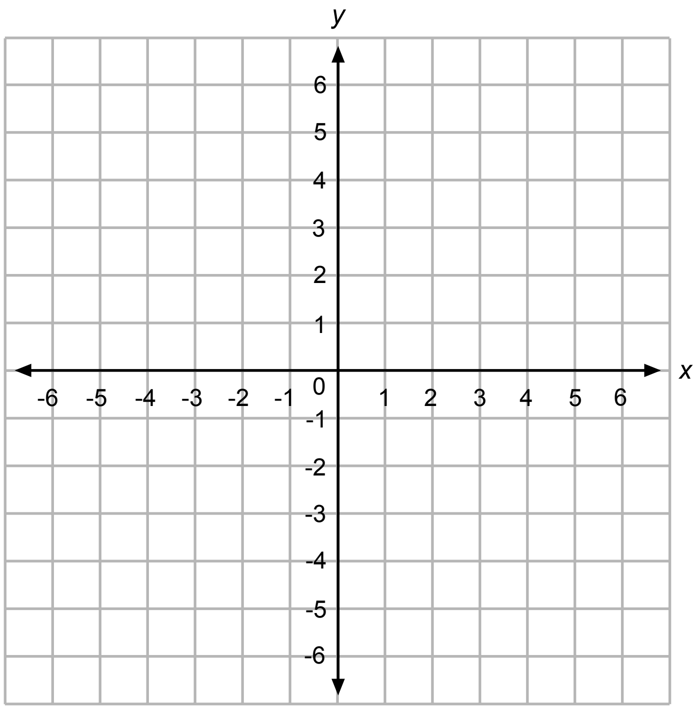
a. axis of symmetry = ________________________

b. coordinates of vertex = ________________________

c. maximum or minimum = ________________________

d. graph

Graph of $g(x) = -x^2 + x$



For each **equation**, *use your* **graphing calculator** *to do the following.*

- Find the axis of symmetry.
- Find the x-intercepts.
- Graph the function.
- Find the solutions.

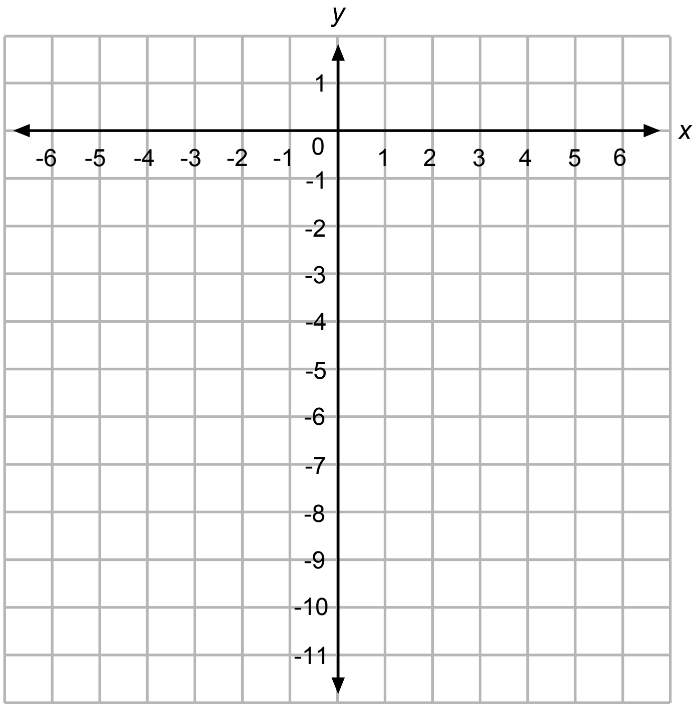
20. $x^2 + 4x - 5 = 0$

a. axis of symmetry = ____________________________

b. x-intercepts = ____________________________

c. graph

Graph of $f(x) = x^2 + 4x - 5$



d. solutions = ____________________________

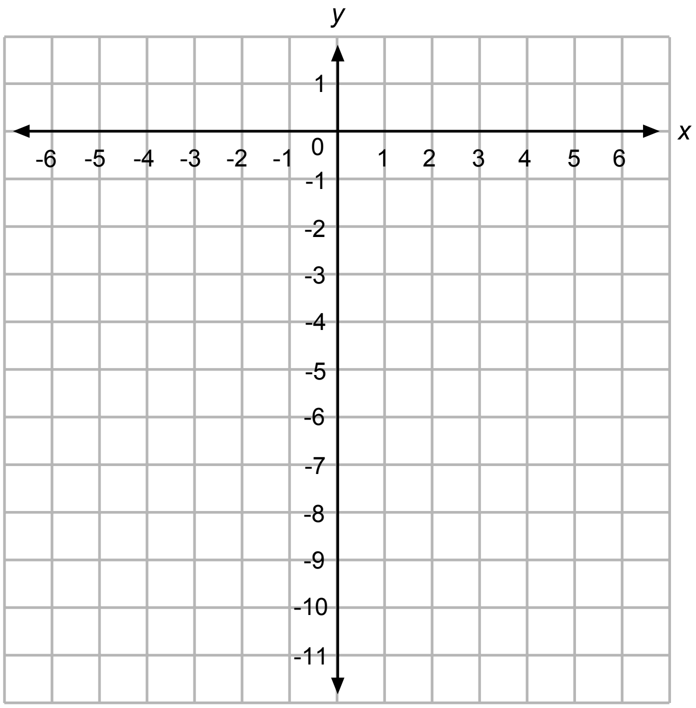
21. $x^2 + 2x - 6 = 0$

a. axis of symmetry = ______________________

b. x-intercepts = ______________________

c. graph

Graph of $f(x) = x^2 + 2x - 6$



d. solutions = ______________________

Unit 10: *X* or (*X, Y*) Marks the Spot!

This unit shows students how to solve equations algebraically and graphically.

Unit Focus

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Algebra Body of Knowledge

Standard 1: Real and Complex Number Systems

- MA.912.A.1.8
  Use the zero product property of real numbers in a variety of contexts to identify solutions to equations.

Standard 3: Linear Equations and Inequalities

- MA.912.A.3.2
  Identify and apply the distributive, associative, and commutative properties of real numbers and the properties of equality.

- MA.912.A.3.5
  Symbolically represent and solve multi-step and real-world applications that involve linear equations and inequalities.

- MA.912.A.3.12
  Graph a linear equation or inequality in two variables with and without graphing technology. Write an equation or inequality represented by a given graph.

- MA.912.A.3.13
  Use a graph to approximate the solution of a system of linear equations or inequalities in two variables with and without technology.

- MA.912.A.3.14
  Solve systems of linear equations and inequalities in two and three variables using graphical, substitution, and elimination methods.

- MA.912.A.3.15
  Solve real-world problems involving systems of linear equations and inequalities in two and three variables.

Standard 4: Polynomials

- MA.912.A.4.3
  Factor polynomial expressions.

Standard 7: Quadratic Equations

- MA.912.A.7.2
  Solve quadratic equations over the real numbers by factoring, and by using the quadratic formula.

- MA.912.A.7.8
  Use quadratic equations to solve real-world problems.

Standard 10: Mathematical Reasoning and Problem Solving

- MA.912.A.10.1
  Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing- and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.

- MA.912.A.10.2
  Decide whether a solution is reasonable in the context of the original situation.

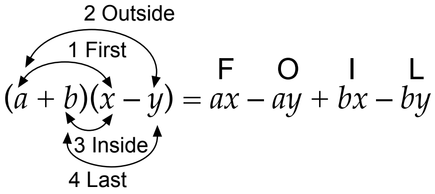
- MA.912.A.10.3
  Decide whether a given statement is always, sometimes, or never true (statements involving linear or quadratic expressions, equations, or inequalities, rational or radical expressions, or logarithmic or exponential functions).

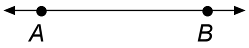
Vocabulary

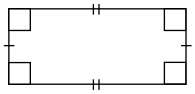
Use the vocabulary words and definitions below as a reference for this unit.

area (*A*)	the measure, in square units, of the inside region of a closed two-dimensional figure; the number of square units needed to cover a surface *Example*: A rectangle with sides of 4 units by 6 units has an area of 24 square units.
axes (of a graph)	the horizontal and vertical number lines used in a coordinate plane system; (singular: *axis*)
coefficient	the number that multiplies the variable(s) in an algebraic expression *Example*: In 4xy, the coefficient of xy is 4. If no number is specified, the coefficient is 1.
consecutive	in order *Example*: 6, 7, 8 are consecutive whole numbers and 4, 6, 8 are consecutive even numbers.
coordinate grid or **plane**	a two-dimensional network of horizontal and vertical lines that are parallel and evenly spaced; especially designed for locating points, displaying data, or drawing maps
coordinates	numbers that correspond to points on a coordinate plane in the form (x, y), or a number that corresponds to a point on a number line

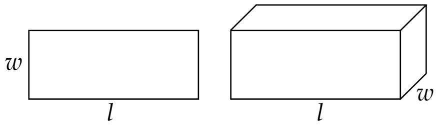
distributive property	the product of a number and the sum or difference of two numbers is equal to the sum or difference of the two products *Examples*: $x(a + b) = ax + bx$ $5(10 + 8) = 5 \bullet 10 + 5 \bullet 8$
equation	a mathematical sentence stating that the two expressions have the same value *Example*: $2x = 10$
equivalent expressions	expressions that have the same value but are presented in a different format using the properties of numbers
even integer	any integer divisible by 2; any integer with the digit 0, 2, 4, 6, or 8 in the units place; any integer in the set {… , -4, -2, 0, 2, 4, …}
factor	a number or expression that divides evenly into another number; one of the numbers multiplied to get a product *Examples*: 1, 2, 4, 5, 10, and 20 are factors of 20 and $(x + 1)$ is one of the factors of $(x^2 - 1)$.
factored form	a number or expression expressed as the product of prime numbers and variables, where no variable has an exponent greater than 1
factoring	expressing a polynomial expression as the product of monomials and polynomials *Example*: $x^2 - 5x + 4 = 0$ $(x - 4)(x - 1) = 0$

FOIL method	a pattern used to multiply two binomials; multiply the first, outside, inside, and last terms: **F** **F**irst terms **O** **O**utside terms **I** **I**nside terms **L** **L**ast terms. *Example*:  $(a + b)(x - y) = ax - ay + bx - by$
formula	a way of expressing a relationship using variables or symbols that represent numbers
fraction	any part of a whole *Example*: One-half written in fractional form is $\frac{1}{2}$
graph	a drawing used to represent data *Example*: bar graphs, double bar graphs, circle graphs, and line graphs
graph of an equation	all points whose coordinates are solutions of an equation
inequality	a sentence that states one expression is greater than (>), greater than or equal to (≥), less than (<), less than or equal to (≤), or not equal to (≠) another expression *Examples*: $a \neq 5$ *or* $x < 7$ *or* $2y + 3 \geq 11$
infinite	having no boundaries or limits
integers	the numbers in the set {… , -4, -3, -2, -1, 0, 1, 2, 3, 4, …}

intersect	to meet or cross at one point
intersection	the point at which lines or curves meet
length (*l*)	a one-dimensional measure that is the measurable property of line segments
line (↔)	a collection of an infinite number of points forming a straight path extending in opposite directions having unlimited length and no width 
monomial	a number, variable, or the product of a number and one or more variables; a polynomial with only *one* term *Examples*: 8 x $4c$ $2y^2$ -3 $\frac{xyz^2}{9}$
negative integers	integers less than zero
negative numbers	numbers less than zero
odd integer	any integer not divisible by 2; any integer with the digit 1, 3, 5, 7, or 9 in the units place; any integer in the set {… , -5, -3, -1, 1, 3, 5, …}
ordered pair	the location of a single point on a rectangular coordinate system where the first and second values represent the position relative to the x-axis and y-axis, respectively *Examples*: (x, y) *or* (3, -4)
parallel (||)	being an equal distance at every point so as to never intersect
point	a specific location in space that has no discernable length or width

polynomial	a monomial or sum of monomials; any rational expression with no variable in the denominator *Examples*: $x^3 + 4x^2 - x + 8$ $5mp^2$ $-7x^2y^2 + 2x^2 + 3$
positive integers	integers greater than zero
product	the result of multiplying numbers together *Example*: In 6 x 8 = 48, the product is 48.
quadratic equation	an equation in the form of $ax^2 + bx + c = 0$
quadratic formula	formula used to solve any quadratic equation; if $ax^2 + bx + c = 0$ and $a \neq 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
rectangle	a parallelogram with four right angles 
simplify an expression	to perform as many of the indicated operations as possible
solution	any value for a variable that makes an equation or inequality a true statement *Example*: In $y = 8 + 9$ $y = 17$ 17 is the solution.
solution set ({ })	the set of values that make an equation or inequality true *Example*: {5, -5} is the solution set for $3x^2 = 75$.
solve	to find all numbers that make an equation or inequality true

standard form (of a quadratic equation)	$ax^2 + bx + c = 0$, where a, b, and c are integers (not multiples of each other) and $a > 0$
substitute	to replace a variable with a numeral *Example*: $8(a) + 3$ $8(5) + 3$
substitution	a method used to solve a system of equations in which variables are replaced with known values or algebraic expressions
sum	the result of adding numbers together *Example*: In $6 + 8 = 14$, the sum is 14.
system of equations	a group of two or more equations that are related to the same situation and share variables *Example*: The solution to a system of equations is an ordered number set that makes all of the equations true.
table (or chart)	a data display that organizes information about a topic into categories
term	a number, variable, product, or quotient in an expression *Example*: In the expression $4x^2 + 3x + x$, the terms are $4x^2$, $3x$, and x.
value (of a variable)	any of the numbers represented by the variable
variable	any symbol, usually a letter, which could represent a number

Venn diagram	overlapping circles used to illustrate relationships among sets
vertical	at right angles to the horizon; straight up and down 
width (*w*)	a one-dimensional measure of something side to side 
***x*-intercept**	the value of x at the point where a line or graph intersects the x-axis; the value of y is zero (0) at this point
***y*-intercept**	the value of y at the point where a line or graph intersects the y-axis; the value of x is zero (0) at this point
zero product property	for all numbers a and b, if $ab = 0$, then $a = 0$ and/or $b = 0$

Unit 10: X or (X, Y) Marks the Spot!

Introduction

In this unit, we will expand our knowledge of problem solving to find solutions to a variety of equations, inequalities, systems of equations, and real-world situations.

Lesson One Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

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Standard 7: Quadratic Equations

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  Solve quadratic equations over the real numbers by factoring, and by using the quadratic formula.

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Standard 10: Mathematical Reasoning and Problem Solving

- MA.912.A.10.1
  Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing- and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.

- MA.912.A.10.2
  Decide whether a solution is reasonable in the context of the original situation.

Quadratic Equations

When we **solve** an **equation** like $x + 7 = 12$, we remember that we must subtract 7 from both sides of the equal sign.

$$\begin{aligned} x + 7 &= 12 \\ x + 7 - 7 &= 12 - 7 \quad \text{subtract 7 from both sides} \\ x &= 5 \end{aligned}$$

That leaves us with $x = 5$. We know that 5 is the only **solution** or **value** that can replace x and make the $x + 7 = 12$ true.

$$\begin{aligned} \text{If } x + 7 &= 12, \text{ and} \\ x &= 5 \text{ is true, then} \\ 5 + 7 &= 12. \end{aligned}$$

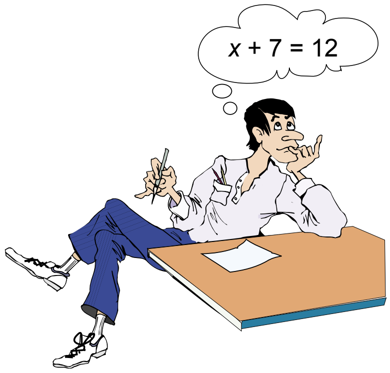
Suppose you have an *equation* that looks like $(x + 7)(x - 3) = 0$. This means there are two numbers, one in each set of parentheses, that when multiplied together, have a **product** of 0. What kinds of numbers can be multiplied and equal 0?

Look at the following options.

$$2 \times -2 = -4 \qquad \frac{1}{5} \times 5 = 1 \qquad -\frac{4}{7} \times \frac{7}{4} = -1$$

The only way for numbers to be multiplied together with a result of zero is if one of the numbers is a 0.

$$a \times 0 = 0$$



Looking back at $(x + 7)(x - 3) = 0$, we understand that there are two **factors**, $(x + 7)$ and $(x - 3)$. The only way to multiply them and get a *product* of 0 is if one of them is equal to zero.

This leads us to a way to *solve* the equation. Since we don't know which of the terms equals 0, we cover all the options and assume either could be equal to zero.

$$\begin{aligned} \text{If } x + 7 &= 0, \\ \text{then } x &= -7. \end{aligned}$$

$$\begin{aligned} \text{If } x - 3 &= 0, \\ \text{then } x &= 3. \end{aligned}$$

We now have two options which could replace x in the original equation and make it true. Let's replace x with -7 and 3, one at a time.

$$\begin{aligned} (x + 7)(x - 3) &= 0 \\ (-7 + 7)(-7 - 3) &= 0 \\ (0)(-10) &= 0 \\ 0 &= 0 \end{aligned} \qquad \begin{aligned} (x + 7)(x - 3) &= 0 \\ (3 + 7)(3 - 3) &= 0 \\ (10)(0) &= 0 \\ 0 &= 0 \end{aligned}$$

Therefore, because either *value* of x gives us a true statement, we see that the **solution set** for $(x + 7)(x - 3) = 0$ is $\{-7, 3\}$.

Now you try the items in the following practice.

Practice

Find the **solution sets**. *Refer to pages 715 and 716 as needed.*

1. $(x + 4)(x - 2) = 0$ { ______ , ______ }

2. $(x - 5)(x + 3) = 0$ { ______ , ______ }

3. $(x - 5)(x - 7) = 0$ { ______ , ______ }

4. $(x + 6)(x + 1) = 0$ { ______ , ______ }

5. $(x - 2)(x - 2) = 0$ { ______ , ______ }

6. $(x + 18)(x - 23) = 0$ { ______ , ______ }

7. $x(x - 16) = 0$ { ______ , ______ }

8. $(x - 5)(2x + 6) = 0$ { ______ , ______ }

9. $(3x - 5)(5x + 10) = 0$ { ______ , ______ }

10. $(10x - 4)(x + 5) = 0$ { ______ , ______ }

Factoring to Solve Equations

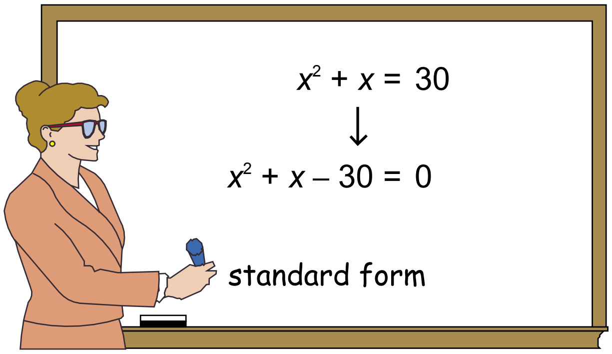
Often, equations are not given to us in **factored form** like those on the previous pages. Looking at $x^2 + x = 30$, we notice the x^2 **term** which tells us this is a **quadratic equation** (an equation in the form $ax^2 + bx + c = 0$). This term also tells us to be on the lookout for two answers in our *solution set*.

You may solve this problem by trial and error. However, we can also solve $x^2 + x = 30$ using a format called **standard form (of a quadratic equation).** This format is written with the *terms* in a special order:

- the x^2 term first
- then the x-term
- then the numerical term followed by $= 0$.

For our original equation,

$$\begin{aligned} x^2 + x &= 30 && \longleftarrow \text{put in \textit{standard form}} \\ x^2 + x - 30 &= 30 - 30 && \longleftarrow \text{subtract 30 from both sides} \\ x^2 + x - 30 &= 0 && \swarrow \end{aligned}$$



Now that we have the proper format, we can factor the quadratic **polynomial**.

Remember: Factoring expresses a *polynomial* as the product of **monomials** and polynomials.

Example 1

Solve by factoring

$x^2 + x - 30 = 0$ ← factor

$(x + 6)(x - 5) = 0$

Set each factor equal to 0

If $x + 6 = 0$, then ← **zero product property**

$x = -6.$ ← add -6 to each side

If $x - 5 = 0$, then ← zero product property

$x = 5.$ ← add 5 to each side

Therefore, the solution set is {-6, 5}.

Example 2

Solve by factoring

$x^2 = 5x - 4$ ← put in standard form

$x^2 - 5x + 4 = 0$ ← factor

$(x - 4)(x - 1) = 0$

Set each factor equal to 0

If $x - 4 = 0$, then ← zero product property

$x = 4.$ ← add -4 to each side

If $x - 1 = 0$, then ← zero product property

$x = 1.$ ← add -1 to each side

{1, 4} ← write the solution set

Now it's your turn to practice on the following page.

Practice

Find the **solution sets**. *Refer to pages 719 and 720 as needed.*

1. $x^2 - x = 42$ { ______ , ______ }

2. $x^2 - 5x = 14$ { ______ , ______ }

3. $x^2 = -5x - 6$ { ______ , ______ }

4. $x^2 - x = 12$ { ______ , ______ }

5. $x^2 = 2x + 8$ { ______ , ______ }

6. $x^2 - 2x = 15$ { _______ , _______ }

7. $x^2 + 8x = -15$ { _______ , _______ }

8. $x^2 - 3x = 0$ { _______ , _______ }

9. $x^2 = -5x$ { _______ , _______ }

10. $x^2 - 4 = 0$ { _______ , _______ }

11. $x^2 = 9$ { ______ , ______ }

12. $3x^2 - 3 = 0$ { ______ , ______ }

13. $2x^2 = 18$ { ______ , ______ }

Practice

Use the list below to write the correct term for each definition on the line provided.

equation
factor
product
solution
solve
value (of a variable)

________________________ 1. a mathematical sentence stating that the two expressions have the same value

________________________ 2. to find all numbers that make an equation or inequality true

________________________ 3. any of the numbers represented by the variable

________________________ 4. any value for a variable that makes an equation or inequality a true statement

________________________ 5. the result of multiplying numbers together

________________________ 6. a number or expression that divides evenly into another number; one of the numbers multiplied to get a product

Practice

Match each definition with the correct term. Write the letter on the line provided.

_______ 1. an equation in the form of $ax^2 + bx + c = 0$

_______ 2. a monomial or sum of monomials; any rational expression with no variable in the denominator

_______ 3. the set of values that make an equation or inequality true

_______ 4. expressing a polynomial expression as the product of monomials and polynomials

_______ 5. a number, variable, or the product of a number and one or more variables; a polynomial with only *one* term

_______ 6. a number or expression expressed as the product of prime numbers and variables, where no variable has an exponent greater than 1

A. factored form

B. factoring

C. monomial

D. polynomial

E. quadratic equation

F. solution set ({ })

Solving Word Problems

We can also use the processes on pages 715-716 and 719-720 to solve word problems. Let's see how.

Example 1

Two **consecutive** (in order) **positive integers** (integers greater than zero) have a product of 110. Find the integers.

$$\begin{aligned}
\text{let the } 1^{\text{st}} \text{ integer} &= x \\
\text{and the } 2^{\text{nd}} \text{ integer} &= x + 1
\end{aligned}$$

$$\begin{aligned}
x(x + 1) &= 110 \\
x^2 + x &= 110 \\
x^2 + x - 110 &= 0 \\
(x - 10)(x + 11) &= 0 \\
x - 10 &= 0 \quad \text{or} \quad x + 11 = 0 \\
x &= 10 \quad \text{or} \quad x = -11
\end{aligned}$$

Since the problem asked for *positive integers*, we must eliminate -11 as an answer. Therefore, the two integers are $x = 10$ and $x + 1 = 11$.

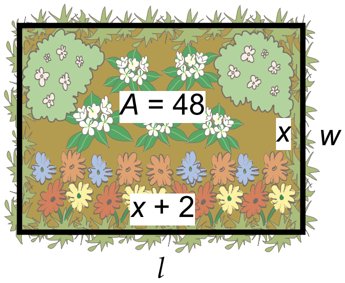
Remember: Integers are the numbers in the set
{… , -4, -3, -2, -1, 0, 1, 2, 3, 4, …}.

Example 2

Billy has a garden that is 2 feet longer than it is wide. If the **area** ***(A)*** of his garden is 48 square feet, what are the dimensions of his garden?

If we knew the **width** ***(w)***, we could find the **length** ***(l)***, which is 2 feet longer. Since we don't know the *width*, let's represent it with x. The *length* will then be $x + 2$.

$$\begin{aligned} \text{width} &= x \\ \text{length} &= x + 2 \end{aligned}$$



The *area* (A) of a **rectangle** can be found using the **formula** length (l) times width (w).

$$\begin{aligned} A &= lw \\ A &= 48 \end{aligned}$$

$$\begin{aligned} \text{So,}\quad x(x + 2) &= 48 \\ x^2 + 2x &= 48 \\ x^2 + 2x - 48 &= 0 \\ (x + 8)(x - 6) &= 0 \\ x + 8 &= 0 \quad \text{or} \quad x - 6 = 0 \\ x &= -8 \quad \text{or} \quad x = 6 \end{aligned}$$

A garden cannot be -8 feet long, so we must use only the 6 as a value for x.

So, the width of the garden is 6 feet and the length is 8 feet.

Practice

Solve each problem. Refer to pages 726 and 727 as needed.

1. The product of two consecutive positive integers is 72. Find the integers.

   Answer: ____________

2. The product of two consecutive positive integers is 90. Find the integers.

   Answer: ____________

3. The product of two consecutive **negative odd integers** is 35. Find the integers.

   Answer: ____________

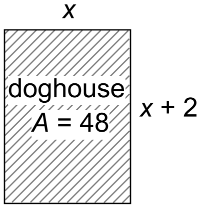
4. The product of two consecutive *negative odd integers* is 143. Find the integers.

   Answer: ____________

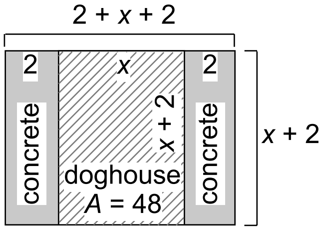
5. Sara's photo is 5 inches by 7 inches. When she adds a frame, she gets an area of 63 square inches. What is the width of the frame?

Answer: ____________ inches

6. Bob wants to build a doghouse that is 2 feet longer than it is wide. He's building it on a concrete slab that will leave 2 feet of concrete slab visible on each side. If the area of the doghouse is 48 square feet, what are the dimensions of the concrete slab?

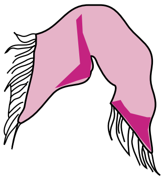


Answer: ____________ feet x ____________ feet



7. Marianne's scarf is 5 inches longer than it is wide. If the area of her scarf is 84 square inches, what are its dimensions?

Answer: ____________ inches x ____________ inches



Practice

Use the list below to complete the following statements.

area (A)
consecutive integers
length (l)
negative integers
odd integers
positive integers
rectangle
width (w)

1. Integers that are less than zero are ________________ .

2. Integers that are greater than zero are ________________ .

3. Integers that are *not* divisible by 2 are ________________ .

4. The measure in square units of the inside region of a closed two-dimensional figure is called the ________________ .

5. A parallelogram with four right angles is called a(n) ________________ .

6. Numbers in order are ________________ .

7. To find the area (A) of a rectangle, you multiply the ________________ by the ________________ .

8. Numbers in the set {… , -4, -3, -2, -1, 0, 1, 2, 3, 4, …} are called ________________ .

Answer the following.

9. The length of a rectangle can ________________ (always, sometimes, never) be a negative number.

Using the Quadratic Formula

Sometimes an equation seems difficult to factor. When this happens, you may need to use the **quadratic formula**. Remember that quadratic equations use the format below.

$$ax^2 + bx + c = 0$$

The *quadratic formula* uses the information from the equation and looks like the following.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Compare the equation with the formula and notice how all the same letters are just in different places.

Let's see how the quadratic formula is used to solve the following equation.

In the equation, $a = 4$, $b = 1$, and $c = -5$. These values are **substituted** into the quadratic formula below.

$4x^2 + x - 5 = 0$ ← original equation

$\boxed{a = 4 \quad b = 1 \quad c = -5}$ ← values

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ← quadratic formula

$x = \frac{-1 \pm \sqrt{(1)^2 - 4(4)(-5)}}{2(4)}$ ← values $a = 4$, $b = 1$, and $c = -5$ *substituted*

$x = \frac{-1 \pm \sqrt{1 - (-80)}}{8}$ ← simplify

$x = \frac{-1 \pm \sqrt{81}}{8}$

$x = \frac{-1 \pm 9}{8}$

$x = \frac{-1 + 9}{8}$ or $x = \frac{-1 - 9}{8}$

$x = \frac{8}{8}$ or $x = \frac{-10}{8}$

$x = 1$ or $x = \frac{-5}{4}$ ← the solution set is $\{1, \frac{-5}{4}\}$

Remember: The symbol ± means plus or minus. Therefore, ± means we have two factors. One is found by adding and the other by subtracting.

Let's look at another example.

$2x^2 + 5x + 3 = 0$ ← original equation

$\boxed{a = 2 \quad b = 5 \quad c = 3}$ ← values

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ ← quadratic formula

$x = \frac{-5 \pm \sqrt{(5)^2 - 4(2)(3)}}{2(2)}$ ← values *a* = 2, *b* = 5, and *c* = 3 *substituted*

$x = \frac{-5 \pm \sqrt{25 - 24}}{4}$ ← simplify

$x = \frac{-5 \pm \sqrt{1}}{4}$

$x = \frac{-5 \pm 1}{4}$

$x = \frac{-5 + 1}{4}$ or $x = \frac{-5 - 1}{4}$

$x = \frac{-4}{4}$ or $x = \frac{-6}{4}$

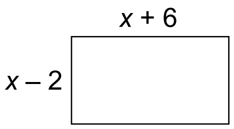
$x = -1$ or $x = \frac{-3}{2}$ ← the solution set is $\{-1, \frac{-3}{2}\}$

Often your answer will not **simplify** all the way to a **fraction** or *integer*. Some answers will look like $\frac{-3 \pm \sqrt{13}}{6}$. You can check your work by using a graphing calculator or another advanced-level calculator.

We can even use the quadratic formula when solving word problems. However, before you start using the quadratic formula, it is important to remember to put the equation you are working with in the correct format. The equation must look like the following.

$$ax^2 + bx + c = 0$$

Look at this example.



If a rectangle has an area of 20 and its dimensions are as shown, find the actual length and width of the rectangle.

$(x + 6)(x - 2) = 20$ ← set up the equation

$x^2 + 4x - 12 = 20$ ← **FOIL**—First, Outside, Inside, Last

$x^2 + 4x - 32 = 0$ ← format ($ax^2 + bx + c = 0$)

$(x + 8)(x - 4) = 0$ ← factor

$x + 8 = 0$ ***or*** $x - 4 = 0$ ← solve

$x = -8$ ***or*** $x = 4$

$x \neq -8$ because that would result in negative lengths. ← first check to see *if* solutions are reasonable

Remember: The symbol ≠ means is ***not*** equal to.

$x + 6 \longrightarrow 4 + 6 = 10$

$x - 2 \longrightarrow 4 - 2 = 2$

← then check answer by replacing x with 4

The length and width of the rectangle are 10 and 2.

Practice

Use the **quadratic formula** *below to solve the following* **equations**.

Check your work using a calculator.

quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

1. $2x^2 + 5x + 3 = 0$

2. $2x^2 + 3x + 1 = 0$

3. $4x^2 + 3x - 1 = 0$

4. $6x^2 + 5x + 1 = 0$

5. $4x^2 - 11x + 6 = 0$

6. $2x^2 - x - 3 = 0$

7. $2x^2 - 3x + 1 = 0$

8. $9x^2 - 3x - 5 = 0$

9. $8x^2 - 6x - 2 = 0$

10. $9x^2 + 9x - 4 = 0$

11. Jacob wants to build a deck that is $(x + 7)$ units long and $(x + 3)$ units wide. If the area of his deck is 117 square units, what are the dimensions of his deck?

Answer: ____________ units x ____________ units

12. Cecilia and Roberto are thinking of 2 consecutive odd integers whose product is 143. Find the integers.

First use the guess and check method. Then check by solving.

Answer: ____________ and ____________

Practice

Use the **quadratic formula** *below to solve the following* **equations***.*

Show work and check your work with a calculator.

quadratic formula

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

1. $2x^2 + 7x + 6 = 0$

2. $5x^2 + 16x + 3 = 0$

3. $6x^2 + 5x + 1 = 0$

4. $9x^2 + 9x + 2 = 0$

5. $10x^2 + 7x + 1 = 0$

6. $10x^2 - 7x + 1 = 0$

7. $x^2 + 2x - 3 = 0$

8. $x^2 + 2x - 15 = 0$

9. $x^2 - 5x + 6 = 0$

10. $x^2 - 9x + 20 = 0$

11. If the sides of a rectangular walkway are $(x + 3)$ units and $(x - 6)$ units, and the area is 10 square units, find the dimensions of the walkway.

Answer: ____________ units x ____________ units

12. Jordan is thinking of 2 consecutive even integers. If the product of her integers is 168, find the numbers.

Hint: You could use the guess and check problem-solving strategy to solve.

Answer: ____________ and ____________

Lesson Two Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.

- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.

- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Writing Process Strand

Standard 3: Prewriting

- LA.910.3.1.3
  The student will prewrite by using organizational strategies and tools (e.g., technology, spreadsheet, outline, chart, table, graph, Venn diagram, web, story map, plot pyramid) to develop a personal organizational style.

Algebra Body of Knowledge

Standard 3: Linear Equations and Inequalities

- MA.912.A.3.12
  Graph a linear equation or inequality in two variables with and without graphing technology. Write an equation or inequality represented by a given graph.

- MA.912.A.3.13
  Use a graph to approximate the solution of a system of linear equations or inequalities in two variables with and without technology.

- MA.912.A.3.14
  Solve systems of linear equations and inequalities in two and three variables using graphical, substitution, and elimination methods.

- MA.912.A.3.15
  Solve real-world problems involving systems of linear equations and inequalities in two and three variables.

Standard 10: Mathematical Reasoning and Problem Solving

- MA.912.A.10.1
  Use a variety of problem-solving strategies, such as drawing a diagram, making a chart, guessing- and-checking, solving a simpler problem, writing an equation, working backwards, and creating a table.

Systems of Equations

When we look at an equation like $x + y = 5$, we see that because there are two **variables**, there are many possible solutions. For instance,

- if $x = 5$, then $y = 0$
- if $x = 2$, then $y = 3$
- if $x = -4$, then $y = 9$
- if $x = 2.5$, then $y = 2.5$, etc.

Another equation such as $x - y = 1$ allows a specific solution to be determined. Taken together, these two equations help to limit the possible solutions.

When taken together, we call this a **system of equations**. A *system of equations* is a group of two or more equations that are related to the same situation and share the same variables. Look at the equations below.

$$\begin{aligned} x + y &= 5 \\ x - y &= 1 \end{aligned}$$

One possible way to solve the system of equations above is to **graph each equation** on the same set of **axes**. Use a **table** *of values* like those on the following page to help determine two possible **points** for each **line (↔)**.

Table of Values

$x + y = 5$	
x	y
0	5
5	0

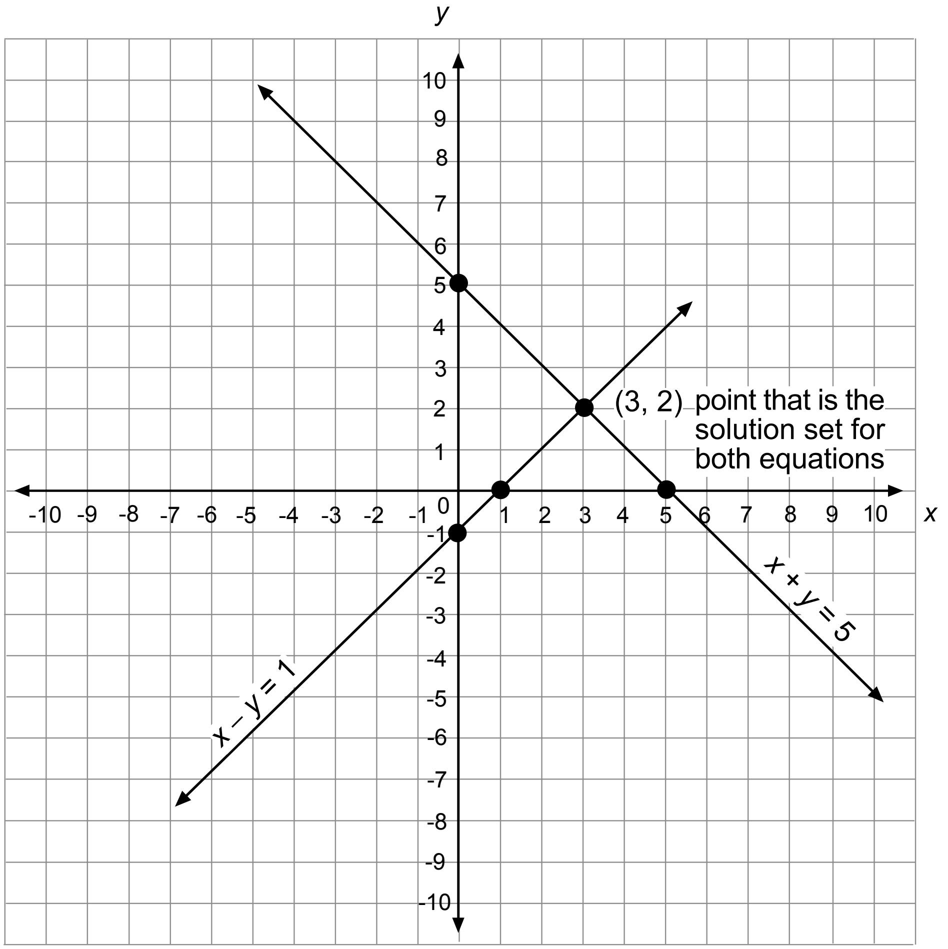
Table of Values

$x - y = 1$	
x	y
0	-1
1	0

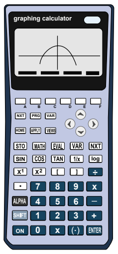
Notice that the values in the table represent the ***x*-intercepts** and ***y*-intercepts**.

Plot the *points* for the first equation on the **coordinate grid** or **plane** below, then draw a *line* connecting them. Do the same for the second set of points.

Graph of $x + y = 5$ and $x - y = 1$



We see from the **graph** above that the two lines **intersect** or *cross at a point*. That point (3, 2) is the solution set for both equations. It is the only point that makes both equations true. You can check your work by replacing x with 3 and y with 2 in both equations to see if they produce true statements.



You can also produce this graph on your *graphing calculator*. To closely estimate the **coordinates** of the points of the graph, move the cursor, the blinking dot, along one line until it gets to the point of **intersection**.

Although graphing is one way to deal with systems of equations; however, it is *not* always the most accurate method. If our graph paper is *not* perfect, our pencil is *not* super-sharp, or the point of *intersection* is *not* at a corner on the grid, we may *not* get the correct answer.

The system can also be solved algebraically with more accuracy. Let's see how that works.

We know from past experience that we can solve problems more easily when there is only one *variable*. So, our job is to eliminate a variable. If we look at the two equations **vertically** (straight up and down), we see that, by adding in columns, the y's will disappear.

$$\begin{array}{rcl} x + y & = & 5 \\ x - y & = & 1 \\ \hline 2x + 0 & = & 6 \end{array}$$

This leaves us with a new equation to solve:

$$\begin{aligned} 2x + 0 &= 6 \\ 2x &= 6 \\ \tfrac{2x}{2} &= \tfrac{6}{2} \longleftarrow \text{divide both sides by 2} \\ x &= 3 \end{aligned}$$

We've found the value for x; now we must find the value of y. Use either of the original equations and replace the x with 3. The example below uses the first one.

$$\begin{aligned} x + y &= 5 \\ 3 + y &= 5 \\ 3 - 3 + y &= 5 - 3 \longleftarrow \text{subtract 3 from both sides} \\ y &= 2 \end{aligned}$$

So, our solution set is {3, 2}.

Let's try another! We'll solve and then graph this time.

$$
\begin{array}{rcll}
2x + y & = & 6 & \\
\underline{-2x + 2y} & = & \underline{-12} & \longleftarrow \text{add to eliminate the } x\text{'s} \\
0 + 3y & = & -6 & \longleftarrow \text{solve} \\
\frac{\cancel{3}y}{\cancel{3}} & = & \frac{-\cancel{6}}{\cancel{3}} = \frac{-2}{1} & \swarrow \\
y & = & -2 &
\end{array}
$$

$$
\begin{array}{rcll}
2x + -2 & = & 6 & \longleftarrow \text{replace } y \text{ with -2 in one equation and solve for } x \\
2x + -2 + 2 & = & 6 + 2 & \\
2x & = & 8 & \\
\frac{\cancel{2}x}{\cancel{2}} & = & \frac{\cancel{8}}{\cancel{2}} = \frac{4}{1} & \swarrow \\
x & = & 4 &
\end{array}
$$

Our solution set is {4, -2}.

Now let's see how graphing the two equations is done on the following page.

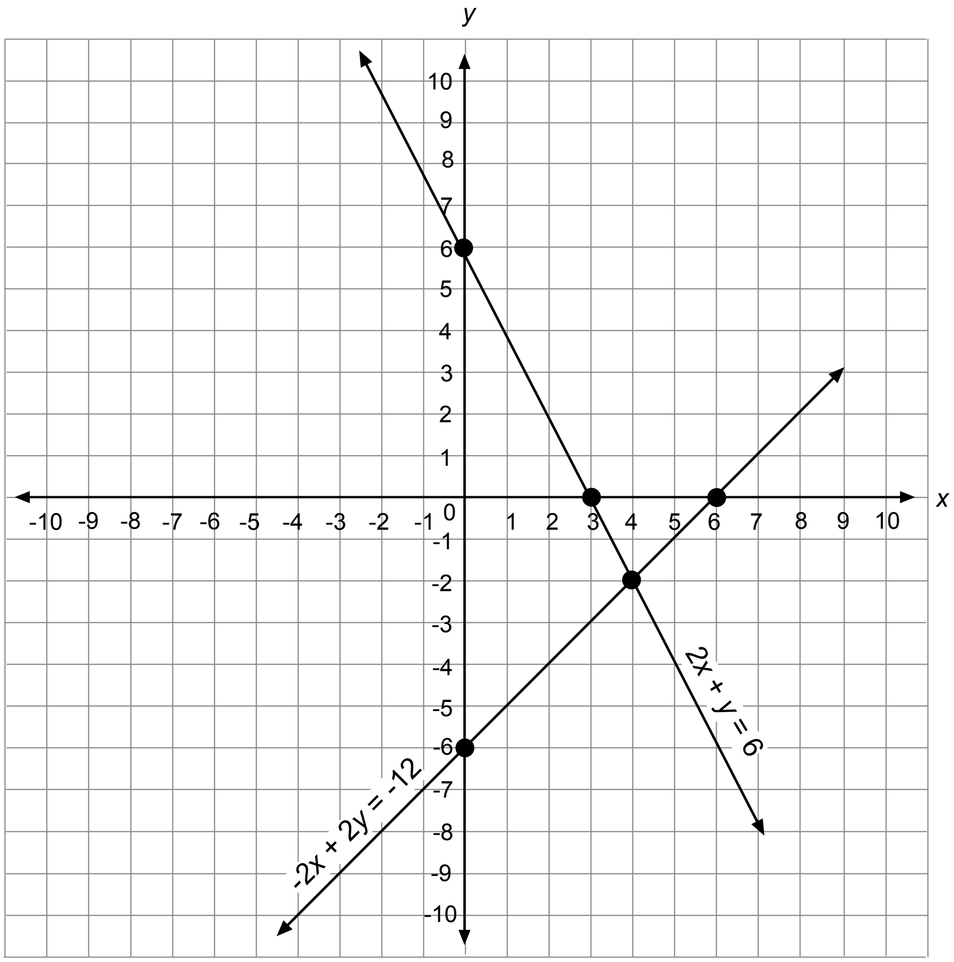
Table of Values

$2x + y = 6$	
x	y
0	6
3	0

Table of Values

$-2x + 2y = -12$	
x	y
0	-6
6	0

Graph of $2x + y = 6$ and $-2x + 2y = -12$



Note: Watch for these special situations.

- If the graphs of the equations are the same line, then the two equations are *equivalent* and have an **infinite** (that is, limitless) number of possible solutions.

- If the graphs do not *intersect* at all, they are **parallel (||)**, and are an equal distance at every point. They have *no* possible solutions. The solution set would be empty—{ }.

Practice

Solve each **system of equations** *algebraically. Use the* **table of values** *to solve and* **graph** *both equations on the graphs provided. Refer to pages 748-752 as needed.*

Check your work with a **graphing calculator** by replacing *x* and *y* in both equations with the coordinates of the point of intersection if one exists.

Hint: Two of the following sets of equations are **equivalent expressions** and will have the same line with an *infinite* number of possible solutions. See note on the previous page.

1. $x - y = -1$
   $x + y = 7$

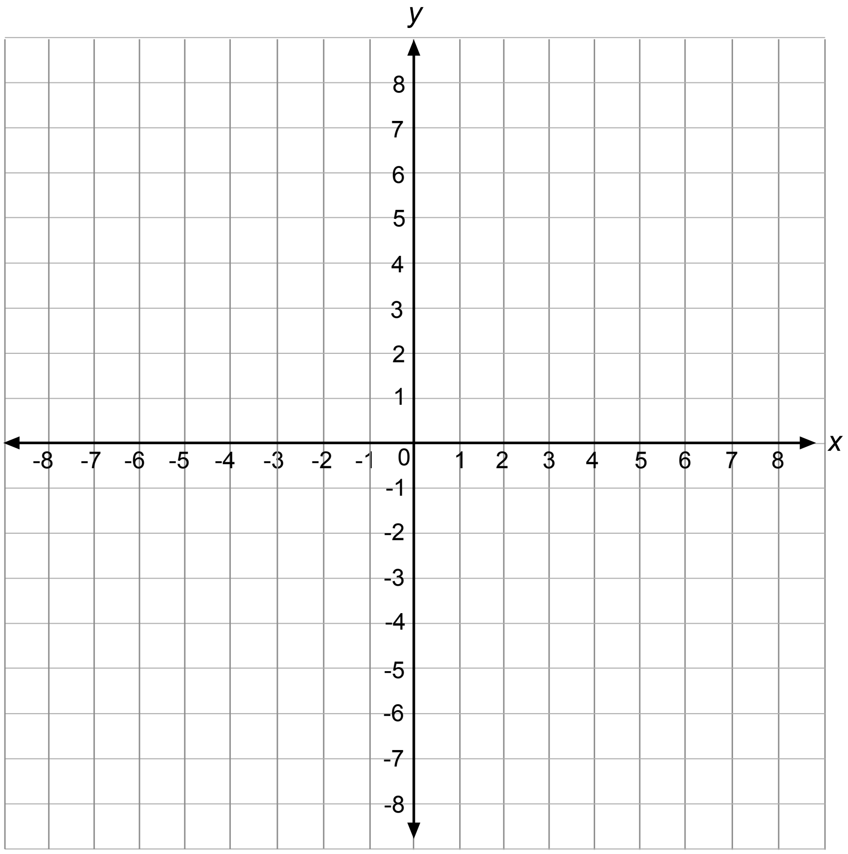
Table of Values

$x - y = -1$	
x	**y**

Table of Values

$x + y = 7$	
x	**y**

Graph of $x - y = -1$ and $x + y = 7$



2. $2x - y = 4$
$x + y = 5$

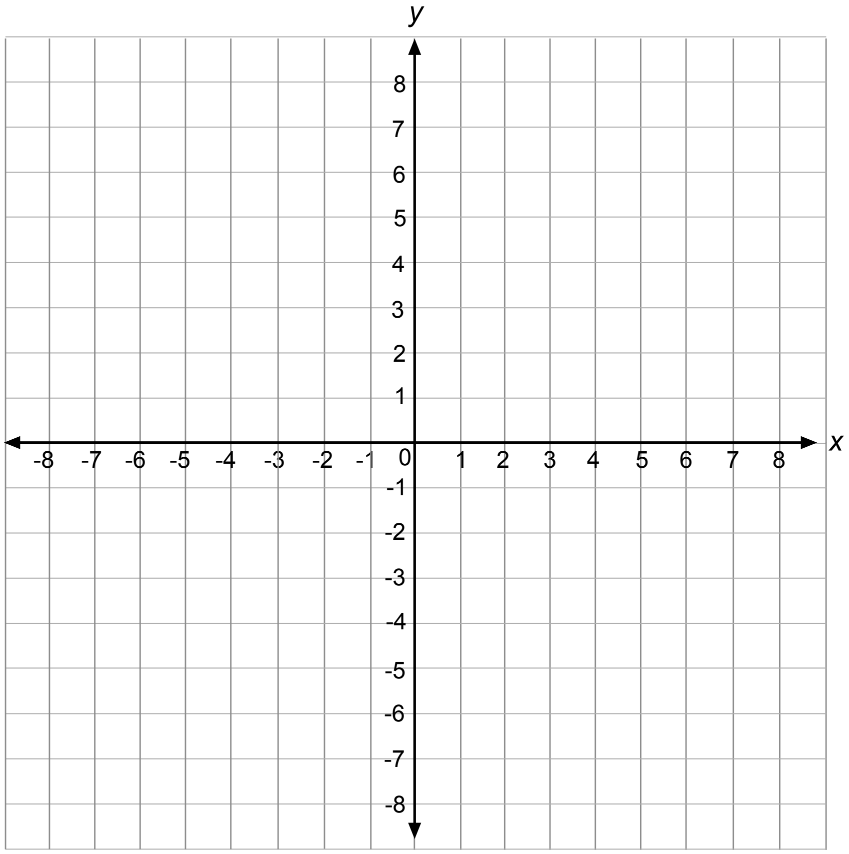
Table of Values

$2x - y = 4$	
x	y

Table of Values

$x + y = 5$	
x	y
0	5
5	0

Graph of $2x - y = 4$ and $x + y = 5$



3. $4x - y = 2$
$-2x + y = 0$

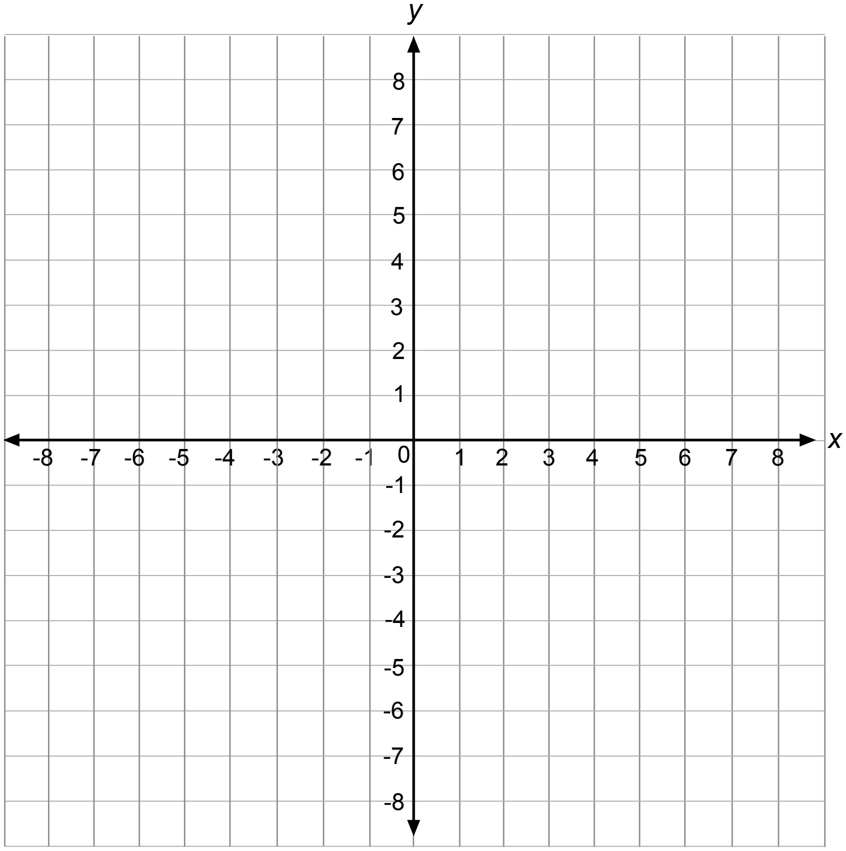
Table of Values

4*x* – *y* = 2	
x	*y*

Table of Values

-2*x* + *y* = 0	
x	*y*

Graph of 4*x* – *y* = 2 and -2*x* + *y* = 0



4. $$\begin{aligned} x - 2y &= 4 \\ 2x - 4y &= 8 \end{aligned}$$

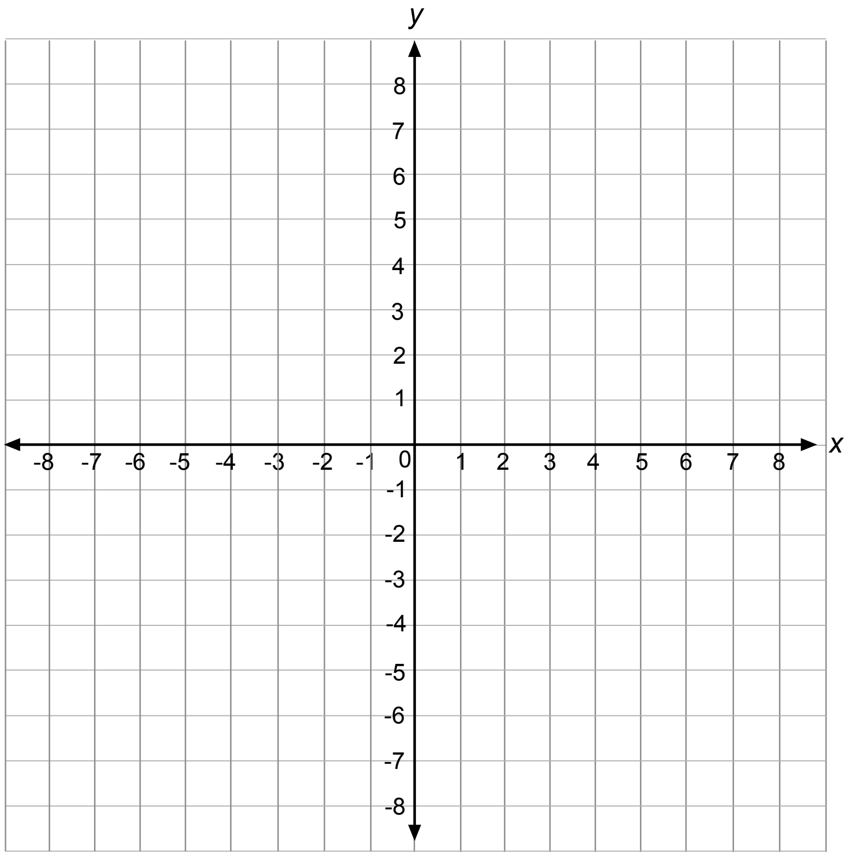
Table of Values

$x - 2y = 4$	
x	***y***

Table of Values

$2x - 4y = 8$	
x	***y***

Graph of $x - 2y = 4$ and $2x - 4y = 8$



5. $2x + y = 8$
$-2x + y = -4$

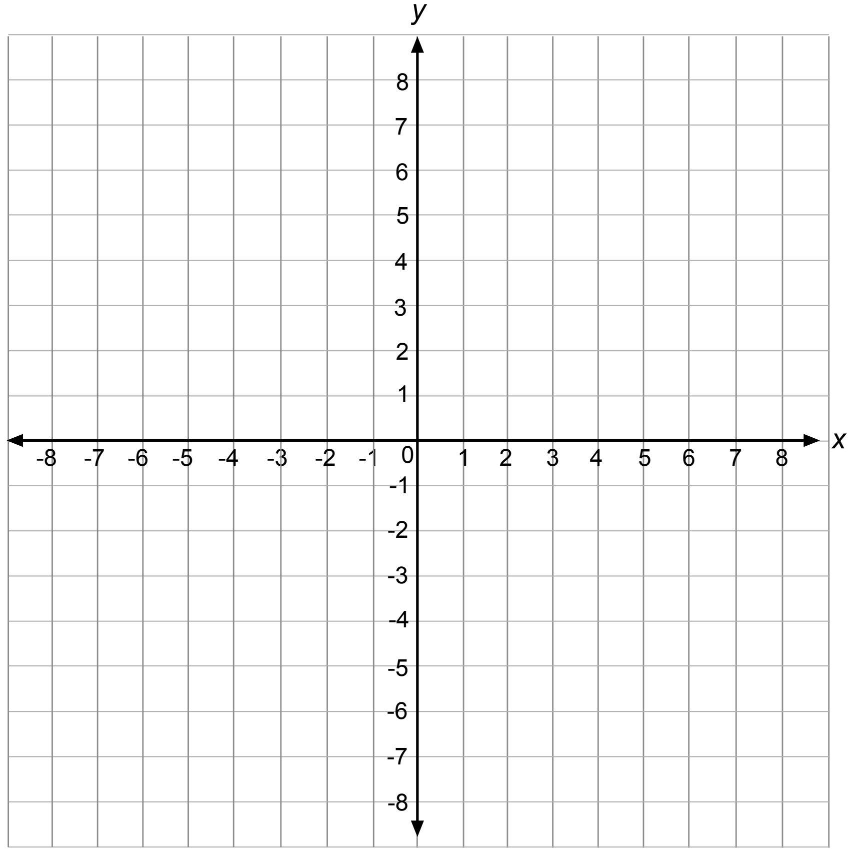
Table of Values

$2x + y = 8$	
x	***y***

Table of Values

$-2x + y = -4$	
x	***y***

Graph of 2*x* + *y* = 8 and -2*x* + *y* = -4



6. $3x - 2y = -1$
$-6x + 4y = 2$

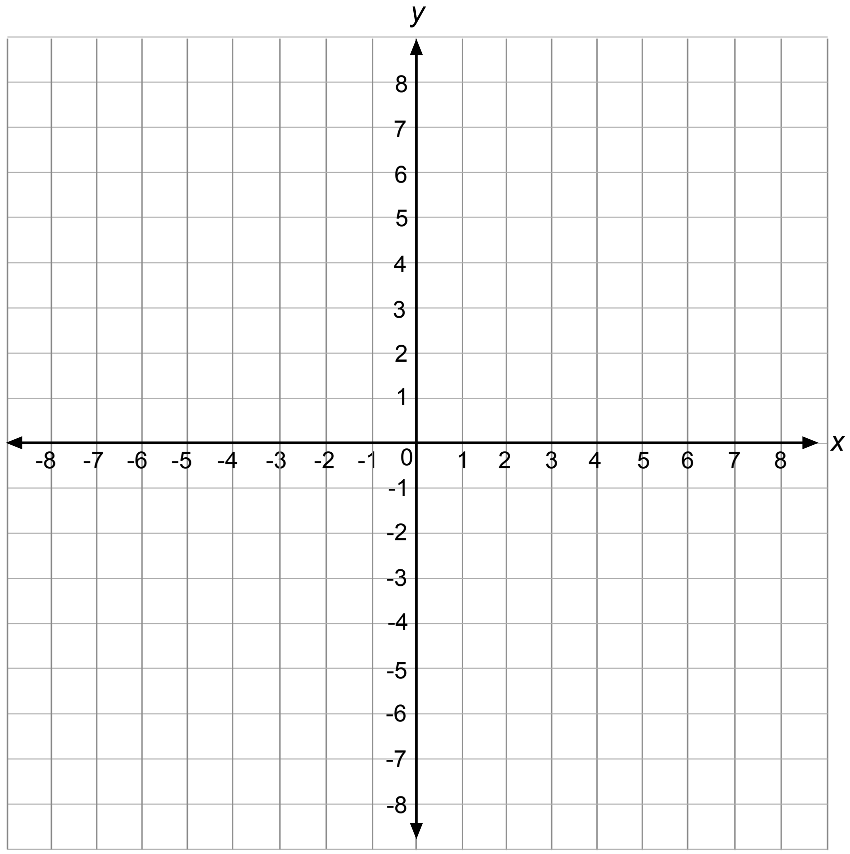
Table of Values

$3x - 2y = -1$	
x	y

Table of Values

$-6x + 4y = 2$	
x	y

Graph of $3x - 2y = -1$ and $-6x + 4y = 2$



Using Substitution to Solve Equations

There are other processes we can use to solve systems of equations. Let's take a look at some of the options.

Example 1

Suppose our two equations are as follows.

$$\begin{aligned} 2x + 3y &= 14 \\ x &= 4 \end{aligned}$$

To solve this system, we could use a method called **substitution**. We simply put the value of x from the second equation in for the x in the first equation.

$$\begin{aligned} 2x + 3y &= 14 && \leftarrow \text{substitute 4 for } x \\ 2(4) + 3y &= 14 && \leftarrow \text{simplify} \\ 8 + 3y &= 14 && \\ 8 - 8 + 3y &= 14 - 8 && \leftarrow \text{subtract} \\ 3y &= 6 && \leftarrow \text{divide} \\ \frac{3y}{3} &= \frac{6}{3} && \\ y &= 2 && \end{aligned}$$

The solution set is {4, 2}.

Example 2

This one is a little more complex.

Below are our two equations.

$$\begin{aligned} 4x - y &= -2 \\ x &= y + 4 \end{aligned}$$

We can substitute $(y + 4)$ from the second equation in for x in the first equation.

$$\begin{array}{rcll} 4x - y &=& -2 & \longleftarrow \text{substitute } (y + 4) \text{ for} \\ 4(y + 4) - y &=& -2 & \longleftarrow \textbf{distribute} \\ 4y + 16 - y &=& -2 & \longleftarrow \text{simplify} \\ 3y + 16 &=& -2 & \longleftarrow \text{subtract} \\ 3y &=& -18 & \longleftarrow \text{divide} \\ y &=& -6 & \end{array}$$

Notice that $(y + 4)$ is in parentheses. This helps us remember to *distribute* when the time comes.

Now we must find the value of x. Use an original equation and substitute -6 for y and then solve for x.

$$\begin{array}{rcll} 4x - y &=& -2 & \longleftarrow \text{original equation} \\ 4x - (-6) &=& -2 & \longleftarrow \text{substitute } (-6) \text{ for } y \\ 4x + 6 &=& -2 & \longleftarrow \text{simplify} \\ 4x &=& -8 & \longleftarrow \text{subtract} \\ x &=& -2 & \longleftarrow \text{divide} \end{array}$$

Now try the practice on the following page.

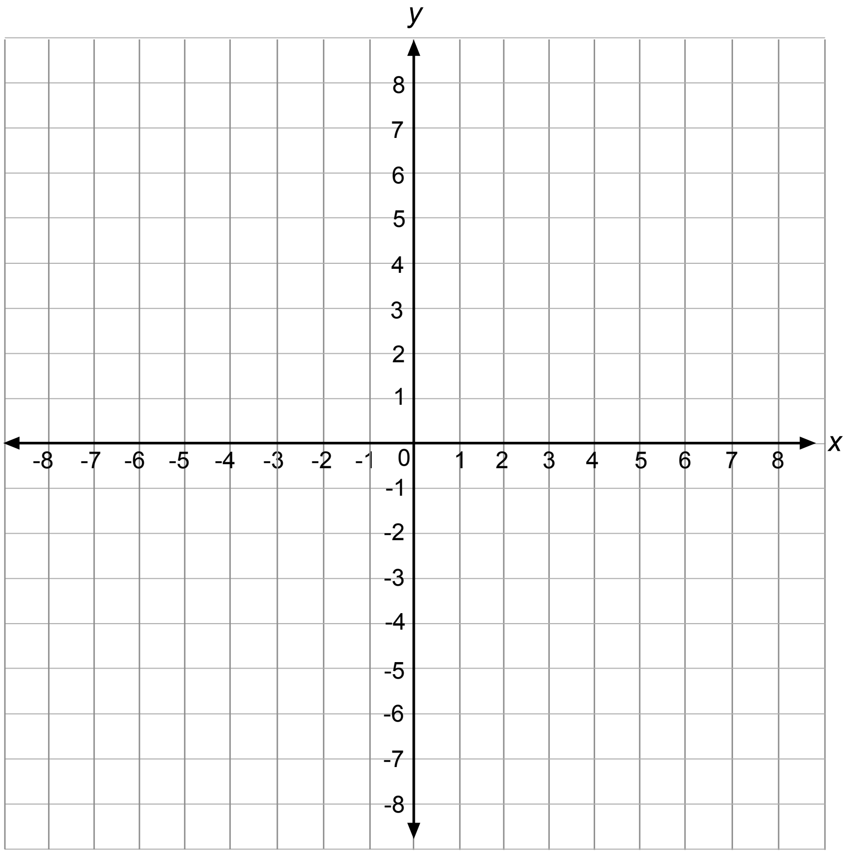
Practice

Solve each **system of equations** *algebraically. Use the* **substitution method** *to solve and* **graph** *both equations on the graphs provided. Refer to pages 759 and 760 as needed.*

Check your work by graphing on a calculator or by replacing *x* and *y* with the coordinates of your solution.

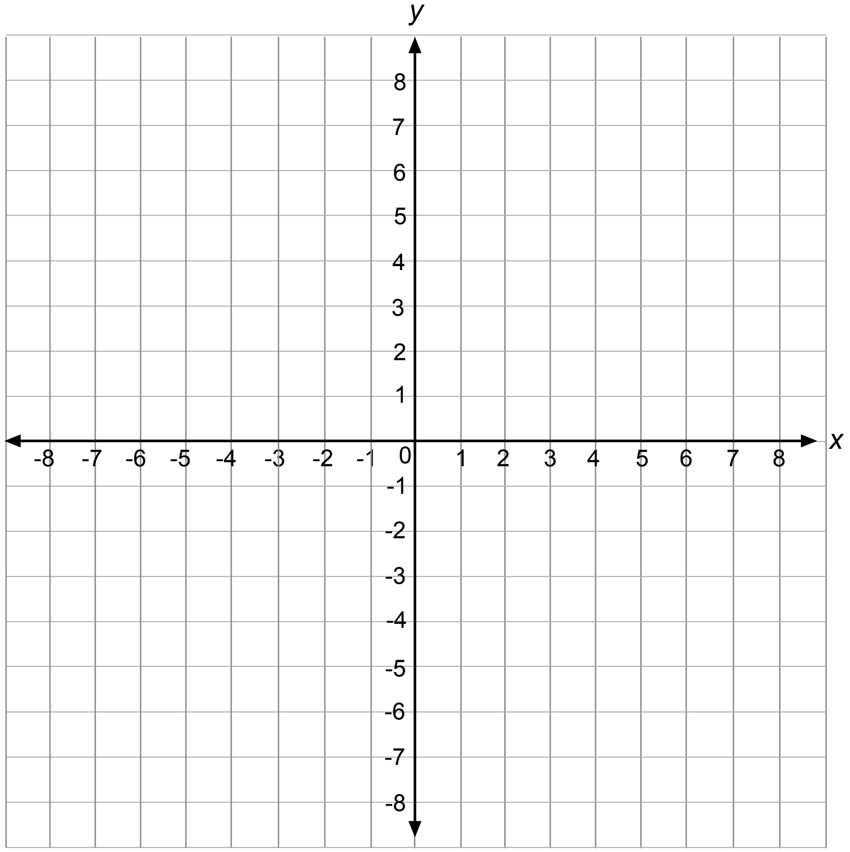
1. $3x - 2y = 6$
   $x = 4$

Graph of 3*x* – 2*y* = 6 and *x* = 4



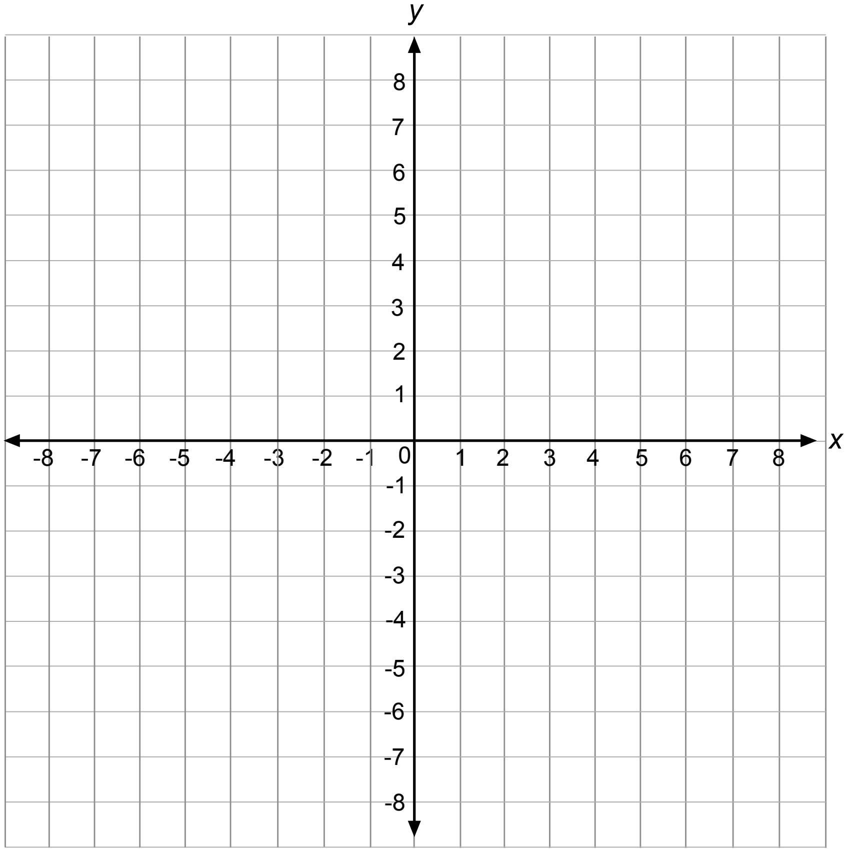
2. $5x - y = 9$
$x = 2y$

Graph of 5*x* – *y* = 9 and *x* = 2*y*



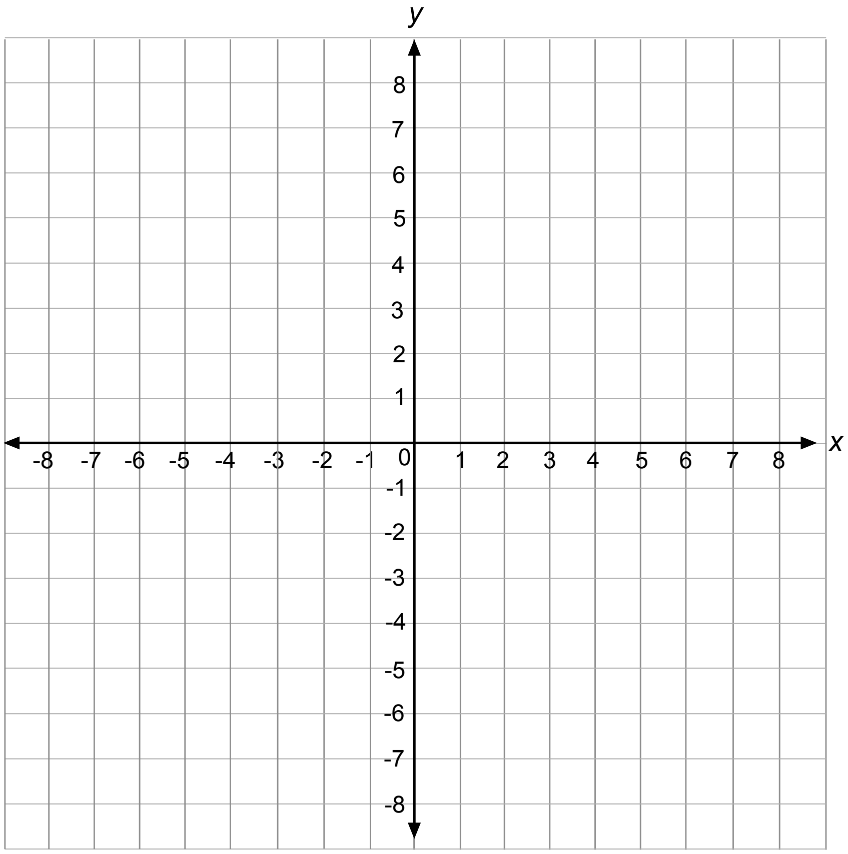
3. $x + y = 5$
$x = y + 1$

Graph of *x* + *y* = 5 and *x* = *y* + 1



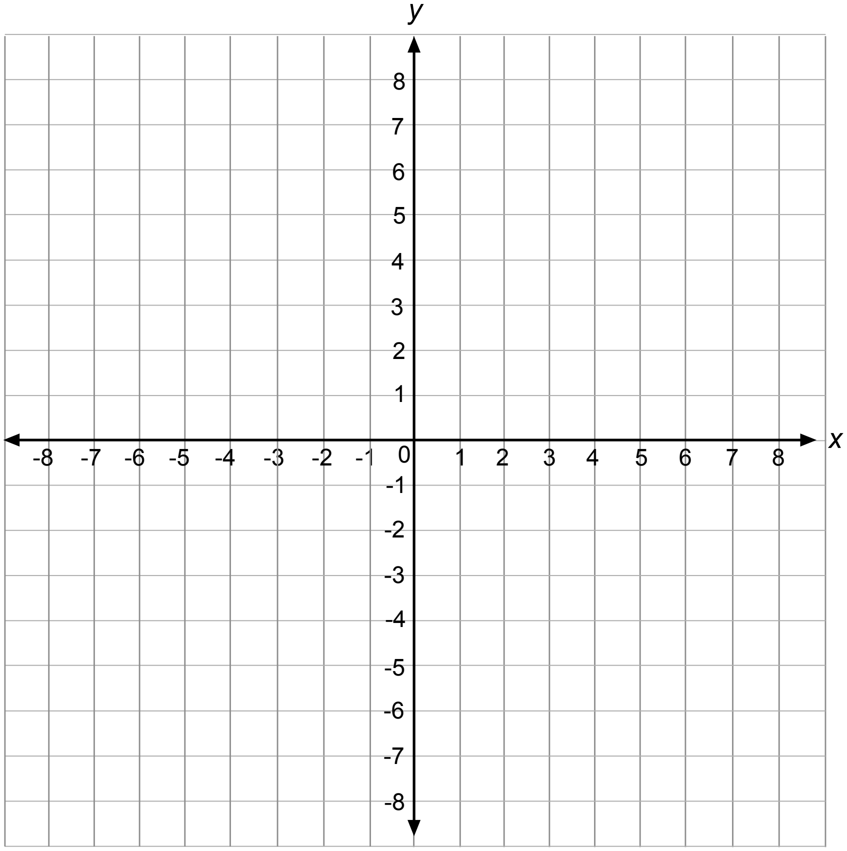
4. $5x + y = -15$
$y = 1 - x$

Graph of 5*x* + *y* = -15 and *y* = 1 – *x*



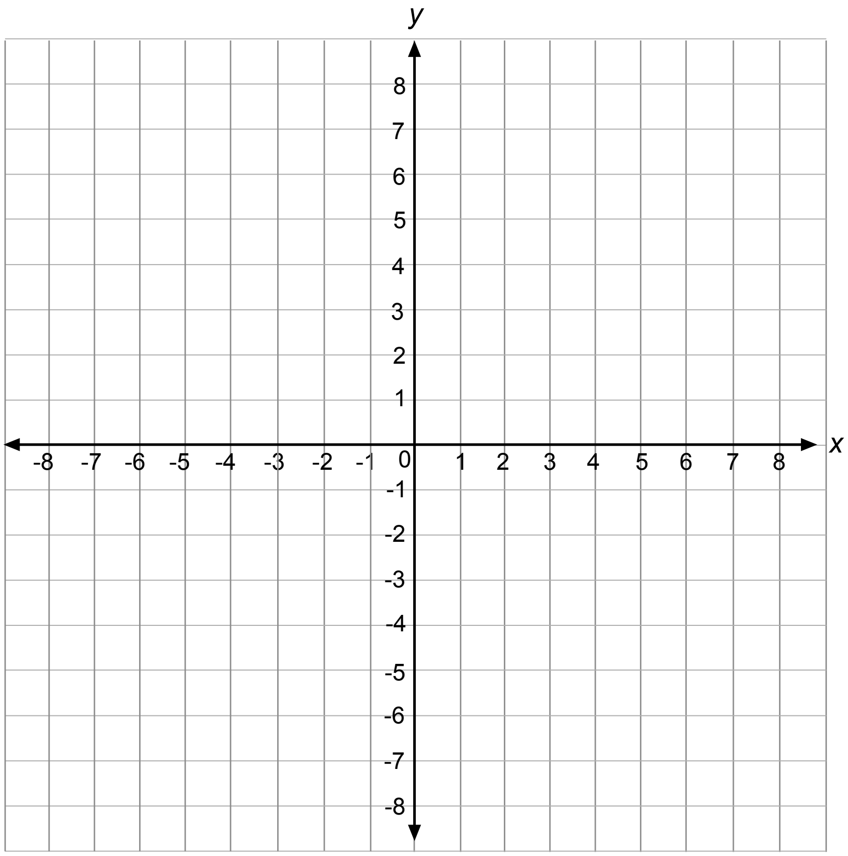
5. $x = 2y + 15$
$4x + 2y = 10$

Graph of $x = 2y + 15$ and $4x + 2y = 10$



6. $x + 2y = 14$
$x = 3y - 11$

Graph of $x + 2y = 14$ and $x = 3y - 11$



Using Magic to Solve Equations

There are times when neither the algebraic or substitution method seems like a good option. If the equations should look similar to these, we have another option.

Example 1

$$\begin{aligned} 5x + 12y &= 41 \\ 9x + 4y &= 21 \end{aligned}$$

We have to perform a little "math-magic" to solve this problem. When looking at these equations, you should see that if the $4y$ were $-12y$ instead, we could add vertically and the y's would disappear from the equation.

So, our job is to make that $4y$ into $-12y$. We could do that by multiplying $4y$ by -3. The only catch is that we must multiply the whole equation by -3 to keep everything balanced.

$$\begin{array}{rcll} 9x + 4y &=& 21 & \leftarrow \text{original equation} \\ -3(9x + 4y &=& 21) & \leftarrow \text{multiply equation by -3} \\ -27x + (-12y) &=& -63 & \leftarrow \text{new 2nd equation} \end{array}$$

Now line up the equations, replacing the second one with the new equation.

$$\begin{array}{rcll} 5x + 12y &=& 41 & \leftarrow \text{original 1st equation} \\ -27x + (-12y) &=& -63 & \leftarrow \text{new 2nd equation} \\ \hline -22x + 0 &=& -22 & \leftarrow \text{subtract vertically} \\ -22x &=& -22 & \leftarrow \text{simplify} \\ x &=& 1 & \leftarrow \text{divide} \end{array}$$

Now that we know the value of x, we can replace x with 1 in the original equation and solve for y.

$$\begin{array}{rcll} 5x + 12y &=& 41 & \leftarrow \text{original 1st equation} \\ 5(1) + 12y &=& 41 & \leftarrow \text{substitute (1) for } x \\ 5 + 12y &=& 41 & \leftarrow \text{simplify} \\ 12y &=& 36 & \leftarrow \text{subtract} \\ y &=& 3 & \leftarrow \text{divide} \end{array}$$

Our solution set is {1, 3}. Be sure to put the answers in the *correct order* because they are an **ordered pair**, where the first and second value represent a position on a *coordinate grid* or *system*.

Sometimes you may have to perform "math-magic" on both equations to get numbers to "disappear."

Example 2

$$\begin{aligned} 3x - 4y &= 2 \\ 2x + 3y &= 7 \end{aligned}$$

After close inspection, we see that this will take double magic. If the **coefficients** of the x's could be made into a $6x$ and a $-6x$, this problem might be solvable. Let's try!

Multiply the first equation by 2 and the second equation by -3.

$$\begin{array}{rcrcl} 2(3x - 4y = 2) & \longrightarrow & 6x - 8y = 4 & \longleftarrow & 1^{st} \text{ equation} \bullet 2 \\ -3(2x + 3y = 7) & \longrightarrow & \underline{-6x - 9y = -21} & \longleftarrow & 2^{nd} \text{ equation} \bullet -3 \\ & & 0 - 17y = -17 & & \\ & & -17y = -17 & & \\ & & y = 1 & & \end{array}$$

Use $y = 1$ to find the value of x using an original equation.

$$\begin{array}{rcl} 3x - 4y = 2 & \longleftarrow & \text{original } 1^{st} \text{ equation} \\ 3x - 4(1) = 2 & \longleftarrow & \text{substitute (1) for } y \\ 3x - 4 = 2 & & \\ 3x = 6 & & \\ x = 2 & & \end{array}$$

The solution set is {2, 1}.

Now it's your turn to practice on the next page.

Practice

Solve each of the following **systems of equations**. *Refer to pages 767 and 768 as needed. Check your work.*

1.
$$\begin{aligned} 3x + y &= 7 \\ 2x - 3y &= 12 \end{aligned}$$

2.
$$\begin{aligned} 3x + y &= 11 \\ x + 2y &= 12 \end{aligned}$$

3.
$$\begin{aligned} 9x + 8y &= -45 \\ 6x + y &= 9 \end{aligned}$$

4. $-5x + 4y = 4$
$4x - 7y = 12$

5. $-2x - 11y = 4$
$5x + 9y = 27$

6. $2x + 3y = 20$
$3x + 2y = 15$

Solving More Word Problems

Let's see how we might use the methods we've learned to solve word problems.

Example 1

Twice the **sum** of two integers is 20. The larger integer is 1 more than twice the smaller. Find the integers.

Let S = the smaller integer
Let L = the larger integer

Now, write equations to fit the wording in the problem.

$$2(S + L) = 20 \qquad \text{and} \qquad L = 2S + 1$$

$$\begin{aligned}
2S + 2L &= 20 \longleftarrow \text{simplify} \\
2S + 2(2S + 1) &= 20 \longleftarrow \text{substitute } (2S + 1) \text{ for } L \\
2S + 4S + 2 &= 20 \longleftarrow \text{distribute} \\
6S + 2 &= 20 \longleftarrow \text{simplify} \\
6S &= 18 \longleftarrow \text{subtract} \\
S &= 3 \longleftarrow \text{divide}
\end{aligned}$$

Since the smaller integer is 3, the larger one is 2(3) + 1 or 7. The integers are 3 and 7.

Example 2

Three tennis lessons and three golf lessons cost \$60. Nine tennis lessons and six golf lessons cost \$147. Find the cost of one tennis lesson and one golf lesson.

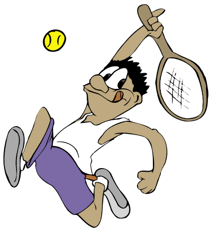
Let T = the cost of 1 tennis lesson
Let G = the cost of 1 golf lesson

Use the variables to interpret the sentences and make equations.

$$\begin{aligned} 3T + 3G &= 60 \\ 9T + 6G &= 147 \end{aligned}$$

Make the *coefficients* of G match by multiplying the first equation by -2.

$$\begin{array}{rcl} -6T + -6G = -120 & \longleftarrow & \text{-2(3}T\text{ + 3}G\text{ = 60)} \\ \underline{9T + 6G = 147} & \longleftarrow & \text{bring in the 2}^{\text{nd}}\text{ equation} \\ 3T + 0 = 27 & \longleftarrow & \text{subtract} \\ 3T = 27 & \longleftarrow & \text{divide} \\ T = 9 & & \end{array}$$



We know that one tennis lesson costs \$9, so let's find the cost of one golf lesson.

$$\begin{array}{rcl} 3T + 3G = 60 & \longleftarrow & \text{original 1}^{\text{st}}\text{ equation} \\ 3(9) + 3G = 60 & \longleftarrow & \text{substitute (9) for }T \\ 27 + 3G = 60 & \longleftarrow & \text{simplify} \\ 3G = 33 & \longleftarrow & \text{divide} \\ G = 11 & & \end{array}$$

So, one tennis lesson costs \$9 and one golf lesson costs \$11.

Now you try a few items on the next page.

Practice

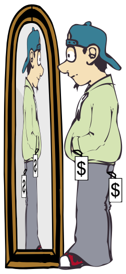
Solve each of the following. Refer to pages 754 and 755 as needed.

1. The sum of two numbers is 35. The larger one is 4 times the smaller one. Find the two numbers.

   Answer: ____________ and ____________

2. A 90-foot cable is cut into two pieces. One piece is 18 feet longer than the shorter one. Find the lengths of the two pieces.

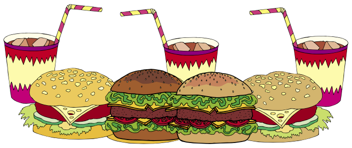
   Answer: ____________ feet and ____________ feet



3. Joey spent \$98 on a pair of jeans and a shirt. The jeans cost \$20 more than the shirt. How much did each cost?

   Answer: jeans = \$ ____________ and shirt = \$ ____________

4. Four sandwiches and three drinks cost $13. Two drinks cost $0.60 more than one sandwich. Find the cost of one drink and one sandwich.



Answer: $ ____________

5. The football team at Leon High School has 7 more members than the team from Central High School. Together the two teams have 83 players. How many players does each team have?

Answer: Central = ____________ and Leon = ____________

6. Andre earns $40 a week less than Sylvia. Together they earn $360 each week. How much does each earn?

Answer: Sylvia = $ ____________ and Andre = $ ____________

Practice

Match each definition with the correct term. Write the letter on the line provided.

_______ 1. a two-dimensional network of horizontal and vertical lines that are parallel and evenly spaced

_______ 2. all points whose coordinates are solutions of an equation

_______ 3. a group of two or more equations that are related to the same situation and share variables

_______ 4. a drawing used to represent data

_______ 5. at right angles to the horizon; straight up and down

_______ 6. any symbol, usually a letter, which could represent a number

_______ 7. the horizontal and vertical number lines used in a coordinate plane system

_______ 8. a data display that organizes information about a topic into categories

_______ 9. to meet or cross at one point

A. axes (of a graph)

B. coordinate grid or plane

C. graph

D. graph of an equation

E. intersect

F. system of equations

G. table (or chart)

H. variable

I. vertical

Practice

Use the list below to write the correct term for each definition on the line provided.

coefficient
infinite
ordered pair
parallel (||)
simplify an expression
substitute
substitution
sum

________________________ 1. a method used to solve a system of equations in which variables are replaced with known values or algebraic expressions

________________________ 2. the result of adding numbers together

________________________ 3. the location of a single point on a rectangular coordinate system where the first and second values represent the position relative to the x-axis and y-axis, respectively

________________________ 4. to perform as many of the indicated operations as possible

________________________ 5. to replace a variable with a numeral

________________________ 6. having no boundaries or limits

________________________ 7. being an equal distance at every point so as to never intersect

________________________ 8. the number that multiplies the variable(s) in an algebraic expression

Lesson Three Purpose

Reading Process Strand

Standard 6: Vocabulary Development

- LA.910.1.6.1
  The student will use new vocabulary that is introduced and taught directly.
- LA.910.1.6.2
  The student will listen to, read, and discuss familiar and conceptually challenging text.
- LA.910.1.6.5
  The student will relate new vocabulary to familiar words.

Algebra Body of Knowledge

Standard 3: Linear Equations and Inequalities

- MA.912.A.3.12
  Graph a linear equation or inequality in two variables with and without graphing technology. Write an equation or inequality represented by a given graph.
- MA.912.A.3.13
  Use a graph to approximate the solution of a system of linear equations or inequalities in two variables with and without technology.
- MA.912.A.3.14
  Solve systems of linear equations and inequalities in two and three variables using graphical, substitution, and elimination methods.

Standard 10: Mathematical Reasoning and Problem Solving

- MA.912.A.10.3
  Decide whether a given statement is always, sometimes, or never true (statements involving linear or quadratic expressions, equations, or inequalities rational or radical expressions or logarithmic or exponential functions).

Graphing Inequalities

When graphing **inequalities**, you use much the same processes you used when graphing equations. The difference is that *inequalities* give you infinitely larger sets of solutions. In addition, your results with inequalities are always expressed using the following terms in relation to another expression:

- greater than (>)
- greater than or equal to (≥)
- less than (<)
- less than or equal to (≤)
- not equal to (≠).

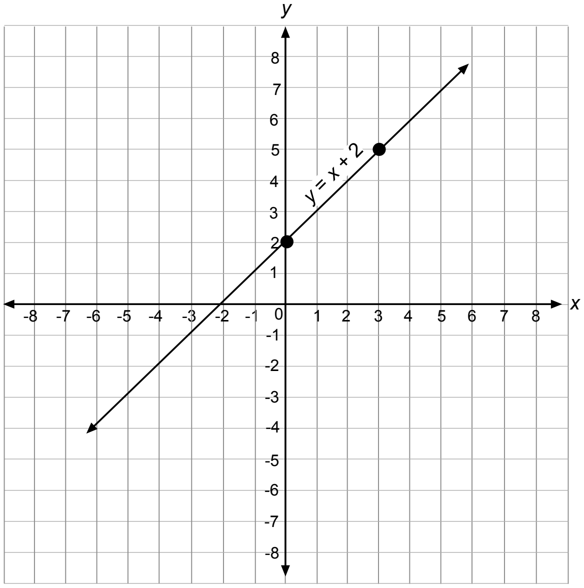
Therefore, we cannot graph an inequality as a line or a point. We must illustrate the entire set of answers by *shading* our graphs.

For instance, when we graph $y = x + 2$ using points, we found by using the table of values below, we get the line seen in Graph 1 below.

Table of Values

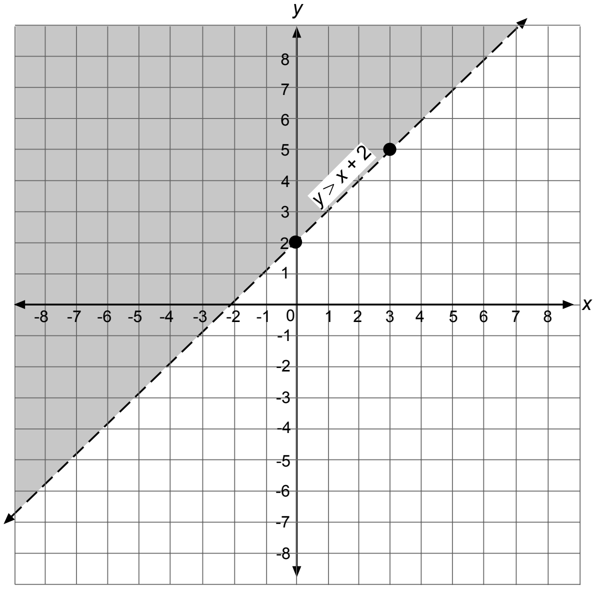
$y = x + 2$	
x	y
0	2
3	5

Graph 1 of $y = x + 2$



But when we graph $y > x + 2$, we use the line we found in Graph 1 as a *boundary*. Since $y \neq x + 2$, we show that by making the boundary line *dotted* (◄- - -►). Then we shade the appropriate part of the grid. Because this is a "greater than" (>) problem, we shade *above* the dotted boundary line. See Graph 2 below.

Graph 2 of $y > x + 2$

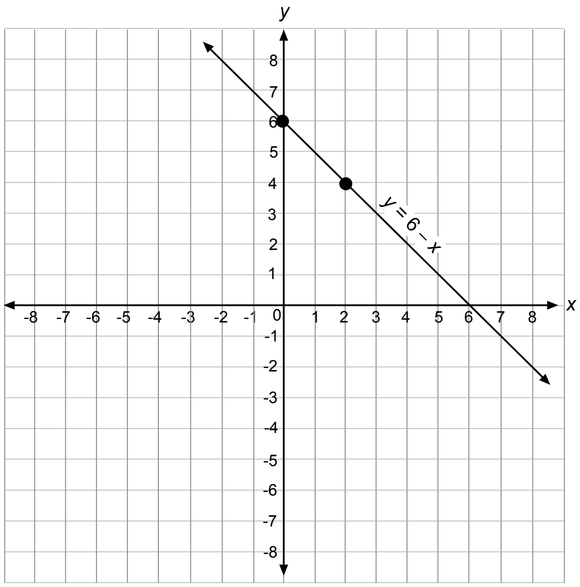


Suppose we wanted to graph $x + y \leq 6$. We first transform the inequality so that y is alone on the left side: $y \leq 6 - x$. We find a pair of points using a table of values, then graph the boundary line. Use the equation $y = 6 - x$ to find two pairs of points in the table of values. Graph the line that goes through points (0, 6) and (2, 4) from the table of values.

Table of Values

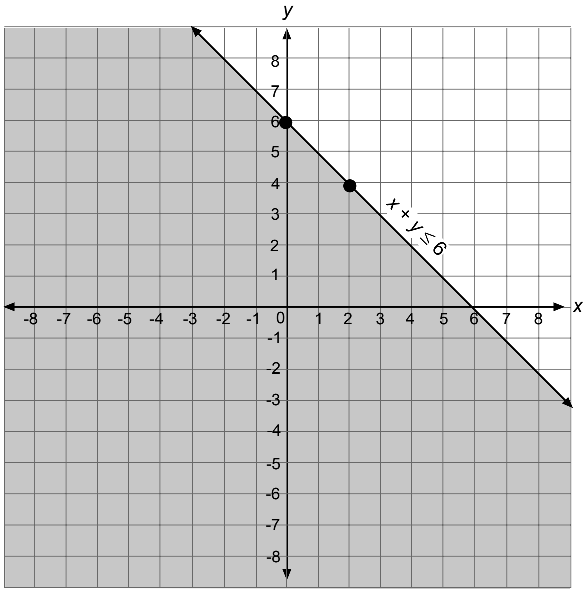
$y = 6 - x$	
x	y
0	6
2	4

Graph 3 of $y = 6 - x$



Now look at the inequality again. The symbol was ≤, so we leave the line solid and shade below the line.

Graph 4 of $x + y \leq 6$



Remember: *Change the inequality sign* whenever you *multiply* or *divide* the inequality by a **negative number**.

$< \rightarrow >$ $\quad\quad$ $\leq \rightarrow \geq$

$> \rightarrow <$ $\quad\quad$ $\geq \rightarrow \leq$

Inequality signs always change when multiplying or dividing by *negative numbers*.

Note:

- **Greater than** (>) means to shade **above** or to the **right** of the line.
- **Less than** (<) means to shade **below** or to the **left** of the line.

Test for Accuracy Before You Shade

You can test your graph for accuracy before you shade by *choosing a point* that *satisfies the inequality*. *Choose a point* that falls in the area you are about to shade. Do *not* choose a point *on the boundary line*.

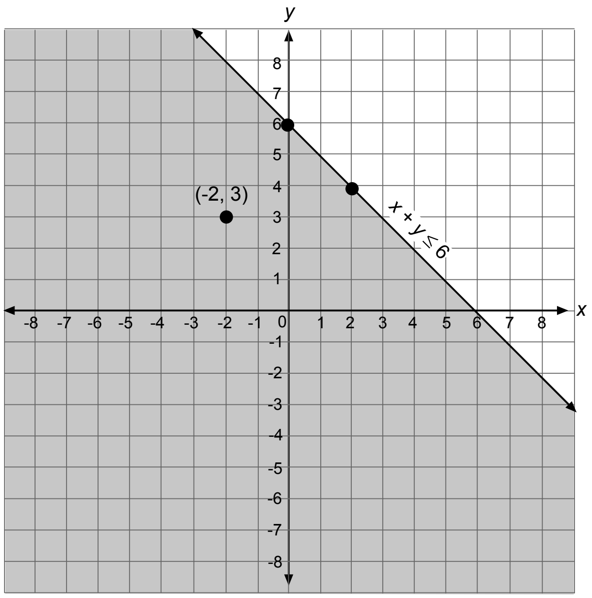
For example, suppose you chose (-2, 3).

$$-2 + 3 \leq 6$$
$$1 \leq 6$$

1. The ordered pair (-2, 3) *satisfies* the inequality.
2. The ordered pair (-2, 3) falls in the area about to be shaded.

Thus, the shaded area for the graph $x + y \leq 6$ is correct.

Graph 4 of $x + y \leq 6$ with Test Point



Practice

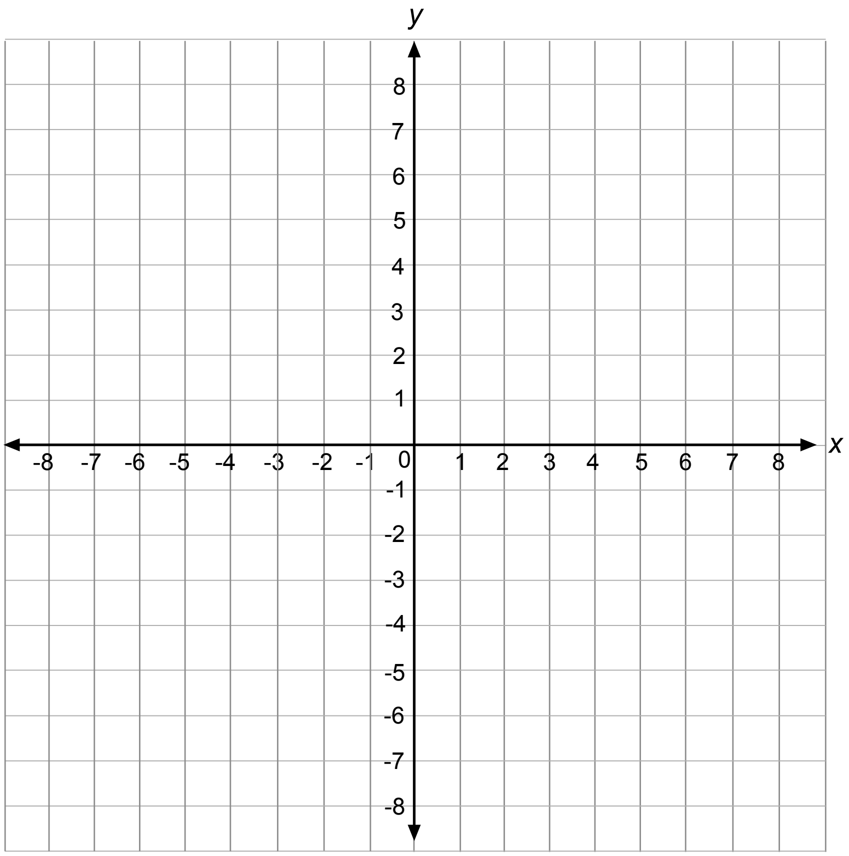
Graph each of the following **inequalities** *on the graphs provided. Refer to pages 778-781 as needed.*

1. $y \geq 2x - 3$

Table of Values

$y \geq 2x - 3$	
x	y

Graph of $y \geq 2x - 3$

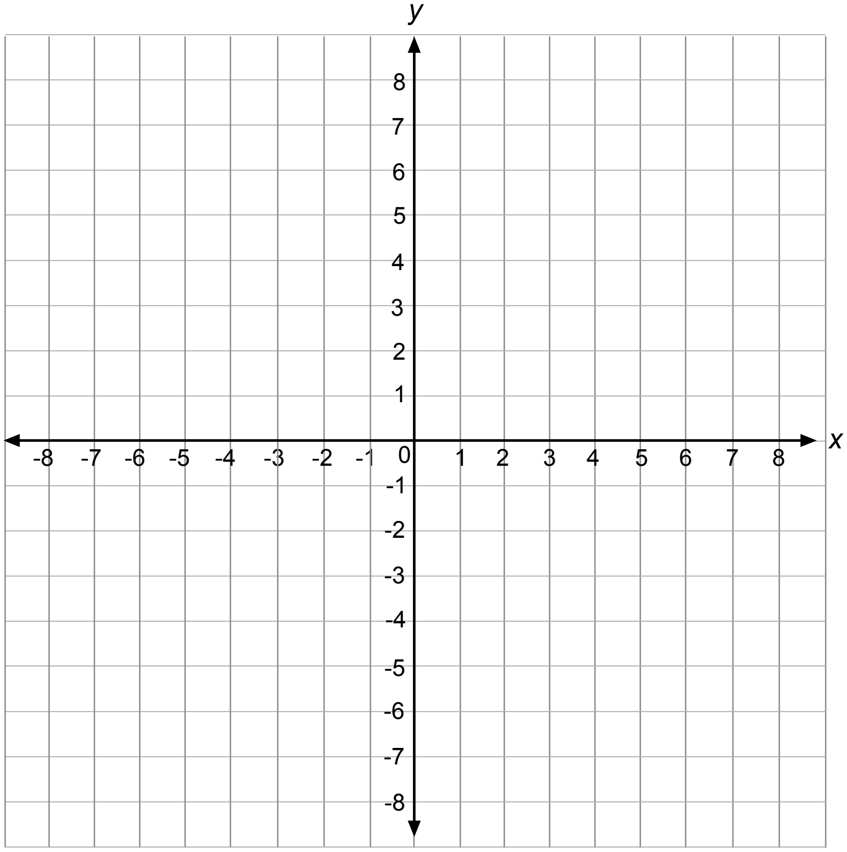


2. $y < x + 4$

Table of Values

$y < x + 4$	
x	y

Graph of $y < x + 4$

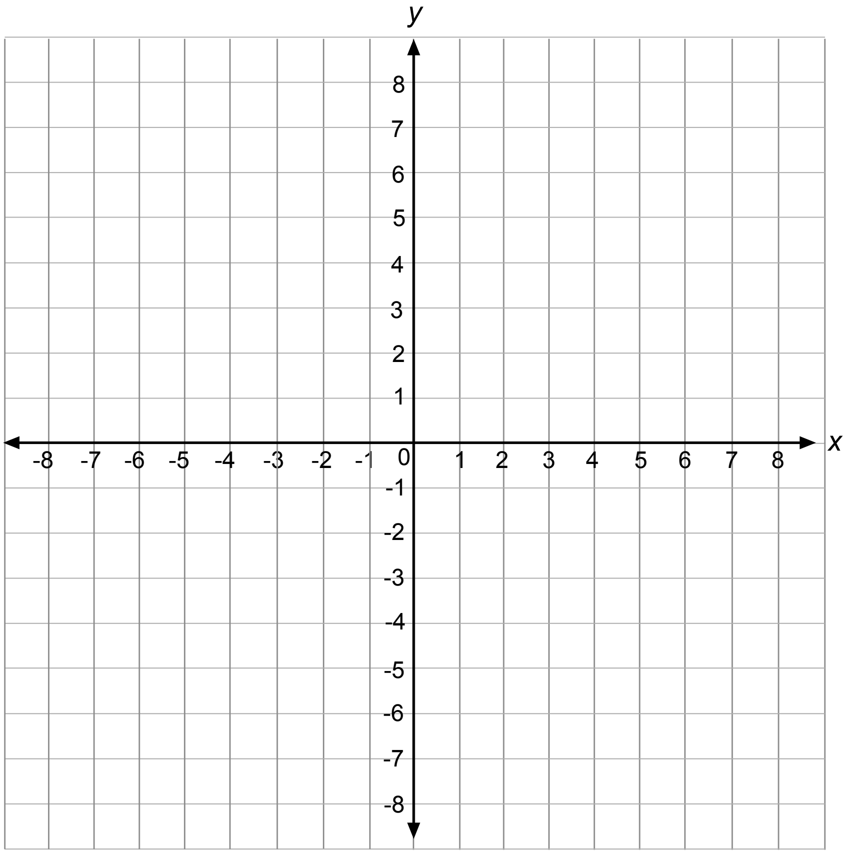


3. $y \leq 3x + 1$

Table of Values

$y \leq 3x + 1$	
x	y

Graph of $y \leq 3x + 1$



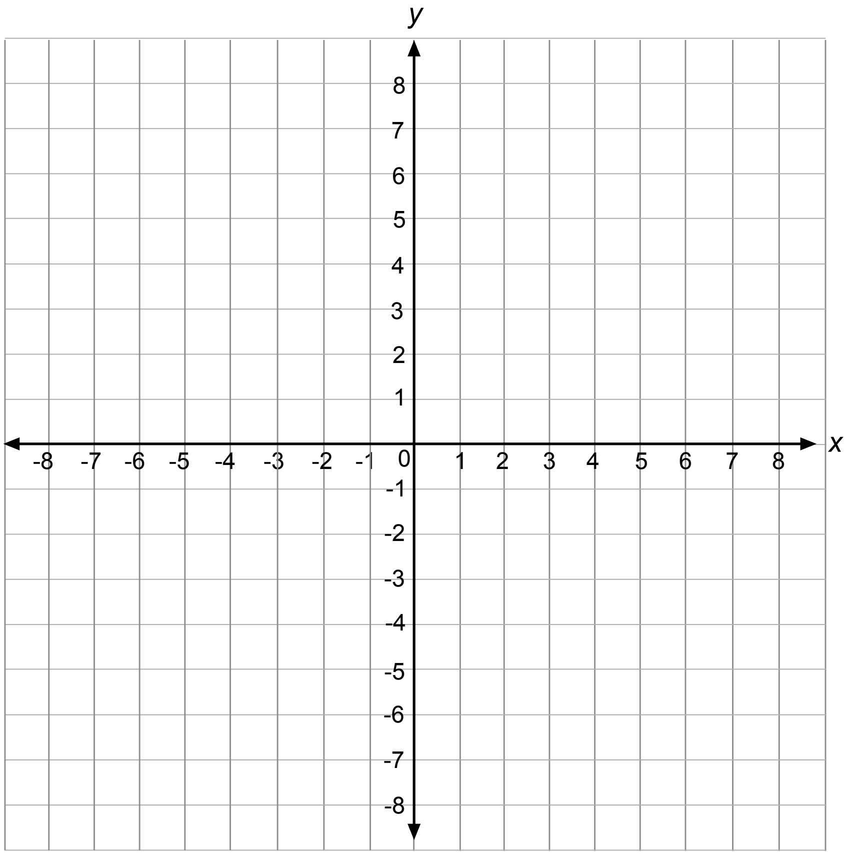
Is the point (-3, 7) part of the solution? ____________

4. $y > x$

Table of Values

$y > x$	
x	y

Graph of $y > x$



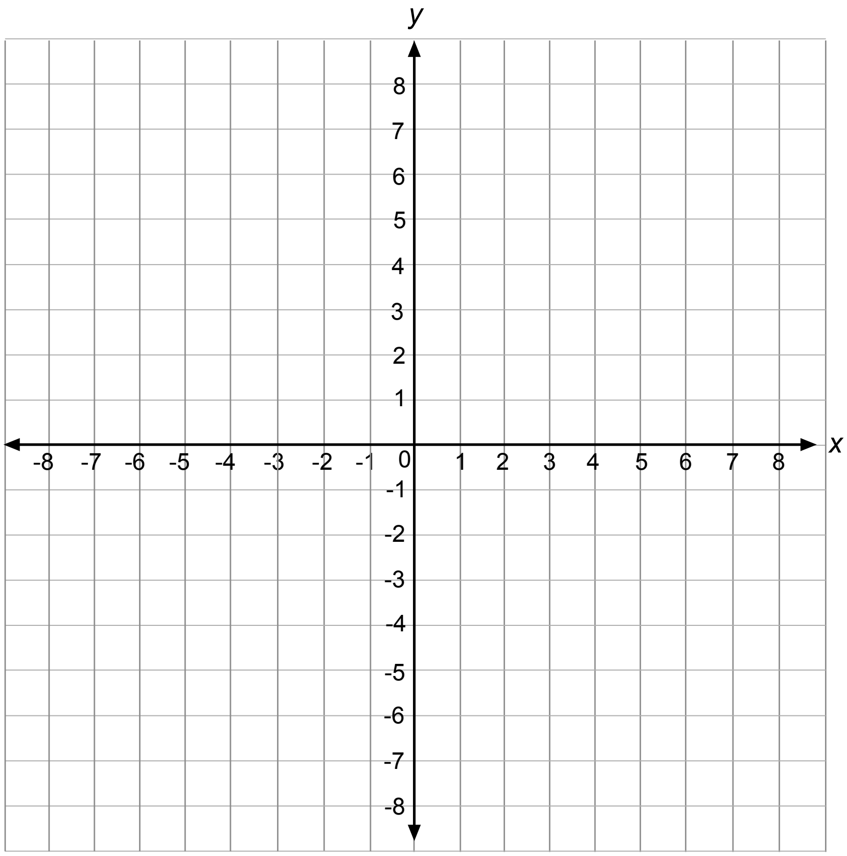
Is the point (3, 6) part of the solution? ____________

5. $x + y < -5$

Table of Values

$x + y < -5$	
x	***y***

Graph of $x + y < -5$

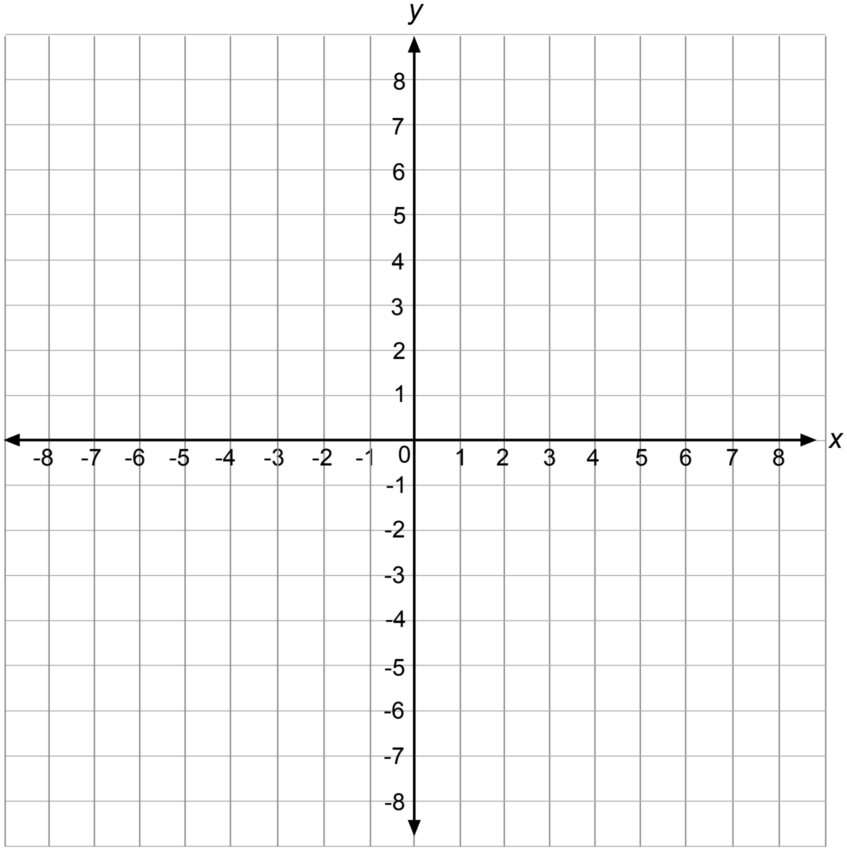


6. $x - 5y \geq 10$

Table of Values

$x - 5y \geq 10$	
x	y

Graph of $x - 5y \geq 10$

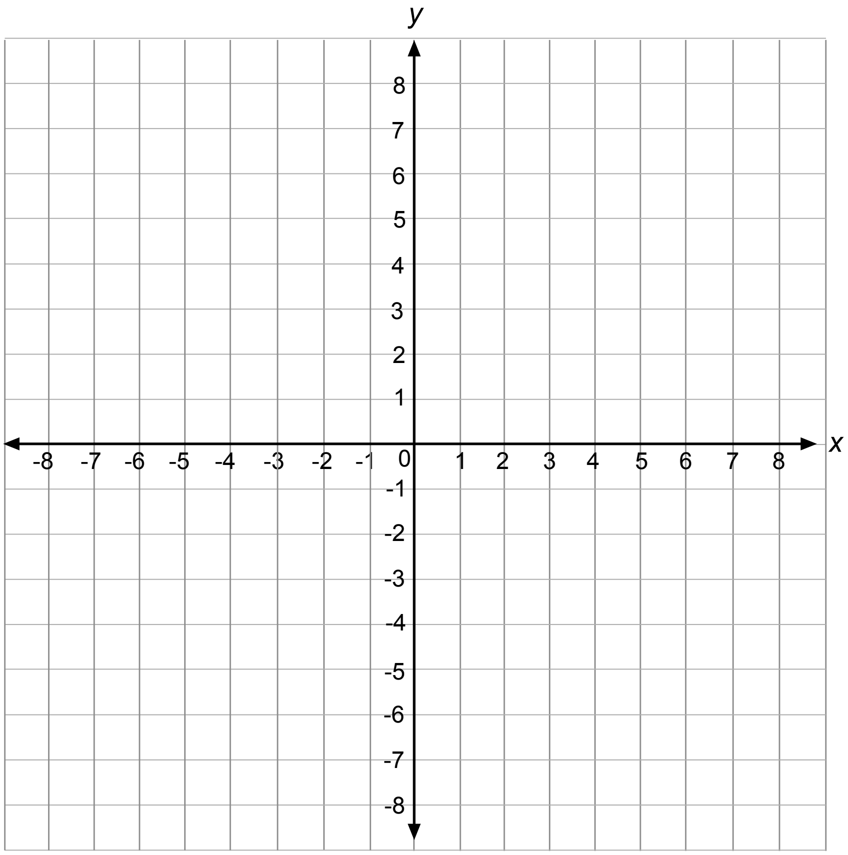


7. $x - 5y \leq 10$

Table of Values

$x - 5y \leq 10$	
x	y

Graph of $x - 5y \leq 10$



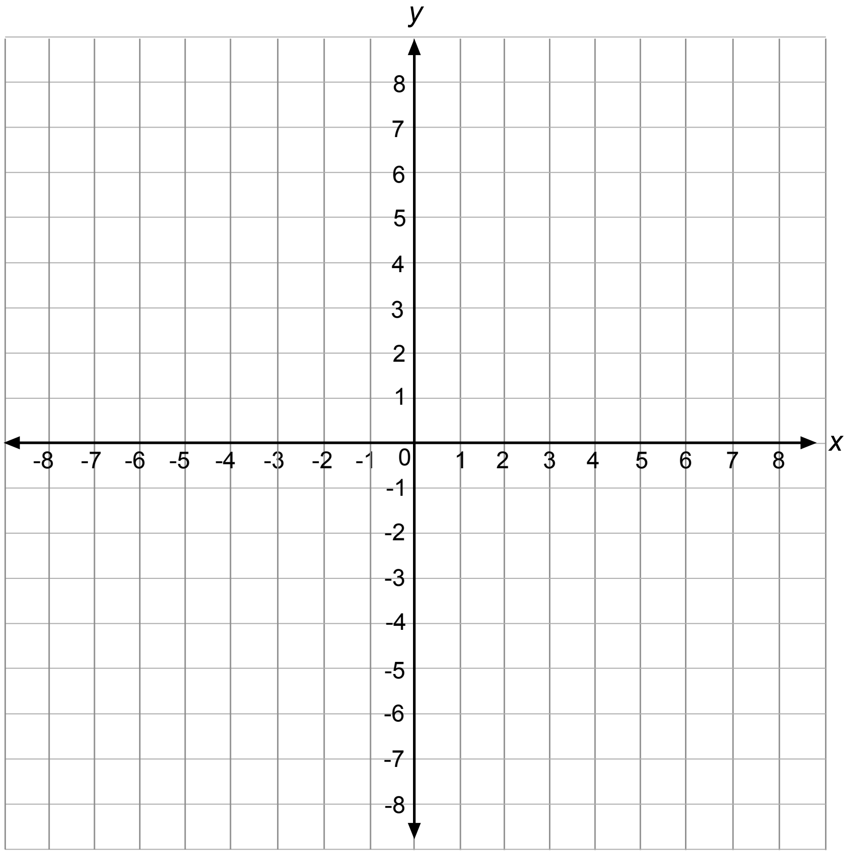
Is the point (0, 0) part of the solution? ____________

8. $y \le -3$

Table of Values

$y \le -3$	
x	***y***

Graph of $y \le -3$



Graphing Multiple Inequalities

We can graph two or more inequalities on the same grid to find which solutions the two inequalities have in common or to find those solutions that work in one inequality or the other. The key words are "and" and "or." Let's see how these small, ordinary words affect our graphing.

Example 1

Graphically show the solutions for $2x + 3y > 6$ **and** $y \leq 2x$.

Note: See how the inequality $2x + 3y > 6$ is transformed in the table of values into the equivalent inequality $y > 2 - \frac{2}{3}x$. Refer to pages 778-781 as needed.

Table of Values

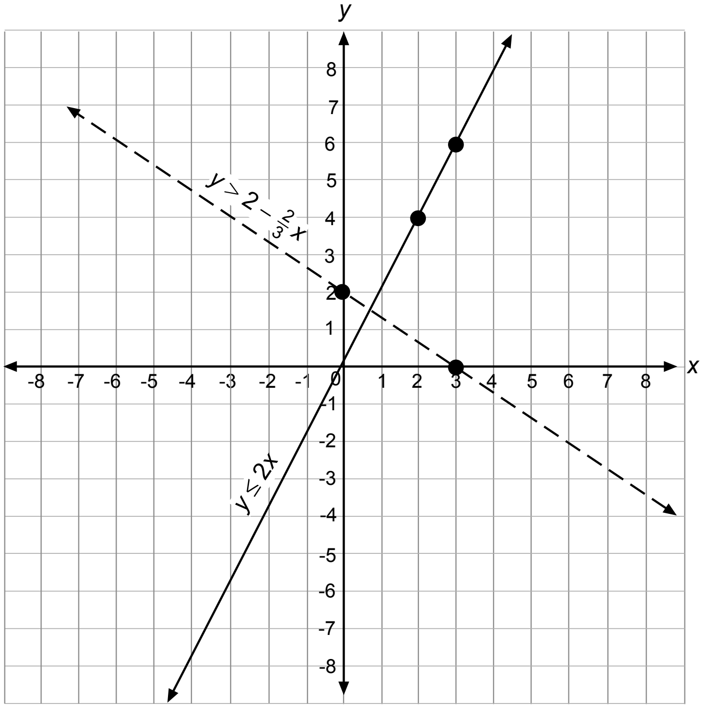
$y > 2 - \frac{2}{3}x$	
x	y
0	2
3	0

Table of Values

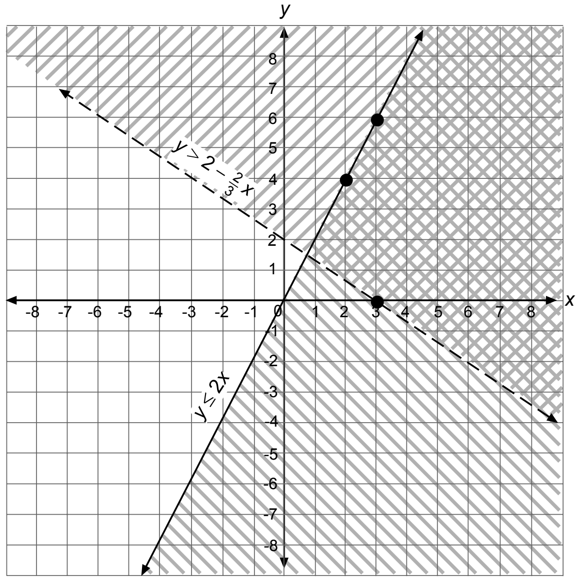
$y \leq 2x$	
x	y
3	6
2	4

Step 1. Find the boundary lines for the two inequalities and draw them. Remember to make the line for the first inequality *dotted*.

Graph Shows Boundary Lines of $2x + 3y > 6$ and $y \leq 2x$



Graph of $2x + 3y > 6$ and $y \le 2x$ with Both Inequalities Shaded



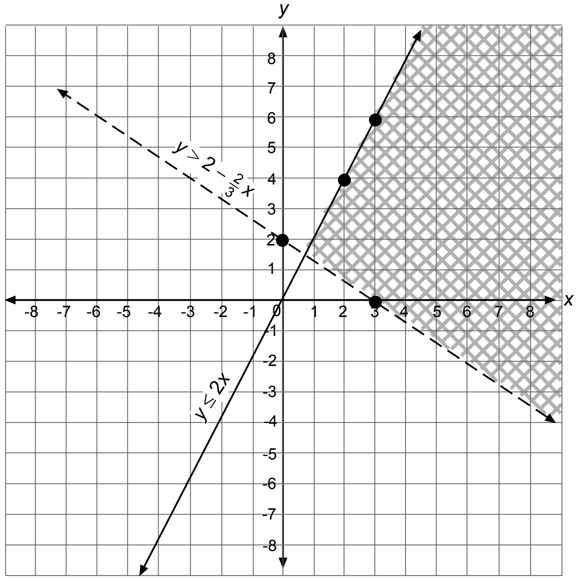
Step 2. Since the 1st inequality is *greater than*, shade *above* the dotted line.

Step 3. Shade the 2nd inequality *below* the solid line using a different type shading or different color.

Step 4. Because this is an "and" problem, we want to have as our solution *only* the parts where both shadings appear at the same time (in other words, where the shadings *overlap*, just as in the **Venn diagrams** in a previous unit). We want to show *only* those solutions that are *valid in both* inequalities *at the same time*.

Step 5. The solution for $2x + 3y > 6$ **and** $y \le 2x$ is shown to the right.

Final Solution for Graph of $2x + 3y > 6$ and $y \le 2x$



Look at the finished graph above.
The point (-1, 1) is *not* in the shaded region. Therefore, the point (-1, 1) is *not* a solution of the intersection of $2x + 3y > 6$ and $y \le 2x$.

Example 2

Let's see how the graph of the solution would look if the problem had been $2x + 3y > 6$ **or** $y \leq 2x$.

We follow the same steps from 1 and 2 of the previous example.

Step 1. Find the boundary lines for the two inequalities and draw them. Remember to make the line for the first inequality *dotted*.

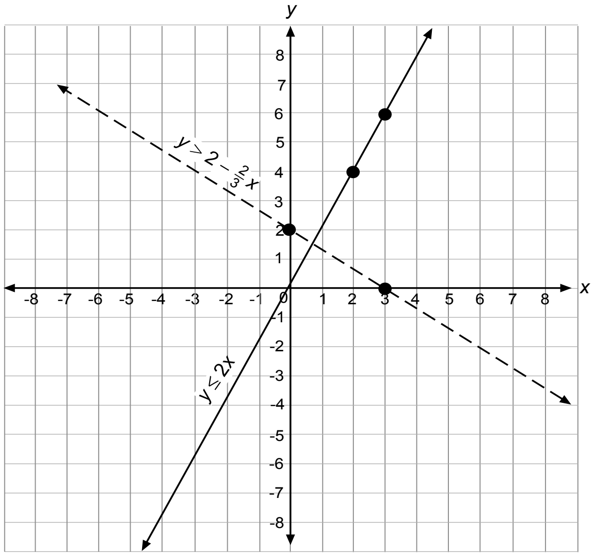
Table of Values

$y > 2 - \frac{2}{3}x$	
x	**y**
0	2
3	0

Table of Values

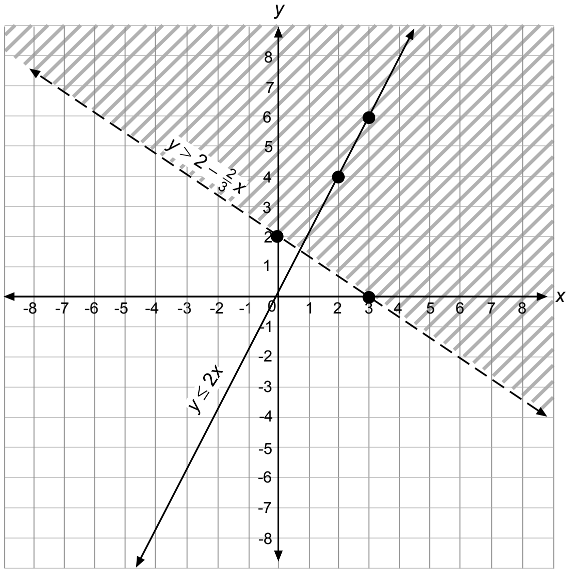
$y \leq 2x$	
x	**y**
3	6
2	4

Graph Shows Boundary Lines of $2x + 3y > 6$ or $y \leq 2x$



Step 2. Since the 1st inequality is *greater than*, shade *above* the dotted line.

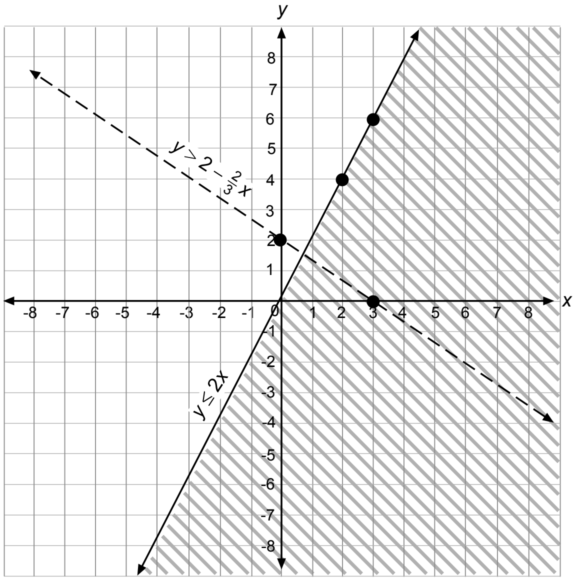
Graph of 2*x* + 3*y* > 6 or *y* ≤ 2*x* with 1st Inequality Shaded



Now we change the process to fit the "*or*."

Step 3. Shade the 2nd inequality *below* the solid line using the same shading as in step 2.

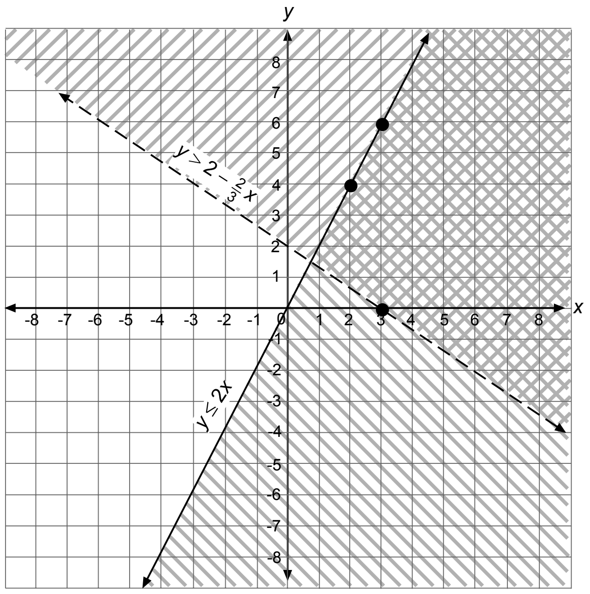
Graph of 2*x* + 3*y* > 6 or *y* ≤ 2*x* with *y* ≤ 2*x* Shaded



Step 4. Because this is now an “or” problem, we want to have as our solution *all* the parts that are shaded. This shows that a solution to *either* inequality is *acceptable*.

Step 5. The solution for $2x + 3y > 6$ **or** $y \leq 2x$ is shown below.

Final Solution for Graph of $2x + 3y > 6$ or $y \leq 2x$



Now it’s your turn to practice.

Practice

Graph the following **inequalities** *on the graphs provided. Refer to pages 778-781 and 790-794 as needed.*

1. $x \geq -2$ **and** $-x + y \geq 1$

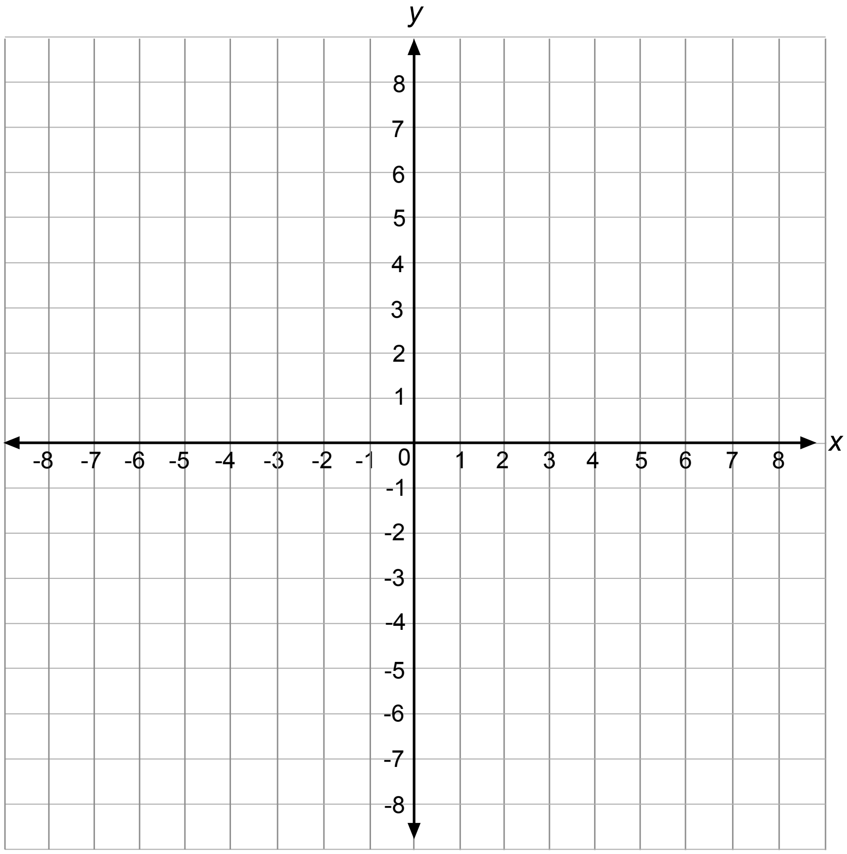
Table of Values

$x \geq -2$	
x	y

Table of Values

$-x + y \geq 1$	
x	y

Graph of $x \geq -2$ and $-x + y \geq 1$



2. $y < -3$ or $y \geq 2$

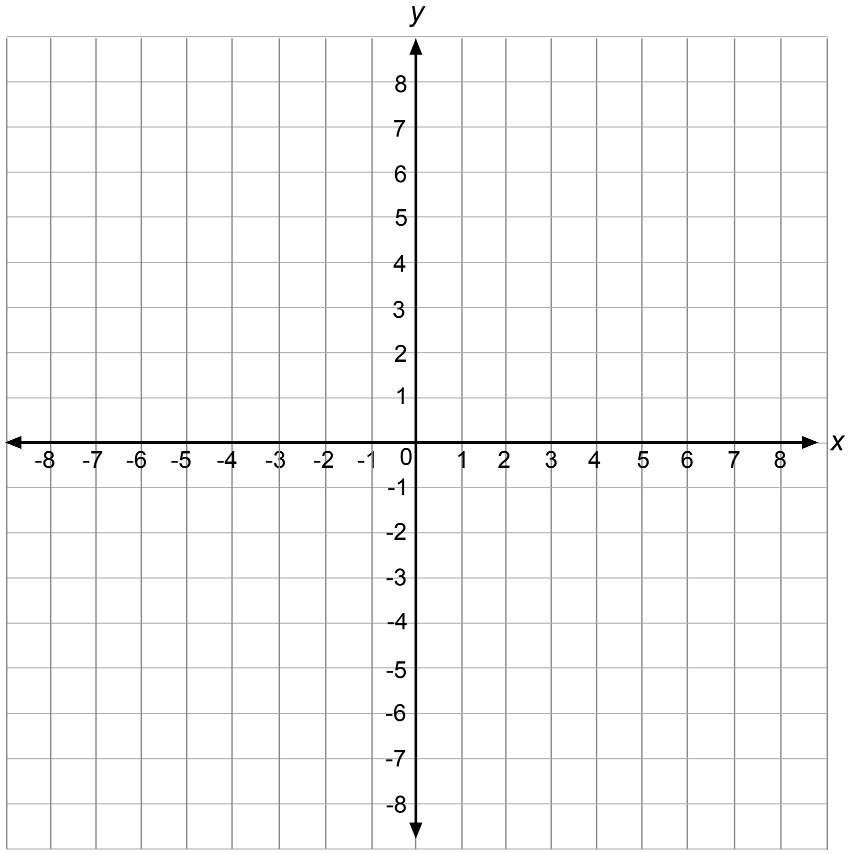
Table of Values

$y < -3$	
x	y

Table of Values

$y \geq 2$	
x	y

Graph of $y < -3$ or $y \geq 2$



3. $x + 2y > 0$ **and** $x - y \leq 5$

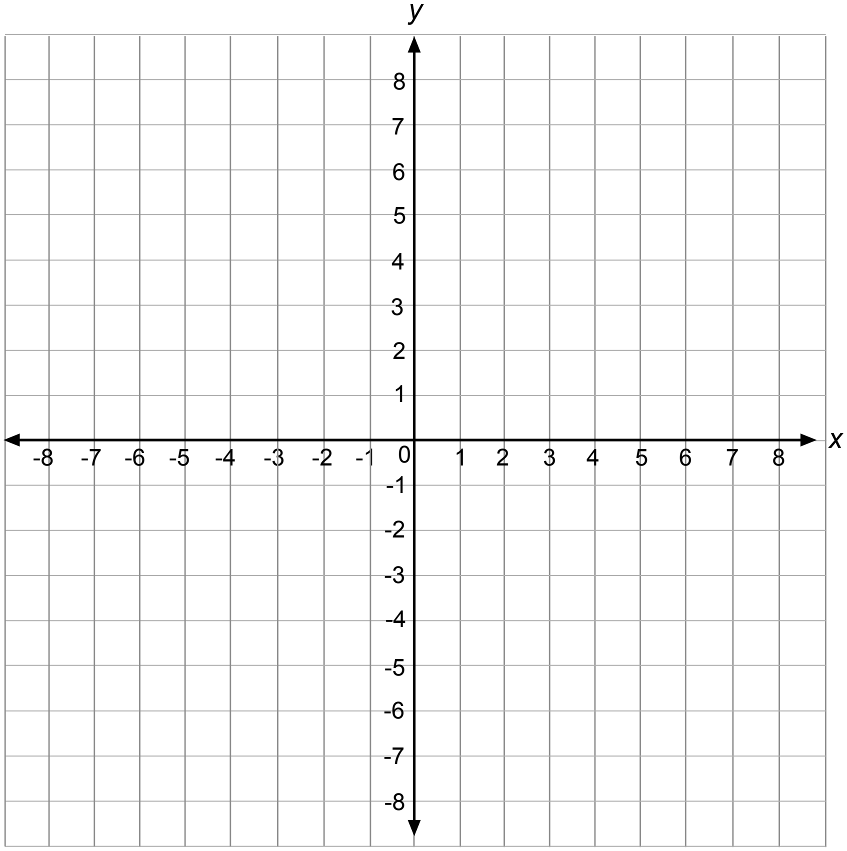
Table of Values

$x + 2y > 0$	
x	y

Table of Values

$x - y \leq 5$	
x	y

Graph of $x + 2y > 0$ and $x - y \leq 5$



Is the point (4, 4) part of the solution? ____________

4. $x + y > 1$ **or** $x - y > 1$

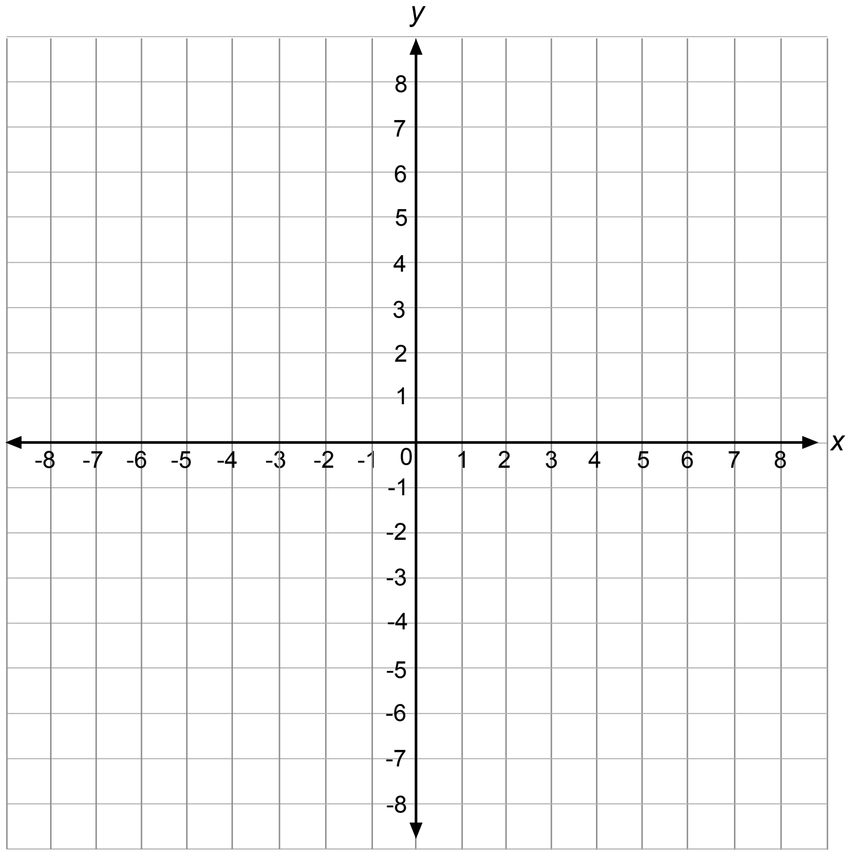
Table of Values

$x + y > 1$	
x	y

Table of Values

$x - y > 1$	
x	y

Graph of $x + y > 1$ or $x - y > 1$



5. $x + y > 1$ **and** $x - y > 1$

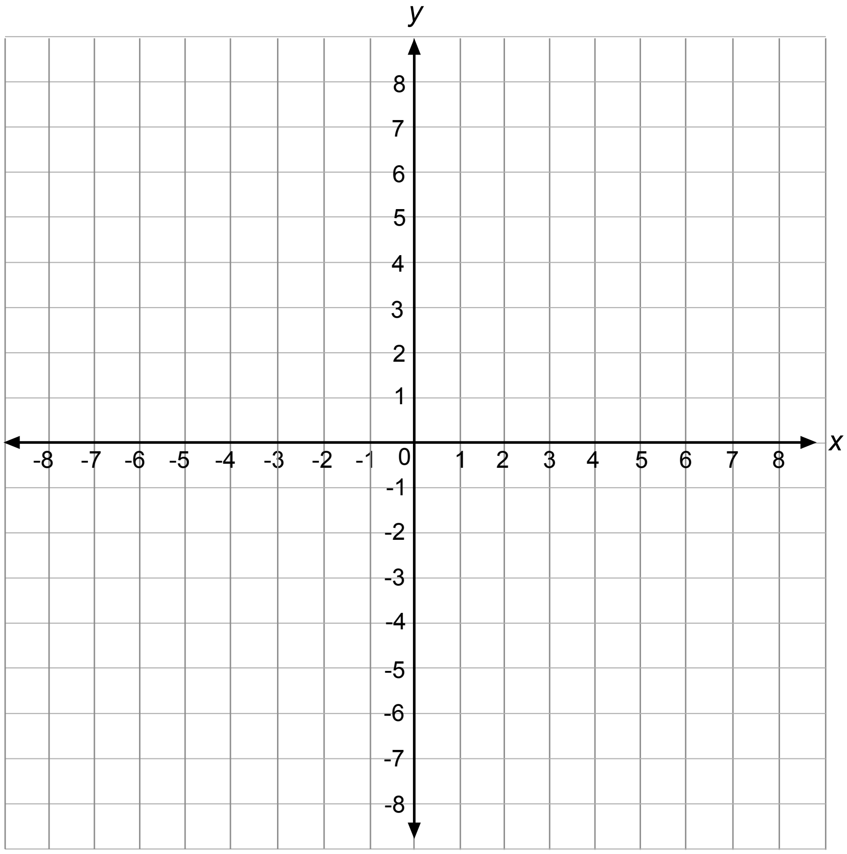
Table of Values

$x + y > 1$	
x	y

Table of Values

$x - y > 1$	
x	y

Graph of $x + y > 1$ and $x - y > 1$



Is the point (0, 0) part of the solution? ____________

6. $y \leq 3$ **or** $x + y > 4$

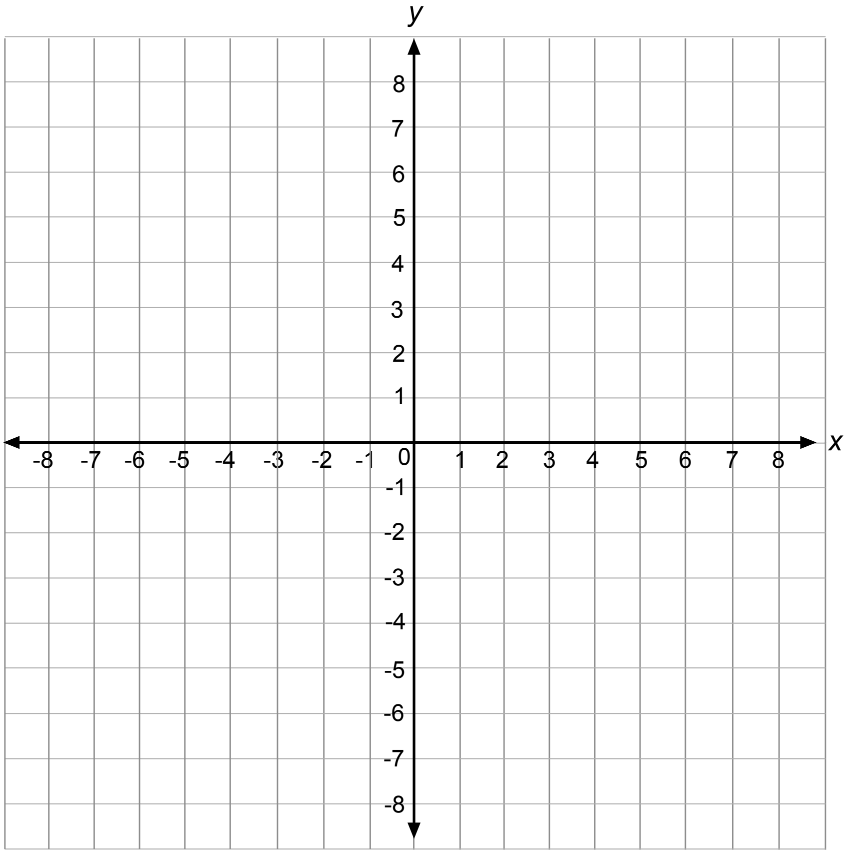
Table of Values

$y \leq 3$	
x	y

Table of Values

$x + y > 4$	
x	y

Graph of $y \leq 3$ and $x + y > 4$



Practice

Match each definition with the correct term. Write the letter on the line provided.

________ 1. a monomial or sum of monomials; any rational expression with no variable in the denominator

________ 2. numbers less than zero

________ 3. a sentence that states one expression is greater than (>), greater than or equal to (≥), less than (<), less than or equal to (≤), or not equal to (≠) another expression

________ 4. a group of two or more equations that are related to the same situation and share variables

________ 5. a method used to solve a system of equations in which variables are replaced with known values or algebraic expressions

________ 6. $ax^2 + bx + c = 0$, where a, b, and c are integers (not multiples of each other) and $a > 0$

________ 7. expressing a polynomial expression as the product of monomials and polynomials

A. factoring

B. inequality

C. negative numbers

D. polynomial

E. standard form (of a quadratic equation)

F. substitution

G. system of equations

Unit Review

Find the **solution sets**.

1. $(x + 5)(x - 7) = 0$ { ______ , ______ }

2. $(3x - 2)(3x - 6) = 0$ { ______ , ______ }

3. $x(x - 7) = 0$ { ______ , ______ }

4. $x^2 + x = 42$ { ______ , ______ }

5. $x^2 - 10x = -16$ { ______ , ______ }

Solve each of the following. **Show all your work.**

6. Max has a garden 4 feet longer than it is wide. If the area of his garden is 96 square feet, find the dimensions of Max's garden.

   Answer: ____________ feet x ____________ feet

7. The product of two consecutive positive **even integers** (integers divisible by 2) is 440. Find the integers.

   Answer: ____________ and ____________

Use the **quadratic formula** *below to solve the following* **equations**.

Check your work using a calculator.

quadratic formula

$$\boxed{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

8. $2x^2 - 7x - 15 = 0$

9. $x^2 + 4x - 30 = -9$

10. The sides of a rose garden are $(x + 8)$ units and $(x - 3)$ units. If the area of the garden is 12 square units, find the dimensions of the rose garden.

    Answer: ____________ units x ____________ units

Solve **algebraically**, *then graph each* **system of equations** *on the graphs provided. Refer to pages 748-752, 759-760, 778-781, and 790-794 as needed.*

11. $2x - y = 6$
$x + y = 9$

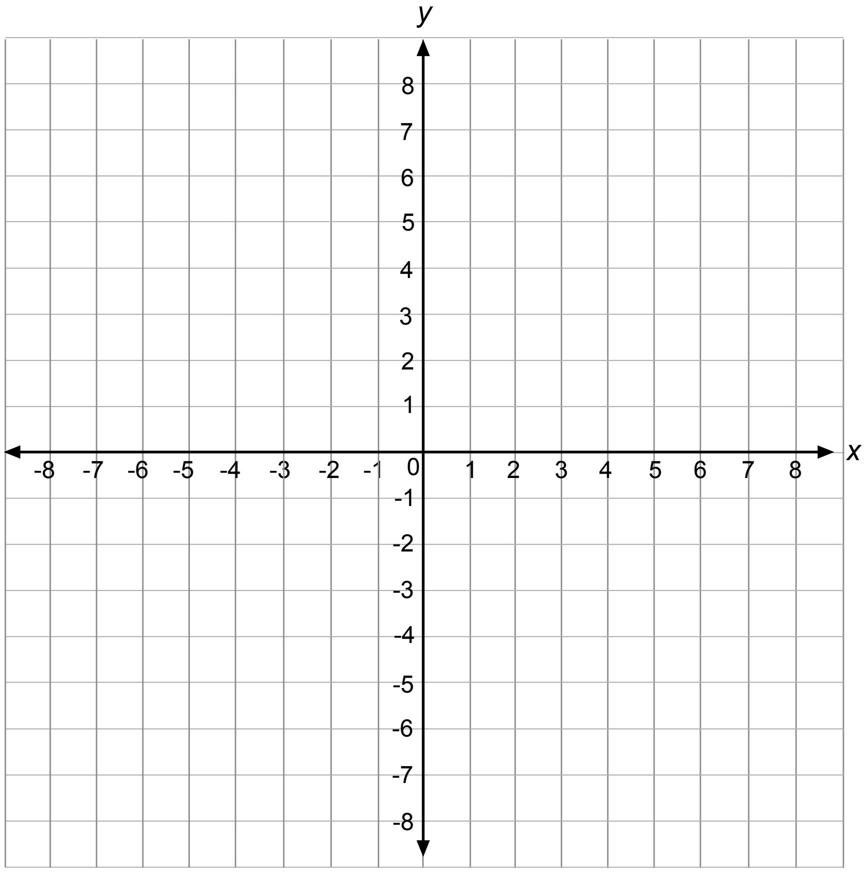
Table of Values

2*x* – *y* = 6	
x	***y***

Table of Values

***x* + *y* = 9**	
x	***y***

Graph of 2*x* – *y* = 6 and *x* + *y* = 9



12. $x + y = 7$
$3x - 4y = 7$

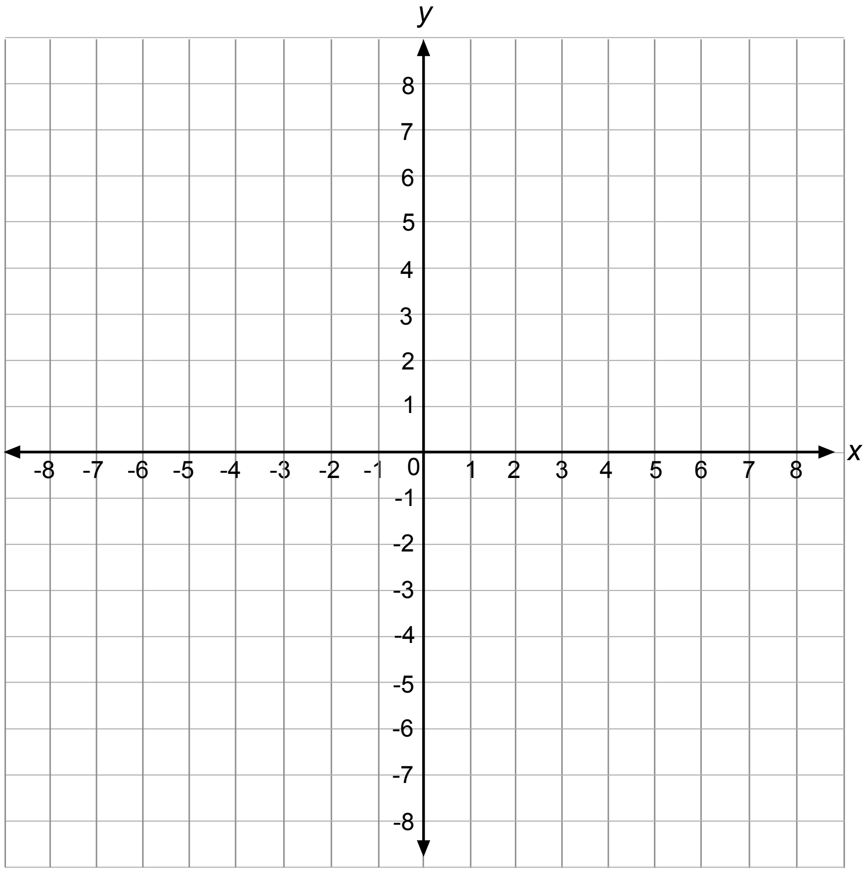
Table of Values

$x + y = 7$	
x	y

Table of Values

$3x - 4y = 7$	
x	y

Graph of $x + y = 7$ and $3x - 4y = 7$



13. $2x - 4y = 8$
$x + 4y = 10$

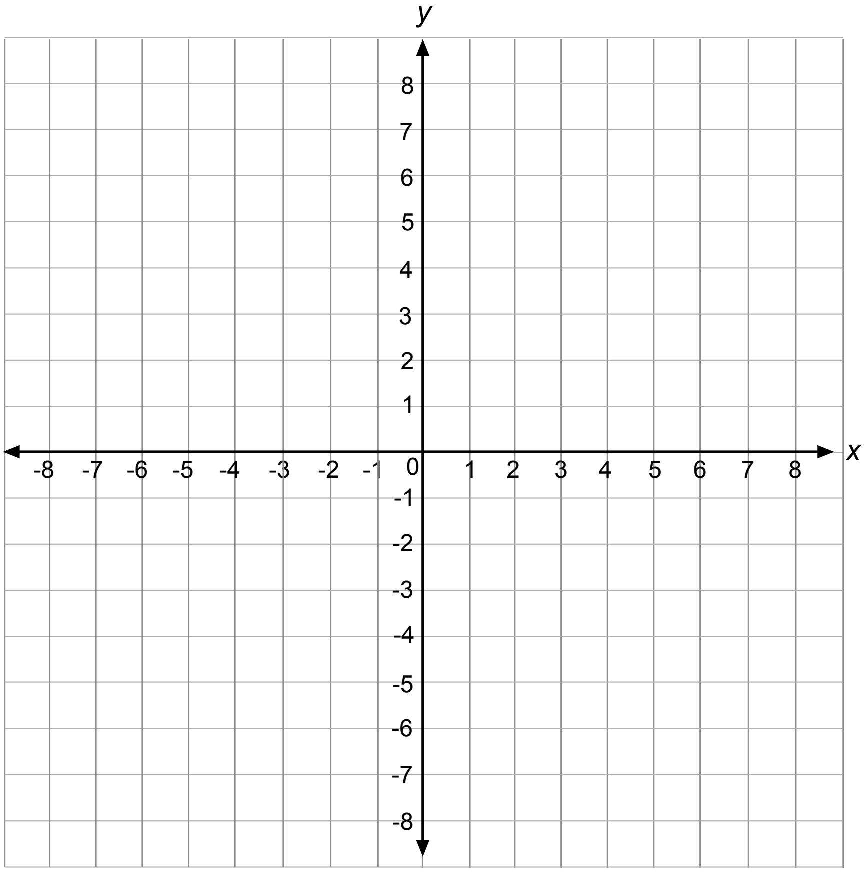
Table of Values

$2x - 4y = 8$	
x	***y***

Table of Values

$x + 4y = 10$	
x	***y***

Graph of 2*x* – 4*y* = 8 and *x* + 4*y* = 10



14. $3x + 2y = 8$
$y = -2$

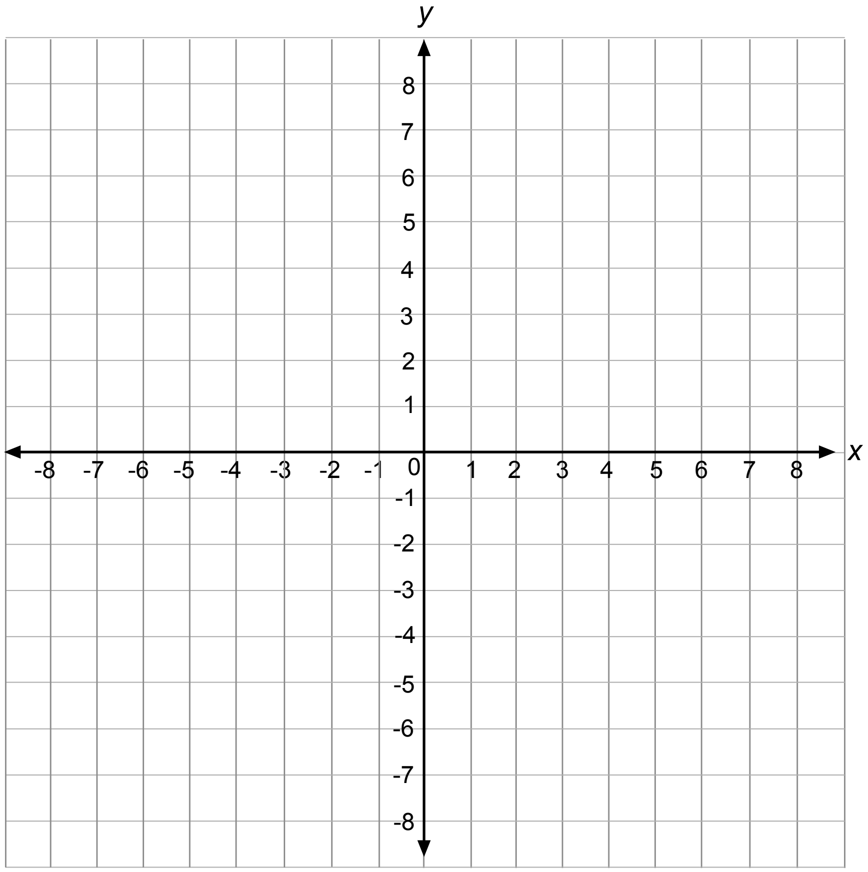
Table of Values

$3x + 2y = 8$	
x	y

Table of Values

$y = -2$	
x	y

Graph of $3x + 2y = 8$ and $y = -2$



15. $3x + 5y = 26$
$2x - 2y = -20$

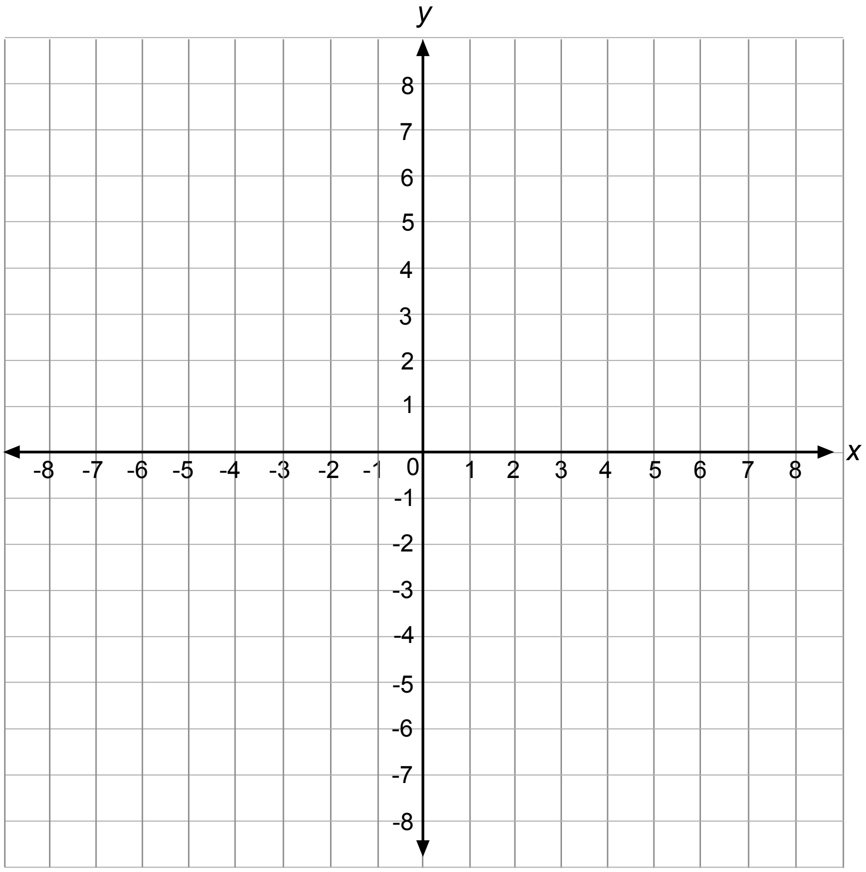
Table of Values

$3x + 5y = 26$	
x	y

Table of Values

$2x - 2y = -20$	
x	y

Graph of $3x + 5y = 26$ and $2x - 2y = -20$



Solve each of the following. **Show all your work.**

16. The sum of two numbers is 52. The larger number is 2 more than 4 times the smaller number. Find the two numbers.

    Answer: ____________ and ____________

17. The band has 8 more than twice the number of students as the chorus. Together there are 119 students in both programs. How many are in each?

    Answer: chorus = ____________ and band = ____________

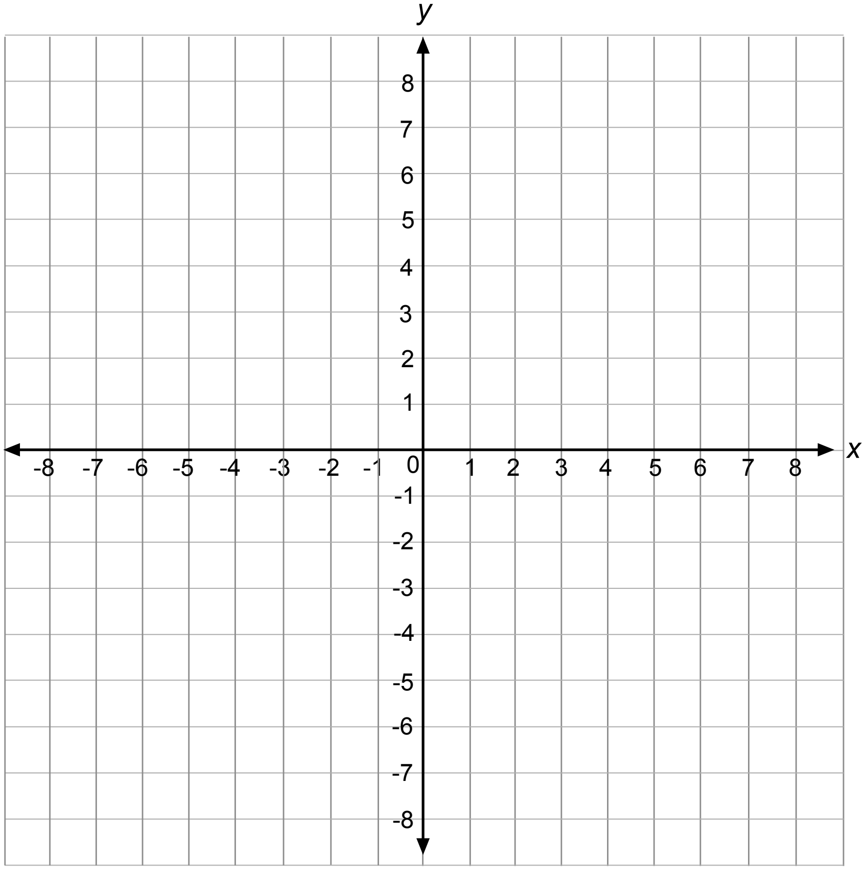
Graph the following **inequalities** *on the graphs provided.*

18. $y > x - 6$

Table of Values

$y > x - 6$	
x	y

Graph of $y > x - 6$

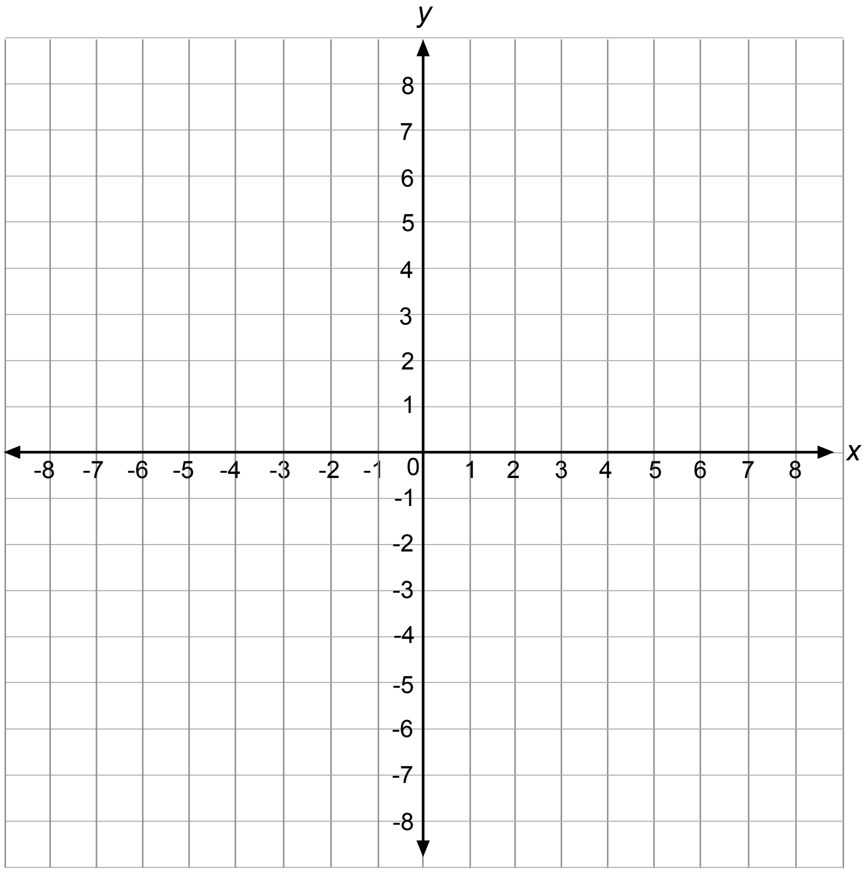


19. $8x - 4y \le 12$

Table of Values

$8x - 4y \le 12$	
x	y

Graph of $8x - 4y \le 12$



Is the point (0, 0) part of the solution? ____________

20. $y \geq 4x - 3$ **and** $x + y < 0$

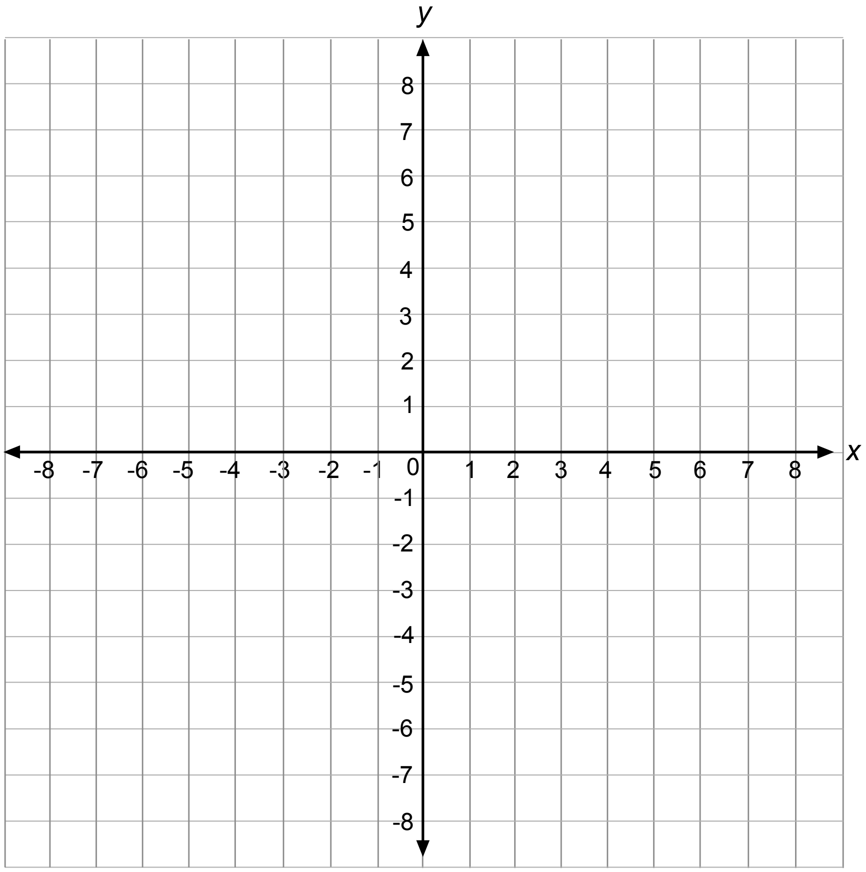
Table of Values

$y \geq 4x - 3$	
x	y

Table of Values

$x + y < 0$	
x	y

Graph of $y \geq 4x - 3$ and $x + y < 0$



21. $x + y > 4$ **or** $y \geq x - 2$

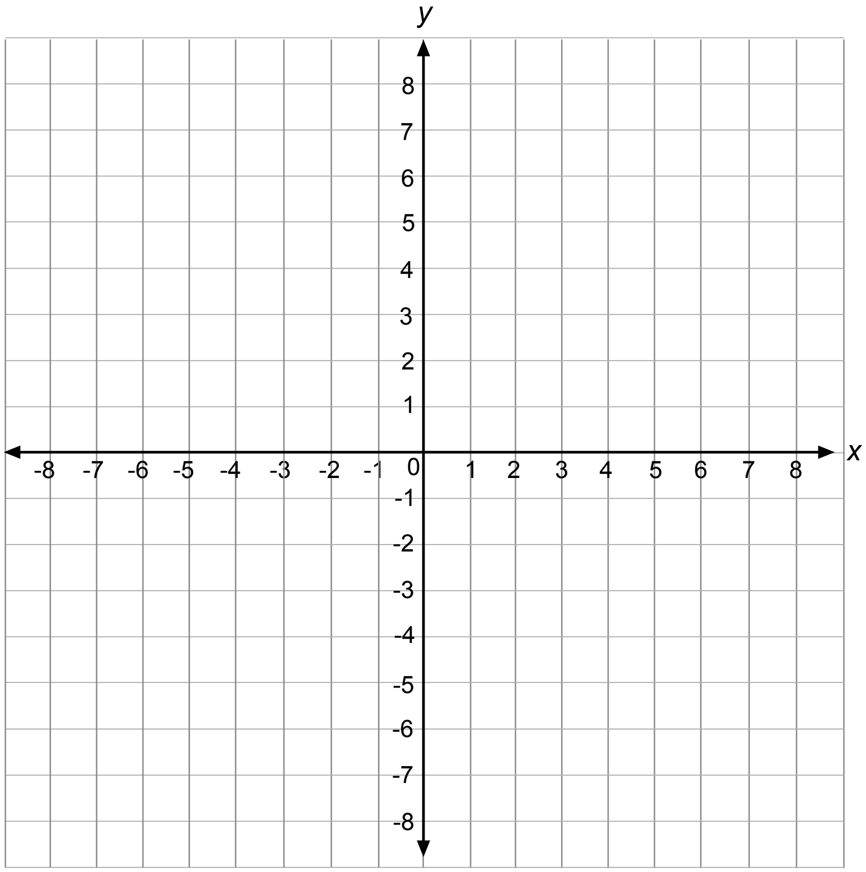
Table of Values

$x + y > 4$	
x	y

Table of Values

$y \geq x - 2$	
x	y

Graph of $x + y > 4$ or $y \geq x - 2$



Is the point (3, -6) part of the solution? ____________

Appendices

Table of Squares and Approximate Square Roots

n	n^2	$\sqrt{n}$
1	1	1.000
2	4	1.414
3	9	1.732
4	16	2.000
5	25	2.236
6	36	2.449
7	49	2.646
8	64	2.828
9	81	3.000
10	100	3.162
11	121	3.317
12	144	3.464
13	169	3.606
14	196	3.742
15	225	3.873
16	256	4.000
17	289	4.123
18	324	4.243
19	361	4.359
20	400	4.472
21	441	4.583
22	484	4.690
23	529	4.796
24	576	4.899
25	625	5.00
26	676	5.099
27	729	5.196
28	784	5.292
29	841	5.385
30	900	5.477
31	961	5.568
32	1,024	5.657
33	1,089	5.745
34	1,156	5.831
35	1,225	5.916
36	1,296	6.000
37	1,369	6.083
38	1,444	6.164
39	1,521	6.245
40	1,600	6.325
41	1,681	6.403
42	1,764	6.481
43	1,849	6.557
44	1,936	6.633
45	2,025	6.708
46	2,116	6.782
47	2,209	6.856
48	2,304	6.928
49	2,401	7.000
50	2,500	7.071
51	2,601	7.141
52	2,704	7.211
53	2,809	7.280
54	2,916	7.348
55	3,025	7.416
56	3,136	7.483
57	3,249	7.550
58	3,364	7.616
59	3,481	7.681
60	3,600	7.746
61	3,721	7.810
62	3,844	7.874
63	3,969	7.937
64	4,096	8.000
65	4,225	8.062
66	4,356	8.124
67	4,489	8.185
68	4,624	8.246
69	4,761	8.307
70	4,900	8.367
71	5,041	8.426
72	5,184	8.485
73	5,329	8.544
74	5,476	8.602
75	5,625	8.660
76	5,776	8.718
77	5,929	8.775
78	6,084	8.832
79	6,241	8.888
80	6,400	8.944
81	6,561	9.000
82	6,724	9.055
83	6,889	9.110
84	7,056	9.165
85	7,225	9.220
86	7,396	9.274
87	7,569	9.327
88	7,744	9.381
89	7,921	9.434
90	8,100	9.487
91	8,281	9.539
92	8,464	9.592
93	8,649	9.644
94	8,836	9.695
95	9,025	9.747
96	9,216	9.798
97	9,409	9.849
98	9,604	9.899
99	9,801	9.950
100	10,000	10.000

Mathematical Symbols

÷ *or* /	divide	∥	is parallel to
x *or* •	times	≈	is approximately equal to
=	is equal to	≅	is congruent to
−	negative	~	is similar to
+	positive	$\sqrt{}$	nonnegative square root
±	positive or negative	%	percent
≠	is not equal to	π	pi
>	is greater than	$\overleftrightarrow{AB}$	line *AB*
<	is less than	$\overline{AB}$	line segment *AB*
≯	is not greater than	$\overrightarrow{AB}$	ray *AB*
≮	is not less than	$\triangle ABC$	triangle *ABC*
≥	is greater than or equal to	$\angle ABC$	angle *ABC*
≤	is less than or equal to	m$\overline{AB}$	measure of line segment *AB*
°	degrees	m$\angle ABC$	measure of angle *ABC*
⊥	is perpendicular to		

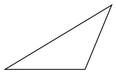
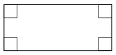
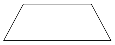
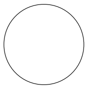
FCAT Mathematics Reference Sheet

Key

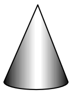
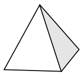
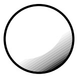
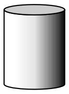
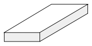
b = base
h = height
l = length
w = width
ℓ = slant height
$S.A.$ = surface area
d = diameter
r = radius
A = area
C = circumference
V = volume

Use 3.14 or $\frac{22}{7}$ for π.

Formulas

	**triangle**	$A = \frac{1}{2}bh$
	**rectangle**	$A = lw$
	**trapezoid**	$A = \frac{1}{2}h(b_1 + b_2)$
	**parallelogram**	$A = bh$
	**circle**	$A = \pi r^2$

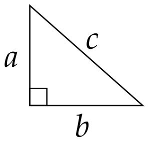
circumference
$C = \pi d$ or $C = 2\pi r$

		Volume	Total Surface Area
	**right circular cone**	$V = \frac{1}{3}\pi r^2 h$	$S.A. = \frac{1}{2}(2\pi r)\ell + \pi r^2$ or $S.A. = \pi r\ell + \pi r^2$
	**square pyramid**	$V = \frac{1}{3}lwh$	$S.A. = 4(\frac{1}{2}l\ell) + l^2$ or $S.A. = 2l\ell + l^2$
	**sphere**	$V = \frac{4}{3}\pi r^3$	$S.A. = 4\pi r^2$
	**right circular cylinder**	$V = \pi r^2 h$	$S.A. = 2\pi rh + 2\pi r^2$
	**rectangular solid**	$V = lwh$	$S.A. = 2(lw) + 2(hw) + 2(lh)$

In the following formulas, n represents the number or sides.

- In a polygon, the sum of the measures of the interior angles is equal to $180(n - 2)$.
- In a regular polygon, the measure of an interior angle is equal to $\frac{180(n-2)}{n}$.

FCAT Mathematics Reference Sheet

Pythagorean theorem:  $a^2 + b^2 = c^2$	Distance between two points $P_1\ (x_1, y_1)$ and $P_2\ (x_2, y_2)$: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Slope-intercept form of an equation of a line: $y = mx + b$ where m = slope and b = the y-intercept.	Midpoint between two points $P_1\ (x_1, y_1)$ and $P_2\ (x_2, y_2)$: $\left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$
Distance, rate, time formula: $d = rt$ where d = distance, r = rate, t = time.	Simple interest formula: $\mathrm{I} = prt$ where p = principal, r = rate, t = time.

Conversions

1 yard = 3 feet = 36 inches
1 mile = 1,760 yards = 5,280 feet
1 acre = 43,560 square feet
1 hour = 60 minutes
1 minute = 60 seconds

1 liter = 1000 milliliters = 1000 cubic centimeters
1 meter = 100 centimeters = 1000 millimeters
1 kilometer = 1000 meters
1 gram = 1000 milligrams
1 kilogram = 1000 gram

1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts

1 pound = 16 ounces
1 ton = 2,000 pounds

Metric numbers with four digits are presented without a comma (e.g., 9960 kilometers). For metric numbers greater than four digits, a space is used instead of a comma (e.g., 12 500 liters).

Index

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Production Software

Adobe InDesign 3.0.1. San Jose, CA: Adobe Systems.

Adobe Photoshop 8.0. Mountain View, CA: Adobe Systems.

Macromedia Freehand 8.0. San Francisco: Macromedia.

Microsoft Office 98. Redmond, WA: Microsoft.