Mathematics 1

Course No. 1205010

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Mathematics 1

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Curriculum Improvement Project
IDEA, Part B, Special Project

LEON COUNTY SCHOOLS

Exceptional Student Education

http://www.leon.k12.fl.us/public/pass/

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Unit 1: Number Sense, Concepts, and Operations

This unit emphasizes how numbers and number operations are used in various ways to solve problems.

Unit Focus

Number Sense, Concepts, and Operations

- Associate verbal names, written word names, and standard numerals with fractions and decimals. (A.1.3.1)
- Understand relative size of fractions and decimals. (A.1.3.2)
- Understand concrete and symbolic representations of rational numbers in real-world situations. (A.1.3.3)
- Understand that numbers can be represented in a variety of equivalent forms, including fractions and decimals. (A.1.3.4)
- Understand and explain the effects of addition, subtraction, multiplication, and division on whole numbers, fractions, and decimals. (A.3.3.1)
- Select the appropriate operation to solve problems involving addition, subtraction, multiplication, and division of rational numbers. (A.3.3.2)
- Add, subtract, multiply, and divide whole numbers, decimals, and fractions, including mixed numbers, to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Use estimation strategies to predict results and to check the reasonableness of results. (A.4.3.1)
- Use concepts about numbers, including primes, factors, and multiples, to build number sequences. (A.5.3.1)

Meter Reader

- reads electric meters and enters the reading on a hand-held computer
- makes a quick check of the condition of the meters and connections

Vocabulary

Study the vocabulary words and definitions below.

chart .... see *table*

common denominator .... a common multiple of two or more denominators
Example: A common denominator for $\frac{1}{4}$ and $\frac{5}{6}$ is 12.

common factor .... a number that is a factor of two or more numbers
Example: 2 is a common factor of 6 and 12.

common multiple .... a number that is a multiple of two or more numbers
Example: 18 is a common multiple of 3, 6, and 9.

decimal number .... any number written with a decimal point in the number
Example: A decimal number falls between two whole numbers, such as 1.5 falls between 1 and 2. Decimal numbers smaller than 1 are sometimes called decimal fractions, such as five-tenths is written 0.5.

decimal point .... the dot dividing a decimal number's whole part from its fractional part

denominator .... the bottom number of a fraction, indicating the number of equal parts a whole was divided into
Example: In the fraction $\frac{2}{3}$ the denominator is 3, meaning the whole was divided into 3 equal parts.

difference .... the result of a subtraction
Example: In 16 - 9 = 7, 7 is the difference.

digit .... any one of the 10 symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9

dividend .... a number that is to be divided by the divisor
Example: In $7\overline{)42}$, $42 \div 7$, $\frac{42}{7}$, 42 is the dividend.

divisor .... a number by which another number, the dividend, is divided
Example: In $7\overline{)42}$, $42 \div 7$, $\frac{42}{7}$, 7 is the divisor.

equivalent
(forms of a number) .... the same number expressed in different forms
Example: $\frac{3}{4}$, 0.75, and 75%

estimation .... the use of rounding and/or other strategies to determine a reasonably accurate approximation without calculating an exact answer

factor .... a number or expression that divides exactly another number
Example: 1, 2, 4, 5, 10, and 20 are factors of 20.

fraction .... any number representing some part of a whole; of the form $\frac{a}{b}$
Example: One-half written in fractional form is $\frac{1}{2}$.

greatest common factor (GCF) .... the largest of the common factors of two or more numbers
Example: For 6 and 8, 2 is the greatest common factor.

inequality .... a sentence that states one expression is greater than, greater than or equal to, less than, less than or equal to, or not equal to another expression
Example: $a \neq 5$ or $x \leq 7$

least common multiple (LCM) ... the smallest of the common multiples of two or more numbers
Examples: For 4 and 6, 12 is the least common multiple.

lowest terms .... see *simplest form*

multiples .... the numbers that result from multiplying a given number by the set of whole numbers
Example: The multiples of 15 are 0, 15, 30, 45, 60, 75, etc.

numerator .... the top number of a fraction, indicating the number of equal parts being considered
Example: In the fraction $\frac{2}{3}$, the numerator is 2.

product .... the result of a multiplication
Example: In $6 \times 8 = 48$, 48 is the product.

quotient .... the result of a division
Example: In $42 \div 7 = 6$, 6 is the quotient.

reciprocals .... two numbers whose product is 1
Example: Since $\frac{3}{4} \times \frac{4}{3} = 1$, the reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$.

rounded number .... a number approximated to a specified place
Example: A commonly used rule to round a number is as follows.

- If the digit in the first place after the specified place is 5 or more, *round up* by adding 1 to the digit in the specified place ($\underline{4}61$ rounded to the nearest hundred is 500).
- If the digit in the first place after the specified place is less than 5, *round down* by *not* changing the digit in the specified place ($\underline{4}41$ rounded to the nearest hundred is 400).

simplest form .... a fraction whose numerator and denominator have no common factor greater than 1
Example: The simplest form of $\frac{3}{6}$ is $\frac{1}{2}$.

sum .... the result of an addition
Example: In 6 + 8 = 14, 14 is the sum.

table (or chart) .... an orderly display of numerical information in rows and columns

whole number .... any number in the set {0, 1, 2, 3, 4 …}

Unit 1: Number Sense, Concepts, and Operations

Introduction

What do you need to be a confident user of mathematics?

You need to understand the following to be a confident user of mathematics:

- the meaning of numbers
- the relative size of numbers
- the various ways of representing numbers
- the effects of operating with numbers

You also need the following to be a confident user of mathematics:

- quick recall of single-digit addition, subtraction, multiplication, and division facts
- accurate and efficient methods to add, subtract, multiply, and divide

In middle school, there is a great deal of focus on fractions, decimals, and percents. This unit will deal with fractions and decimals. Later, work will be done with percents and with scientific notation to write large and small numbers. The use of negative numbers and irrational numbers will also be studied.

Mike and Misty—Old Friends, New Middle School Mathematics Teachers

Mike and Misty remained friends throughout their college days. Both are now recent college graduates. Each will teach middle school mathematics. When they were in middle school, they liked math. Both of them were pretty good at it. They sometimes followed rules in math books that they didn't really understand. Neither of them ever stopped asking why. They never stopped seeking answers to their questions. Sometimes they figured out ways to do problems that were different from the examples in the book. Sometimes they figured out ways to do problems that were different from the methods the teacher used. They liked knowing more than one way to solve problems.

In this book, they will share some traditional approaches with you. They will also share some ways they think helped them to better understand mathematics. You may find other ways. When you do, be sure to share them. The more we know, the more choices we have, and the more interesting and helpful mathematics becomes.

Misty and Mike have enjoyed the way people in the field of technology work to make technology "user friendly." They hope the mathematics you learn in these units will be user friendly, also.

Lesson One Purpose

- Associate verbal names, written word names, and standard numerals with fractions. (A.1.3.1)
- Understand relative size of fractions. (A.1.3.2)
- Understand concrete and symbolic representations of fractions in real-world situations. (A.1.3.3)
- Understand that numbers can be represented in a variety of equivalent forms, including fractions. (A.1.3.4)
- Understand and explain the effects of multiplication and division on whole numbers and fractions. (A.3.3.1)
- Multiply and divide whole numbers and fractions, including mixed numbers, to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)

Fractions! Can They Be User Friendly?

Mike and Misty's parents co-own a candy store. They grew up playing and working in the store. Sometimes a math problem in a book was difficult for them. Their parents would help them make sense of it. At times their parents would use candy to model a math problem. Once the candy was used in the math problem, it was thrown away or eaten. Mike and Misty liked math. They didn't need "a spoonful of

sugar" to make the mathematics go down. However, they didn't back away from sweetening their homework.

Think of some examples in addition to the ones listed below when you use **fractions** or when you may use them in the future.

- She is 5 feet, $5\frac{1}{2}$ inches tall.
- To triple this recipe, I need to use 3 times $\frac{3}{4}$ cup milk.
- He ate a quarter ($\frac{1}{4}$) pound hamburger.
- She is $4\frac{1}{2}$ years old.
- We need to add to the cost of our lunch a 15% tip and a 5% tax which, combined, will be $\frac{1}{5}$ of the total bill.
- She worked $8\frac{3}{4}$ hours and earned \$6.25 per hour.
- The pair of jeans is on sale at $\frac{1}{2}$ off the regular price.
- The box of cereal weighs $18\frac{3}{4}$ ounces.
- To find the Fahrenheit equivalent for a Celsius temperature, we multiply the Celsius temperature by $\frac{9}{5}$ and add 32.
- To find the area of a triangle, we multiply $\frac{1}{2}$ times the base times the height.
- The brownies will be baked in a pan $9\frac{1}{2}$ inches wide and 11 inches long.

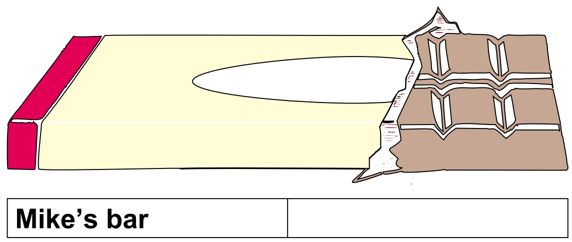
He ate a quarter ($\frac{1}{4}$) pound hamburger.

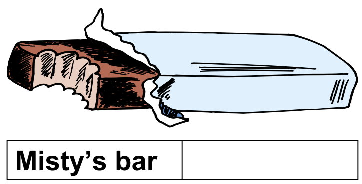
Brownies will be baked in a pan $9\frac{1}{2}$ inches wide and 11 inches long.

The pair of jeans is on sale at $\frac{1}{2}$ off the regular price.

Dividing a Whole Equally

Misty offered Mike half of her candy bar, which was one of his favorite kinds, in exchange for half of his. He agreed and the trade was made. Then Mike complained that Misty did not play fair. He wanted to cancel the deal. What do you think happened?



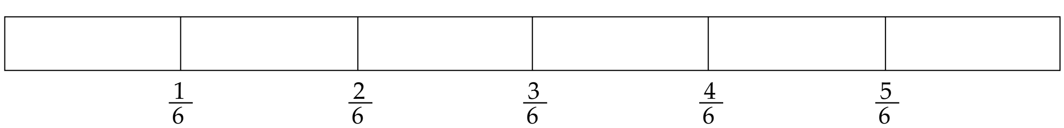


It can easily be seen that Mike's candy bar is larger than Misty's. Therefore, $\frac{1}{2}$ of the larger bar is more that $\frac{1}{2}$ of the smaller bar. Since a fraction is part of a whole, it is important to consider the whole when making comparisons. If you are asked whether $\frac{1}{4}$ or $\frac{1}{5}$ is greater, it is common to assume that these are parts of the same whole or **equivalent** wholes. This will be true in this unit.

The representation of the candy bar below is divided into 6 equal parts. Each part represents $\frac{1}{6}$ of the whole. We could easily cut the bar along the marks to give someone $\frac{1}{6}$, $\frac{2}{6}$, $\frac{3}{6}$, $\frac{4}{6}$, $\frac{5}{6}$, or make *no cuts* and give them *one whole bar*.

The **denominator** tells us the number of equal parts into which the bar was divided, and the **numerator** tells us how many of those parts we are using.

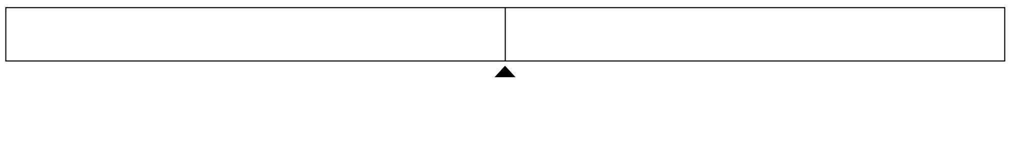
$\frac{1}{6}$ ← numerator / ← denominator



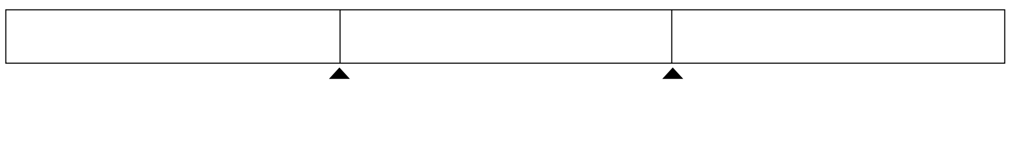
Practice

The candy bars below are all the same size. Each is divided into a certain number of **equal parts**. *Label the* **fractional parts** *of bars A, B, C, and D the way the bar divided into six equal parts was labeled on the previous page. Write the fractional parts on the lines provided.*

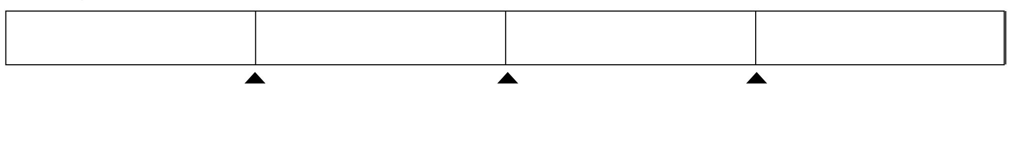
bar A

1. 

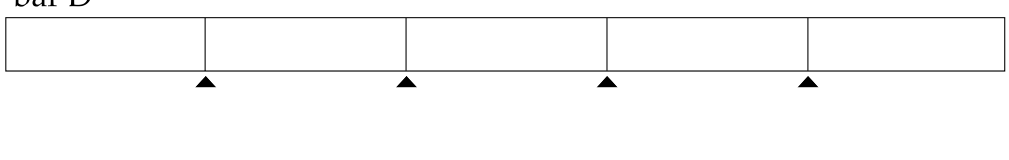
bar B

2. 

bar C

3. 

bar D

4. 

Use the bars on the previous page to **compare the fractional parts of a bar** listed by each number below. Indicate whether the **first fraction is greater than** (>), **less than**(<), or **equal to** (=) the **second fraction**.

(**Remember:** If one value is greater than or less than another, it is called an **inequality**.)

5. $\frac{1}{2}$ ______ $\frac{1}{3}$

6. $\frac{1}{2}$ ______ $\frac{2}{3}$

7. $\frac{1}{2}$ ______ $\frac{1}{4}$

8. $\frac{1}{2}$ ______ $\frac{2}{4}$

9. $\frac{1}{2}$ ______ $\frac{3}{4}$

10. $\frac{1}{2}$ ______ $\frac{1}{5}$

11. $\frac{1}{2}$ ______ $\frac{2}{5}$

12. $\frac{1}{2}$ ______ $\frac{3}{5}$

13. $\frac{1}{2}$ ______ $\frac{4}{5}$

14. $\frac{1}{3}$ ______ $\frac{1}{4}$

15. $\frac{1}{3}$ ______ $\frac{2}{4}$

16. $\frac{1}{3}$ ______ $\frac{3}{4}$

17. $\frac{2}{3}$ ______ $\frac{3}{4}$

18. $\frac{1}{4}$ ______ $\frac{1}{5}$

19. $\frac{2}{4}$ ______ $\frac{1}{2}$

20. $\frac{3}{4}$ ______ $\frac{1}{2}$

21. $\frac{3}{4}$ ______ $\frac{2}{5}$

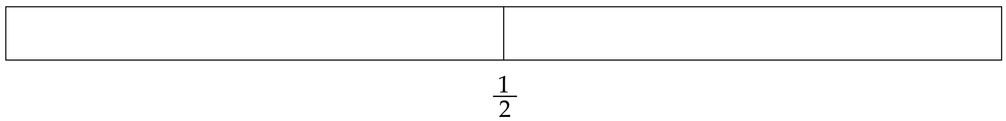
22. $\frac{3}{4}$ ______ $\frac{3}{5}$

23. $\frac{3}{4}$ ______ $\frac{4}{5}$

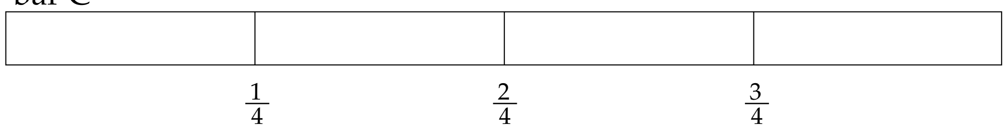
Equivalent Forms of a Fraction

If we divided each of the two halves in bar A into two equal parts, we would have a total of four equal parts.

bar A



bar C

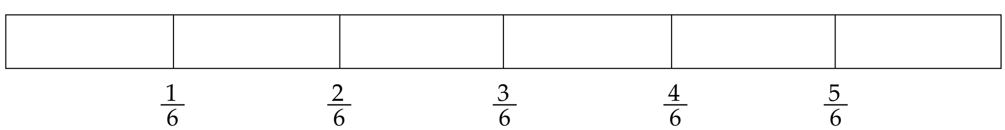


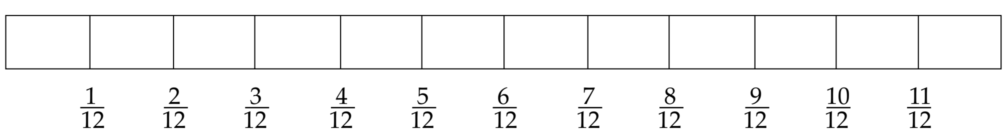
Bar A would then look like bar C on page 11.

If we do this, we can see that the mark for $\frac{1}{2}$ now has another name, which is $\frac{2}{4}$.

Having 1 of 2 equal pieces of the bar is the same as having 2 of 4 equal pieces of the bar. The fractions $\frac{1}{2}$ and $\frac{2}{4}$ are said to be *equivalent*. The fractions are equivalent because they name the same number expressed in a different form.

The bar in the example on page 10 was divided into six equal parts. Let's see what it would look like if we divided each of those six parts into two equal parts to make a total of 12.



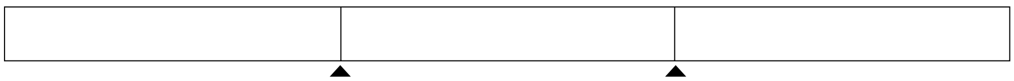


It can be seen that having 1 part of 6 is the same amount as 2 parts of 12. Having 2 parts of 6 is the same as having 4 parts of 12. When we write $\frac{2}{12}$ in simplest form, we write $\frac{1}{6}$. A **factor** divides into another number so that the remainder is zero. Two divides 2 evenly, so 2 is a factor of 2. Two divides 12 evenly, so 2 is a factor of 12. Since 2 and 12 have a **common factor** of 2, we divide the numerator and denominator by the factor 2, and the result is $\frac{1}{6}$. Each original $\frac{1}{6}$ was divided into two equal parts.

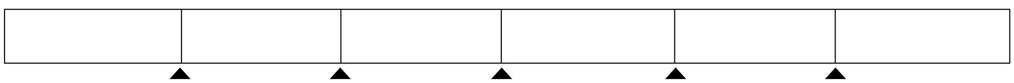
Practice

Bars B, C, and D from the previous practice have been redrawn. Below each one is at least one new bar. The new bars are now called bars BB, BBB, CC, and DD. Each of the **original parts** *of bars B, C, and D has been* **divided into two equal parts**. *The new bar BB has also been divided into two equal parts. Label the markings of each bar. Write the fractional parts on the line provided.*

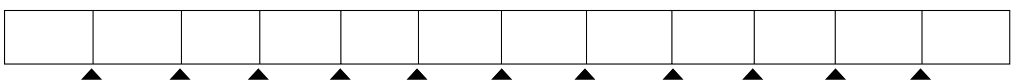
bar B

1. 

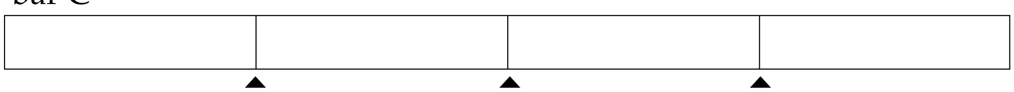
bar BB

2. 

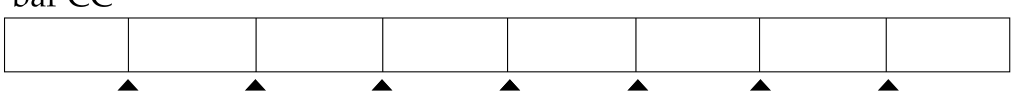
bar BBB

3. 

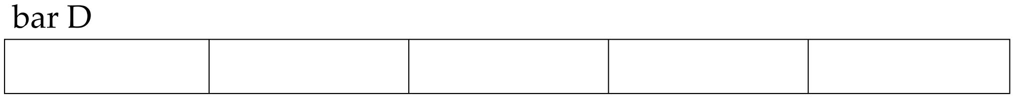
bar C

4. 

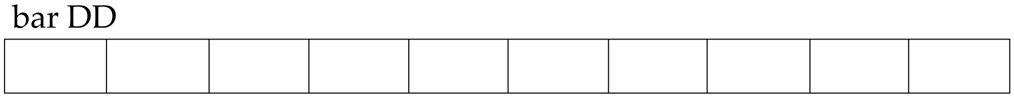
bar CC

5. 

bar D

6. 

bar DD

7. 

Use the bars on the previous page to complete the following.

8. $\frac{1}{2} = \frac{\quad}{4}$ or $\frac{\quad}{6}$ or $\frac{\quad}{8}$ or $\frac{\quad}{10}$ or $\frac{\quad}{12}$

9. $\frac{1}{3} = \frac{\quad}{6}$ or $\frac{\quad}{12}$

10. $\frac{1}{4} = \frac{\quad}{8}$ or $\frac{\quad}{12}$

11. $\frac{1}{5} = \frac{\quad}{10}$

12. $\frac{1}{6} = \frac{\quad}{12}$

13. $\frac{2}{3} = \frac{\quad}{6}$ or $\frac{\quad}{12}$

14. $\frac{3}{4} = \frac{\quad}{8}$ or $\frac{\quad}{12}$

15. $\frac{2}{5} = \frac{\quad}{10}$

16. $\frac{3}{5} = \frac{\quad}{10}$

17. $\frac{4}{5} = \frac{\quad}{10}$

18. $\frac{1}{6} = \frac{\quad}{12}$

19. $\frac{2}{6} = \frac{\quad}{12}$

20. $\frac{3}{6} = \frac{\quad}{12}$

21. $\frac{4}{6} = \frac{\quad}{12}$

22. $\frac{5}{6} = \frac{\quad}{12}$

Practice

Match each definition with the correct term. Write the letter on the line provided.

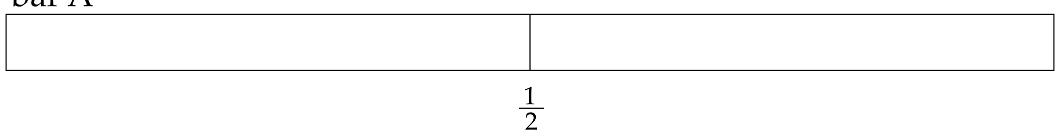
______ 1. any number representing some part of a whole; of the form $\frac{a}{b}$	A. common factor
______ 2. the same number expressed in different forms	B. denominator
______ 3. the bottom number of a fraction, indicating the number of equal parts a whole was divided into	C. equivalent
______ 4. the top number of a fraction, indicating the number of equal parts being considered	D. factor
______ 5. a sentence that states one expression is greater than, greater than or equal to, less than, less than or equal to, or not equal to another expression	E. fraction
______ 6. a number that is a factor of two or more numbers	F. inequality
______ 7. a number or expression that divides exactly another number	G. numerator

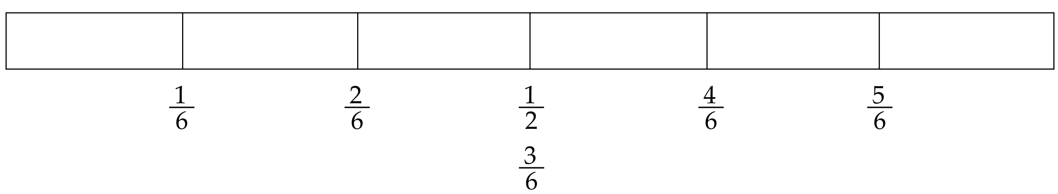
Equivalent Names of Fractions

Fractions can have more than one name.

If each of the two equal parts of bar A were divided into 3 equal parts, 6 parts would result, and each would be $\frac{1}{6}$ of the whole.

bar A





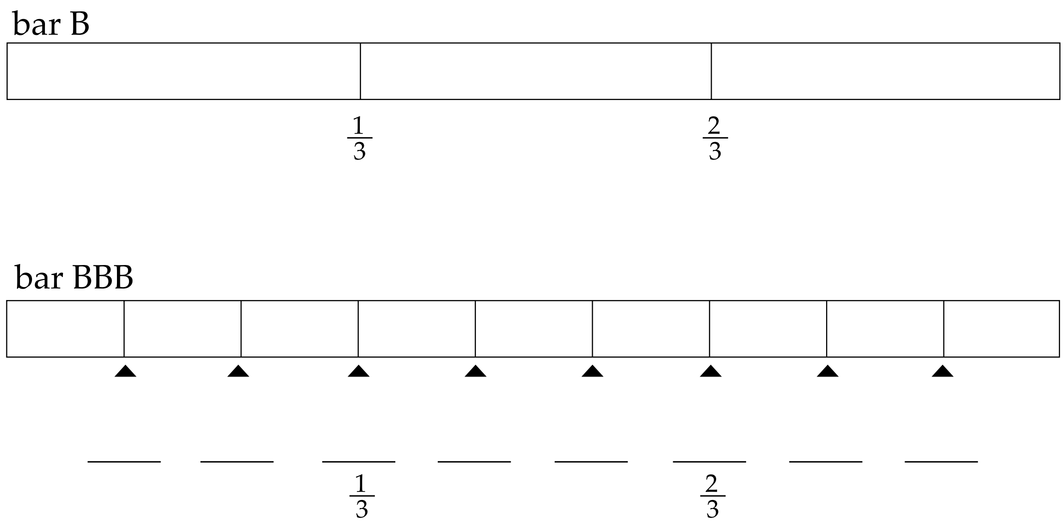
Another name for $\frac{2}{6}$ is $\frac{1}{3}$.

Another name for $\frac{4}{6}$ is $\frac{2}{3}$.

Practice

Complete the following.

If each of the three equal parts of bar B from the first practice were divided into 3 equal parts, 9 parts would result, and each would be $\frac{1}{9}$ of the whole. Label the parts showing the new names.



Dividing the Equal Parts

We could continue to divide the equal parts in bars C and D from the previous practice into three parts. We can see that we could divide *any* of the bars into *any* number of equal parts to create *new* fractional parts. If we wanted 45 equal parts, we could divide each of 3 parts into 15 on bar B. Or we could divide each of 5 parts into 9 on bar D. Fortunately that is not necessary. Fractions can still be user friendly!

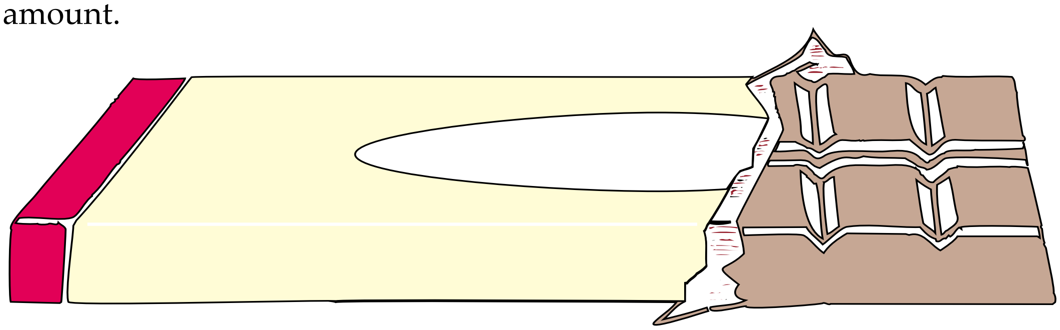
Mike and Misty recall seeing the following example in textbooks when they were young.

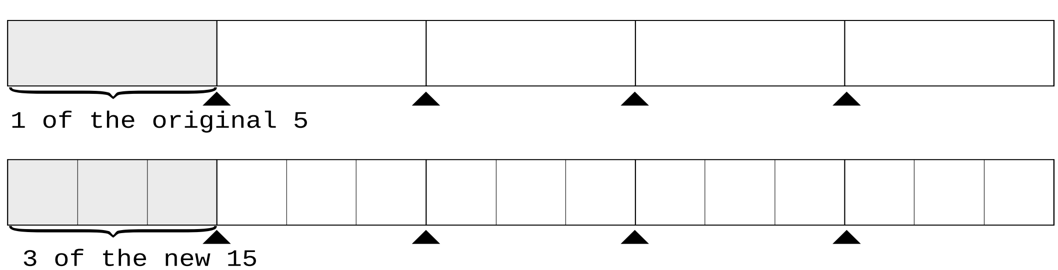
$\frac{1}{5} \text{ x } \frac{3}{3} = \frac{3}{15}$

Seeing Is Believing

They were told that $\frac{3}{3}$ is the same as 1 and that multiplying by 1 gives us the same number. In this case, $\frac{1}{5}$ was multiplied by the fraction $\frac{3}{3}$, which is equal to 1. This resulted in the fraction $\frac{3}{15}$, which is equal to $\frac{1}{5}$.

Mike and Misty would like to model this with a candy bar from their parents' store. They took a bar and cut it into 5 equal parts. Then they made 3 equal parts out of each of the 5. You could have 1 part of the original 5 parts or 3 parts of the new 15 parts. Both add up to the same amount.





Try explaining this to a classmate or friend and see if they understand.

Practice

Complete the following.

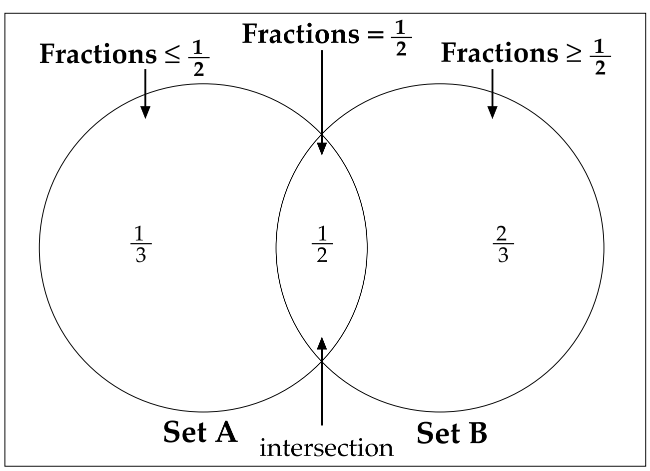
The diagram below is called a *Venn diagram*. This Venn diagram is made up of two overlapping circles placed inside a rectangle. The two overlapping circles show the special characteristics of two sets of items or concepts. The middle part, where the circles overlap, shows what the sets have in common.

- One of the circles contains a set of names of fractions *less than or equal to* (≤) $\frac{1}{2}$.
- The other circle contains a set of names of fractions *greater than or equal to* (≥) $\frac{1}{2}$.
- Since both circles include names of fractions *equal to* (=) $\frac{1}{2}$, those should go in the part of each circle that is *shared* with the other. The part where both circles overlap is called the *intersection*.

Place the following fractions in the Venn diagram so that set A represents fractions **less than or equal to** (≤) $\frac{1}{2}$ and set B represents fractions **greater than or equal to** (≥) $\frac{1}{2}$. Place the fractions **equal to** (=) $\frac{1}{2}$ in the intersection of set A and set B. Your candy bar models may help. The first three fractions are placed for you.

$\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, $\frac{1}{6}$, $\frac{2}{6}$, $\frac{3}{6}$, $\frac{4}{6}$, $\frac{5}{6}$, $\frac{3}{8}$, $\frac{4}{8}$, $\frac{5}{8}$, $\frac{3}{10}$,

$\frac{5}{10}$, $\frac{7}{10}$, $\frac{5}{12}$, $\frac{6}{12}$, $\frac{9}{12}$



Part of a Part!

Misty and Mike now think about giving someone *part of a part* of a candy bar. Think of a candy bar being cut into halves for you and Friend One.

You	Friend One

Suddenly Friend Two and Friend Three appear. You would like to share the candy bar with them. If you and your original friend agree to share, you might do this.

You	Friend Two	Friend One	Friend Three

The four of you would each get $\frac{1}{2}$ of $\frac{1}{2}$ of the candy bar, or $\frac{1}{4}$.

If your friend is reluctant to share his half, you might share yours.

You	Friend Two	Friend Three	Friend One

Friend One would have his original $\frac{1}{2}$ bar. You, Friend Two, and Friend Three would each have $\frac{1}{3}$ of $\frac{1}{2}$ of the bar, which is $\frac{1}{6}$ bar.

Practice

Shade the following and give the **fractional part** *the* **shaded region** *represents.*

1. $\frac{1}{2}$ of $\frac{1}{3}$ = __________

2. $\frac{1}{3}$ of $\frac{1}{3}$ = __________

3. $\frac{2}{3}$ of $\frac{1}{3}$ = __________

4. $\frac{1}{4}$ of $\frac{1}{3}$ = __________

5. $\frac{1}{3}$ of $\frac{1}{4}$ = __________

6. $\frac{2}{3}$ of $\frac{1}{4}$ = __________

7. $\frac{1}{4}$ of $\frac{1}{4}$ = __________

8. $\frac{3}{4}$ of $\frac{1}{4}$ = __________

9. $\frac{1}{2}$ of $\frac{1}{5}$ = __________

10. $\frac{1}{4}$ of $\frac{1}{5}$ = __________

11. $\frac{2}{3}$ of $\frac{1}{5}$ = __________

12. If you give your friend $\frac{1}{3}$ of $\frac{1}{2}$ of a bar, is that the same as giving your friend $\frac{1}{2}$ of $\frac{1}{3}$ of the bar? _______________ (yes, no)

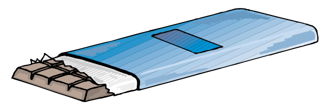
Solve and check your answer by shading.

13. $\frac{1}{3}$ of $\frac{1}{2}$ = __________

14. $\frac{1}{2}$ of $\frac{1}{3}$ = __________

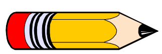
Multiplying Fractions

Mike and Misty enjoyed having the candy bars to model problems like these. They also remembered that when they learned to multiply fractions in middle school, they got the same answers as they found using the *part of a part* method to divide a candy bar.



$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

$\frac{2}{3} \times \frac{1}{12} = \frac{2}{36} = \frac{1}{18}$



(**Remember:** The $\frac{2}{36}$ is simplified because the numerator, 2, and the denominator, 36, have a common factor of 2. When each is divided by 2, the simplified equivalent fraction is $\frac{1}{18}$. A fraction is written in **simplest form** or **lowest terms** when its numerator and denominator have no common factor greater than 1.)

Their teacher gave them some simple steps.

1. Multiply the numerators of the fractions to be multiplied. Write that as the numerator of the **product**.

2. Multiply the denominators of the fractions to be multiplied. Write that as the denominator of the product.

3. Simplify the product, if possible.

Example:

$\frac{1}{4} \times \frac{2}{5} = \frac{1 \times 2}{4 \times 5} = \frac{2}{20} = \frac{1}{10}$

The numerators were multiplied (1 times 2).

The denominators were multiplied (4 x 5).

The product, $\frac{2}{20}$, could be simplified. The numerator and denominator had a common factor of 2. When each was divided by two, the product in *simplest form* or *lowest terms* was $\frac{1}{10}$.

Practice

Complete the following showing your work. Write each answer in **simplest form**.

1. $\frac{1}{2} \times \frac{1}{3} =$ __________
2. $\frac{1}{3} \times \frac{1}{3} =$ __________
3. $\frac{1}{4} \times \frac{1}{3} =$ __________
4. $\frac{2}{3} \times \frac{1}{3} =$ __________
5. $\frac{1}{3} \times \frac{1}{4} =$ __________
6. $\frac{2}{3} \times \frac{1}{4} =$ __________
7. $\frac{1}{4} \times \frac{1}{4} =$ __________
8. $\frac{3}{4} \times \frac{1}{4} =$ __________
9. $\frac{1}{2} \times \frac{1}{5} =$ __________
10. $\frac{1}{3} \times \frac{1}{5} =$ __________
11. $\frac{2}{3} \times \frac{1}{5} =$ __________
12. $\frac{1}{4} \times \frac{1}{5} =$ __________

13. How do these products compare with the fractional part of candy bars found in the previous practice? ____________________________

____

____

____

____

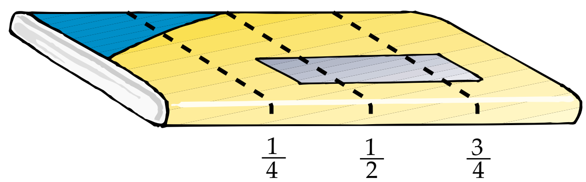
14. Is the product of $\frac{1}{2}$ and $\frac{1}{3}$ the same as $\frac{1}{2}$ of $\frac{1}{3}$ of a bar? ________

Explain. ____

____

Modeling—Seeing Why Rules Are Different

Mike and Misty think that the way their teacher taught them is the quickest way to get an answer. However, they also think that modeling with the candy bars helps them understand why it works. They learned to add fractions before they learned to multiply. When adding, they did not add the numerators and add the denominators. Modeling the problems with candy bars helped them see why the rules are different.



Practice

Write a paragraph to a friend explaining **multiplication of fractions**. *You may use numerical examples, pictorial examples, or both in your explanation.*

Dividing Fractions

Mike remembers getting confused about division of fractions when he was young. As he recalls, he was told *when dividing by a fraction*, turn it upside down and multiply. His teacher really said to multiply the first fraction by the **reciprocal** of the second fraction. His older brother told him that the reciprocal was a big word for turning a fraction *upside down*. He did not understand the rule but followed it.

His problem was that sometimes he became confused and used the rule when multiplying as well as dividing. Sometimes he turned both fractions in a division problem upside down.

Mike and Misty have developed some reasoning to clear up any confusion that may exist when they teach.

- They know that 2 divided by 2 is 1. $2\overline{)2}^{\,1}$ *or* $\frac{2}{1} \div \frac{2}{1}$
- They know that the reciprocal of 2 is $\frac{1}{2}$. $\frac{2}{1} \leftrightarrows \frac{1}{2}$
- They try 2 times $\frac{1}{2}$ and they get $\frac{2}{2}$ or 1. $2 \text{ x } \frac{1}{2} = \frac{2}{1} \text{ x } \frac{1}{2} = \frac{2}{2} = 1$

- They know that 8 divided by 4 is 2. $4\overline{)8}^{\,2}$ *or* $\frac{8}{1} \div \frac{4}{1}$
- They know that the reciprocal of 4 is $\frac{1}{4}$. $\frac{4}{1} \leftrightarrows \frac{1}{4}$
- They try 8 times $\frac{1}{4}$ and they get $\frac{8}{4}$ or 2. $8 \text{ x } \frac{1}{4} = \frac{8}{1} \text{ x } \frac{1}{4} = \frac{8}{4} = 2$

- They know that 5 divided by 3 is 1 with 2 left over or $1\frac{2}{3}$.

$$\begin{array}{r} 1 \\ 3\overline{)5} \\ \underline{3} \\ 2 \end{array} = 1\frac{2}{3} \ \textit{or} \ \frac{5}{1} \div \frac{3}{1}$$

- They know that $\frac{1}{3}$ is the reciprocal of 3.

$$\frac{1}{3} \leftrightarrow \frac{3}{1}$$

- They try $\frac{5}{1}$ times $\frac{1}{3}$ and get $\frac{5}{3}$ or $1\frac{2}{3}$

$$5 \times \frac{1}{3} = \frac{5}{1} \times \frac{1}{3} = \frac{5}{3} = 1\frac{2}{3}$$

- They can see that dividing by 2 is the same as multiplying by $\frac{1}{2}$.

$$\begin{array}{r} 1 \\ 2\overline{)2} \end{array} = \frac{2}{1} \times \frac{1}{2} = \frac{2}{2} = 1$$

- They can see that dividing by 4 is the same as multiplying by $\frac{1}{4}$.

$$\begin{array}{r} 2 \\ 4\overline{)8} \end{array} = \frac{8}{1} \times \frac{1}{4} = \frac{8}{4} = 2$$

- They can see that dividing by 3 is the same as multiplying by $\frac{1}{3}$.

$$\begin{array}{r} 1\frac{2}{3} \\ 3\overline{)5} \end{array} = \frac{5}{1} \times \frac{1}{3} = \frac{5}{3} = 1\frac{2}{3}$$

They can now try dividing by $\frac{1}{2}$. Will it be the same as multiplying by 2?

For example:

4 divided by $\frac{1}{2}$ = ?

This is asking how many one-halves are there in four wholes?

Remember the candy bars. If we have four wholes, how many halves would we have?

We would have eight.

1 whole

1 half	2 halves

$\frac{1}{2}$

2 wholes

3 halves	4 halves

$\frac{1}{2}$

3 wholes

5 halves	6 halves

$\frac{1}{2}$

4 wholes

7 halves	8 halves

$\frac{1}{2}$

There are 8 halves in 4 wholes.

When we divide a **whole number** by a whole number, our **quotient** is less than the **dividend**.

$$\begin{array}{r} \text{quotient} \\ \text{divisor}\overline{)\text{dividend}} \end{array}$$

For example:

8 divided by 2 is 4. $8 \div 2 = 4$

The quotient 4 is less than the dividend 8.

When we divided a whole number by a fraction, our *quotient* was greater than the dividend. When we divide a whole number by a number less than one, but greater than 0, our quotient is greater than our dividend.

4 divided by $\frac{1}{2}$ is 8. $4 \div \frac{1}{2} = \frac{4}{1} \text{ x } \frac{2}{1} = \frac{8}{1} = 8$

The quotient 8 is greater than the dividend 4.

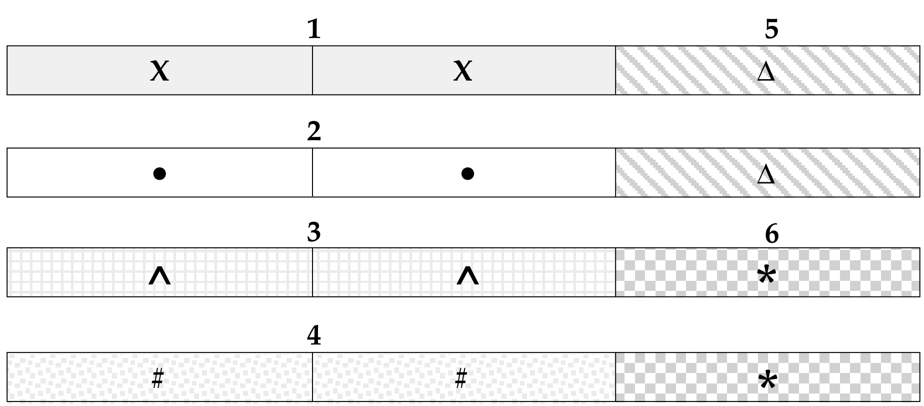
Could you explain to a friend why this is true?

More examples:

$4 \div \frac{2}{3} =$

$\frac{4}{1} \text{ x } \frac{3}{2} = \frac{12}{2}$ or 6

Let's find how many $\frac{2}{3}$ bars we could make from 4 whole bars.



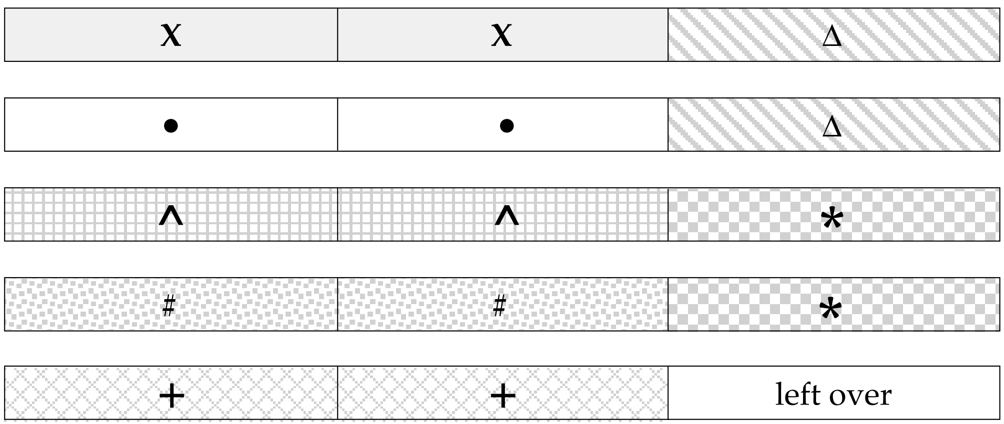
We can see six $\frac{2}{3}$ bars, with each of the two-thirds marked with a different symbol: X, Δ, •, ^, *, and #. We have six $\frac{2}{3}$ bars with none left over.

Misty and Mike wonder if you can rise to another challenge.

$5 \div \frac{2}{3} \ =$

$\frac{5}{1} \text{ x } \frac{3}{2} = \frac{15}{2}$ or $7\frac{1}{2}$

They used bars to model this problem.



We can see seven $\frac{2}{3}$ bars, each marked with a different symbol: X, Δ, •, ^, *, #, and +. We have seven $\frac{2}{3}$ bars with $\frac{1}{3}$ bar left over.

How can this be? When we divided 5 by $\frac{2}{3}$ by multiplying $\frac{5}{1}$ by $\frac{3}{2}$ we got $7\frac{1}{2}$.

How can we have seven $\frac{2}{3}$ bars with $\frac{1}{3}$ bar left over?

Think and think hard. What part of a $\frac{2}{3}$ bar is $\frac{1}{3}$ bar?

Wow! One-third is one-half of a $\frac{2}{3}$ bar! Five whole bars gives us $7\frac{1}{2}$ of these $\frac{2}{3}$ bars.

Practice

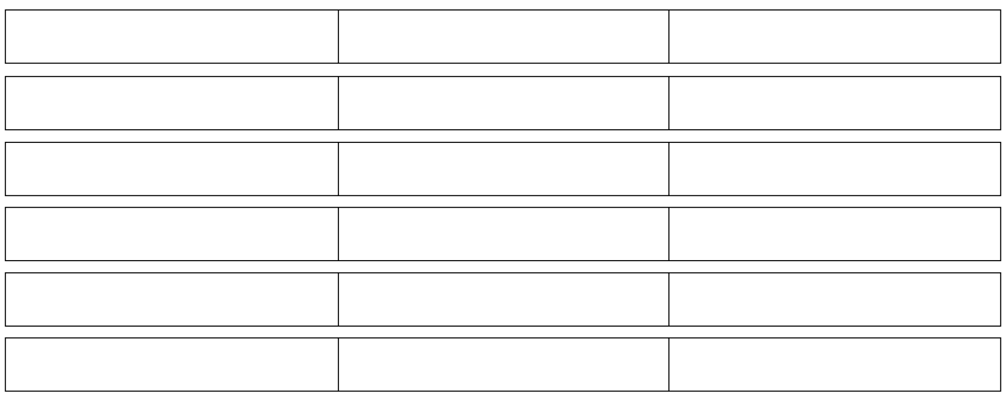
Use **both methods** *below to solve the following showing your work. Write answer in* **simplest form***.*

a. Use the rule: to divide by a fraction, multiply by its reciprocal.
b. Use a model, such as candy bars, to solve the problem.

1. $6 \div \frac{2}{3} =$

   a. ______________________________

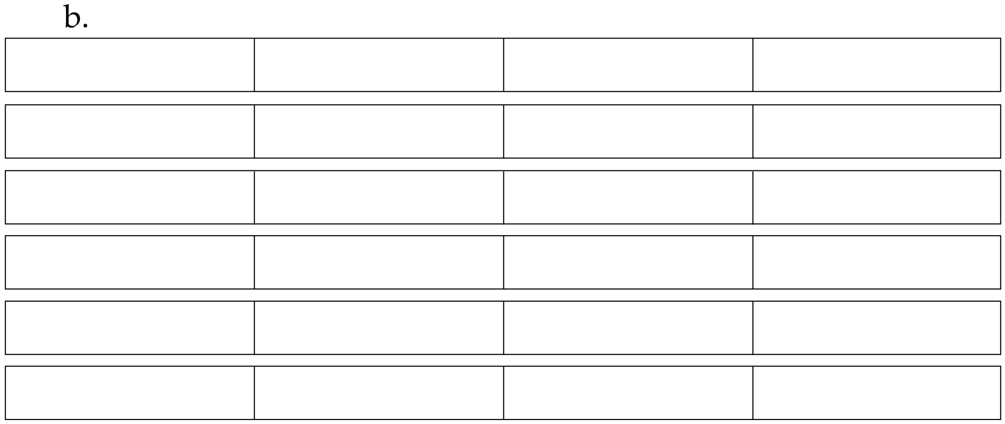
   b.



2. $6 \div \frac{3}{4} =$

   a. ______________________________

   b.



3. $3 \div \frac{3}{5} =$

a. ____

b.

4. Reflect: You are making use of models (candy bars) to make sense of working with fractions. Are the models making fractions more user friendly for you? Why or why not? ____________________

____

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Practice

Use the list below to write the correct term for each definition on the line provided.

______ 1. the result of a multiplication

______ 2. two numbers whose product is 1

______ 3. the result of a division

______ 4. a number that is to be divided by the divisor

______ 5. a number by which another number, the dividend, is divided

______ 6. any number in the set {0, 1, 2, 3, 4 …}

______ 7. a fraction whose numerator and denominator have no common factor greater than 1

A. dividend

B. divisor

C. product

D. quotient

E. reciprocals

F. simplest form

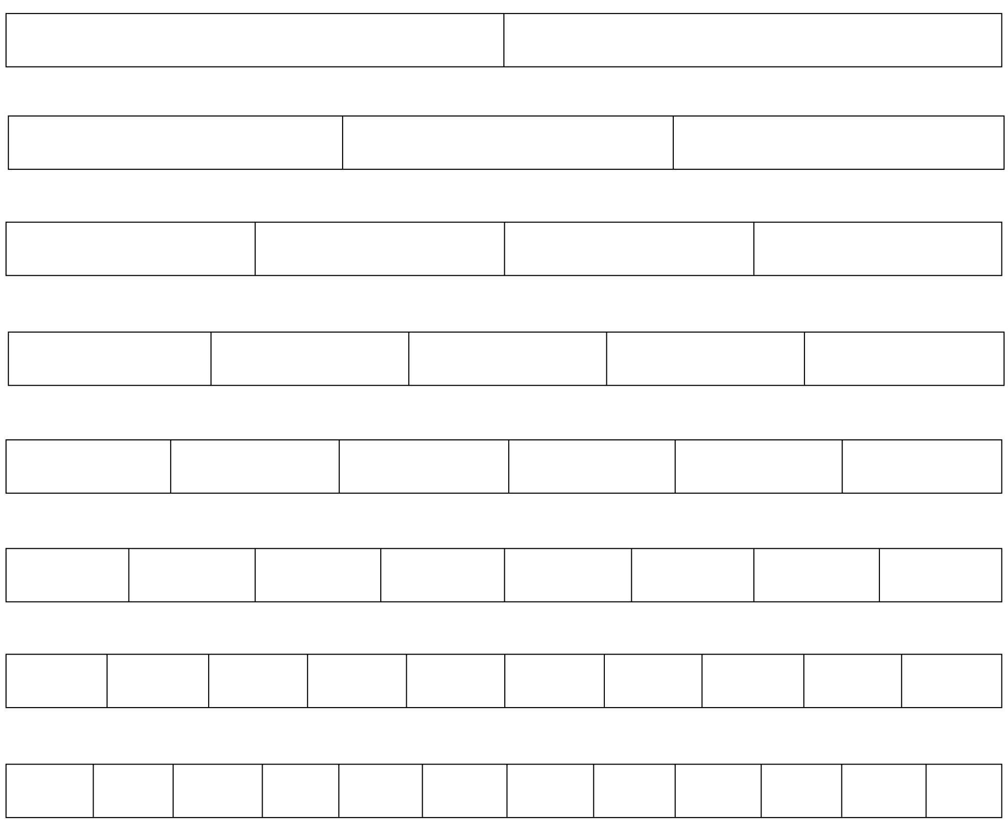
G. whole number

Lesson Two Purpose

- Understand concrete and symbolic representations of rational numbers in real-world situations. (A.1.3.3)
- Understand that numbers can be represented in a variety of equivalent forms, including fractions. (A.1.3.4)
- Add and subtract whole numbers and fractions, including mixed numbers, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Use concepts about numbers, including primes, factors, and multiples to build number sequences. (A.5.3.1)

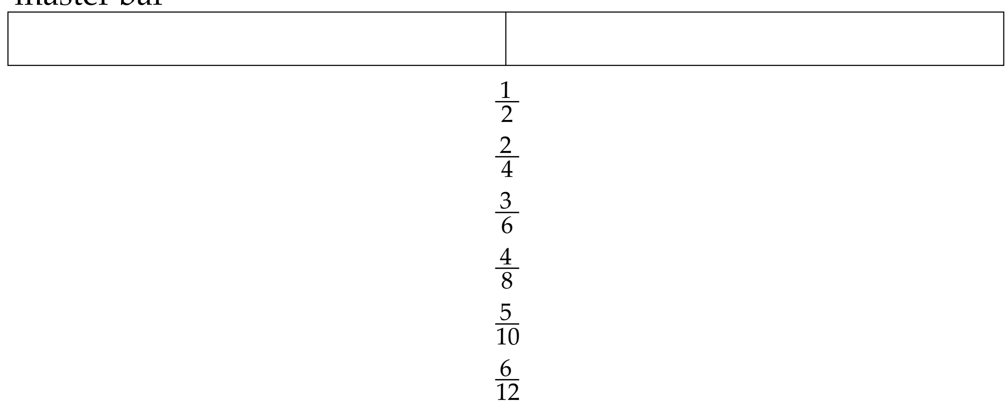
Sums and Differences of Fractional Parts

The candy bar models can be useful in understanding how to find **sums** and **differences** of fractional parts.



Complete the master bar below by transferring all markings and values from the other bars to it. The names you'll have for $\frac{1}{2}$ will also be $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$, and $\frac{6}{12}$ which are shown underneath the master bar.

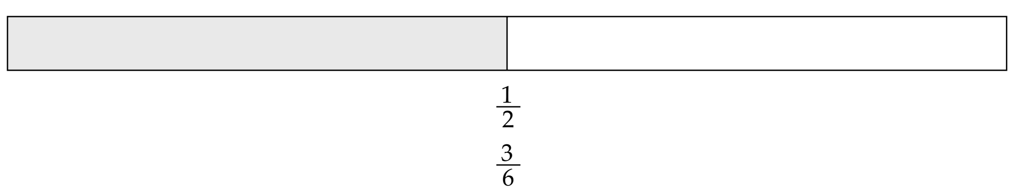
master bar

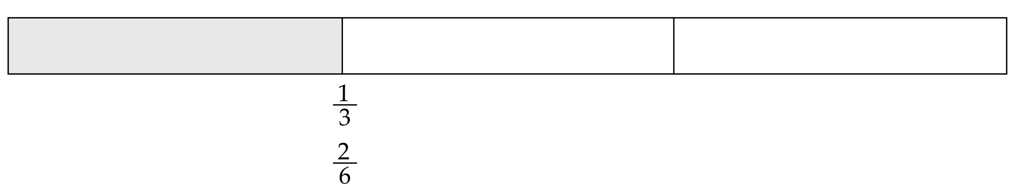


Finding Sums

Find the sum of $\frac{1}{2}$ and $\frac{1}{3}$.

- We know that from 0 to $\frac{1}{2}$ is $\frac{1}{2}$ unit in length.
- We could start at $\frac{1}{2}$ on the master bar. Then measure $\frac{1}{3}$ of a bar to the right of the $\frac{1}{2}$ mark because we are adding $\frac{1}{3}$ bar to the $\frac{1}{2}$ bar. We can see that the $\frac{1}{2}$ unit and the $\frac{1}{3}$ unit laid end to end on the master bar give us a length of $\frac{5}{6}$ unit.





- The denominator of 6 is a **multiple** of 2 and 3. A *multiple* is a number that results from multiplying a given number and any other whole number. And six is the *smallest number* into which 2 and 3 can be evenly divided.

  (**Note:** We can also start at $\frac{1}{2}$ and use a bar divided into 12 equal parts. We can see the $\frac{1}{2}$ unit and the $\frac{1}{3}$ unit laid end to end will also give us a length of $\frac{10}{12}$. We know that $\frac{10}{12}$ is equivalent to $\frac{5}{6}$.)

- We can line up the bars divided into two parts and six parts. We can see that $\frac{1}{2}$ and $\frac{3}{6}$ are the same length. They are *equivalent* fractions.

- We can line up the bars divided into three parts and six parts. We can see that $\frac{1}{3}$ and $\frac{2}{6}$ are the same length. They are *equivalent* fractions.

- $\frac{3}{6} + \frac{2}{6} = \frac{5}{6}$

Mike and Misty remember their textbook showing this problem done this way.

$$\begin{array}{r} \frac{1}{2} \times \frac{3}{3} = \frac{3}{6} \\ + \frac{1}{3} \times \frac{2}{2} = \frac{2}{6} \\ \hline \frac{5}{6} \end{array}$$

The bars helped them understand why they found the **common denominator** of 6 for $\frac{1}{2}$ and $\frac{1}{3}$. They also could better see that $\frac{1}{2}$ and $\frac{3}{6}$ are equal when using the bars. The same was true for $\frac{1}{3}$ and $\frac{2}{6}$.

(**Remember:** A *common denominator* is a **common multiple** of two or more denominators.)

Practice

Use your candy **bar fractional models** *from page 36 to help you find the following* **sums***. Write each answer in the* **simplest form***.*

1. $\frac{1}{2} + \frac{1}{5} =$ ____________
2. $\frac{4}{5} + \frac{1}{10} =$ ____________
3. $\frac{2}{5} + \frac{1}{10} =$ ____________
4. $\frac{3}{5} + \frac{1}{10} =$ ____________
5. $\frac{5}{6} + \frac{1}{12} =$ ____________
6. $\frac{1}{2} + \frac{1}{4} =$ ____________
7. $\frac{1}{2} + \frac{1}{6} =$ ____________
8. $\frac{1}{2} + \frac{1}{8} =$ ____________
9. $\frac{1}{2} + \frac{1}{10} =$ ____________
10. $\frac{1}{3} + \frac{1}{4} =$ ____________
11. $\frac{1}{3} + \frac{1}{6} =$ ____________
12. $\frac{1}{3} + \frac{1}{12} =$ ____________

13. $\frac{1}{4} + \frac{1}{12} =$ ____________

14. $\frac{2}{3} + \frac{1}{4} =$ ____________

15. $\frac{2}{3} + \frac{1}{6} =$ ____________

16. $\frac{2}{3} + \frac{1}{12} =$ ____________

17. $\frac{3}{4} + \frac{1}{6} =$ ____________

Looking for Patterns

Looking for patterns is often helpful in making discoveries and making mathematics more user friendly. Study the following **table**.

Common Denominators

Denominators of Fractions to be Added	Common Denominators
2, 4	4
2, 6	6
2, 8	8
2, 10	10
3, 6	6
3, 12	12
4, 12	12
5, 10	10
6, 12	12

Practice

Use the table on the previous page to answer the following.

1. When the larger denominator of the two fractional parts to be added was a multiple of the smaller denominator, the denominator in the sum was the ________________________ (smaller, larger) denominator.

or

2. When the smaller denominator divided evenly into the larger denominator, the denominator in the sum was the ________________________ (smaller, larger) denominator.

Study the following table to answer the following.

Common Denominator

Denominators of Fractions to be Added	Common Denominator
2, 5	10
3, 4	12
4, 6	12

3. When the larger denominator of the two fractional parts to be added was *not* a multiple of the smaller denominator, the denominator of the sum was the ________________________ (product, sum) of the two denominators if they had *no* common factor, which was true with 2, 5, and 3, 4. It was the ________________________ (product, sum) divided by their **greatest common factor (GCF)** if they have one, which was true with 4, 6.

(**Remember:** The *greatest common factor* is the largest of the *common factors* of two or more numbers.)

General Rule for Adding Fractions

Mike thinks there are too many rules but enjoys looking for patterns to put puzzle pieces together. Misty gives him a general rule that works for all cases of adding fractional parts.

Rule for adding fractional parts: To determine the common denominator for two fractional parts to be added, find the **least common multiple (LCM)** of the denominators of the fractional parts to be added.

(**Remember:** The *least common multiple* is the smallest of the *common multiples* of two or more numbers.)

For example:

Multiples and Common Denominator

Denominators of Fractions to Be Added with Some of Their Multiples		Common Denominator
2:	2, 4, 6, 8,10	10
5:	5, 10	
2:	2, 4, 6, 8	4
4:	4, 8	
2:	2, 4, 6	6
6:	6	
2:	2, 4, 6, 8, 10	10
10:	10	
3:	3, 6, 9, 12	12
4:	4, 8, 12	
3:	3, 6	6
6:	6	
3:	3, 6, 9, 12	12
12:	12	
4:	4, 8, 12	12
6:	6, 12	
4:	4, 8, 12	12
12:	12	
5:	5, 10	10
10:	10	
6:	6, 12	12
12:	12	

Further Illustration of Adding Fractions Rule

If we are adding fractions that may *not* be in simplest form, let's see what can happen.

$\frac{3}{6} + \frac{1}{4} =$

The LCM for 6 and 4 is 12. We rename $\frac{3}{6}$ to $\frac{6}{12}$ and $\frac{1}{4}$ to $\frac{3}{12}$.

$$\begin{array}{r} \frac{3}{6} \times \frac{2}{2} = \frac{6}{12} \\ + \frac{1}{4} \times \frac{3}{3} = \frac{3}{12} \\ \hline \frac{9}{12} \end{array}$$

We then add $\frac{6}{12}$ and $\frac{3}{12}$ producing a sum of $\frac{9}{12}$.

The 12 is the denominator in the *initial sum*, but it can be *simplified*. Both 9 and 12 have a factor of 3.

We know that $\frac{9}{12}$ is equivalent to $\frac{3}{4}$. The initial sum was $\frac{9}{12}$, but when simplified, the sum is $\frac{3}{4}$.

Mike and Misty remember discovering that the *product of the two denominators* of fractional parts to be added would *always* be a *common denominator*. They thought this was sometimes quicker than finding the least common multiple.

An example of this might be as follows.

$$\begin{array}{r} \frac{1}{4} = \frac{6}{24} \\ + \frac{1}{6} = \frac{4}{24} \\ \hline \frac{10}{24} = \frac{5}{12} \end{array}$$

They found the product of 4 and 6 was 24 and used it as their common denominator. Their initial sum of $\frac{10}{24}$ could be simplified.

Had they used the least common multiple of 4 and 6, for their denominator, this would not have been true. In this case, the answer would have already been expressed in lowest terms and would not have needed to be simplified.

$$\begin{array}{r} \frac{1}{4} = \frac{3}{12} \\ + \frac{1}{6} = \frac{2}{12} \\ \hline \frac{5}{12} \end{array}$$

You can see that there are choices in ways to solve problems. Do you have other suggestions? If so, please share them.

Study the following.

$$\begin{array}{rcccl} & \frac{3}{6} & \times & \frac{2}{2} & = \frac{6}{12} \\ + & \frac{1}{4} & \times & \frac{3}{3} & = \frac{3}{12} \\ \hline & & & & \frac{9}{12} = \frac{3}{4} \end{array}$$

With the problem $\frac{3}{6} + \frac{1}{4} =$, I know that $\frac{3}{6} = \frac{1}{2}$, so I change the problem to $\frac{1}{2} + \frac{1}{4} =$.

$$\begin{array}{rcccl} & \frac{1}{2} & \times & \frac{2}{2} & = \frac{2}{4} \\ + & \frac{1}{4} & \times & \frac{1}{1} & = \frac{1}{4} \\ \hline & & & & \frac{3}{4} \end{array}$$

Using Our Understanding of Models

Look at the problems on the next page. The set of candy bar models will only work if additional subdividing on strips is done and values are added to the ones you already have. Understanding the model often helps us find answers without the actual model. We find a common denominator for the two fractional parts being added. Then we use it to make addition of fractions with common denominators possible.

Example:

$$\frac{1}{3} + \frac{1}{5} =$$

If we measure $\frac{1}{5}$ unit from the $\frac{1}{3}$ mark, there is no mark for this value on our candy bar model. We could take the bar divided into 3 equal parts. Then we could divide each of the three into 5 to give us a total of 15. The candy bar model would then provide the sum since $\frac{1}{5} = \frac{3}{15}$ and $\frac{1}{3} = \frac{5}{15}$.

$$\frac{3}{15} + \frac{5}{15} = \frac{8}{15}$$

The same would be true if we started with the $\frac{1}{5}$ mark and measured $\frac{1}{3}$ unit to the right. We could divide each of the 5 equal parts into 3 parts for a total of 15 parts. We again see that $\frac{1}{5}$ bar plus $\frac{1}{3}$ bar is $\frac{8}{15}$ bar.

If we use our understanding of that model, we could do this.

$$\begin{array}{rcccl} & \frac{1}{5} & \times & \frac{3}{3} & = \frac{3}{15} \\ + & \frac{1}{3} & \times & \frac{5}{5} & = \frac{5}{15} \\ \hline & & & & \frac{8}{15} \end{array}$$

Practice

Complete the following showing your work. Refer to previous pages as needed. Write each answer in **simplest form**.

1. $\frac{1}{3} + \frac{1}{8}$

2. $\frac{1}{3} + \frac{1}{10}$

3. $\frac{1}{4} + \frac{1}{5}$

4. $\frac{1}{4} + \frac{1}{10}$

5. $\frac{1}{5} + \frac{1}{6}$

6. $\frac{1}{5} + \frac{1}{8}$

7. $\frac{1}{5} + \frac{1}{12}$

8. $\frac{1}{6} + \frac{1}{8}$

9. $\frac{1}{6} + \frac{1}{10}$

10. $\frac{1}{8} + \frac{1}{10}$

11. $\begin{array}{r} \frac{1}{8} \\ +\ \frac{1}{12} \\ \hline \end{array}$

12. $\begin{array}{r} \frac{1}{10} \\ +\ \frac{1}{12} \\ \hline \end{array}$

13. $\begin{array}{r} \frac{2}{3} \\ +\ \frac{1}{8} \\ \hline \end{array}$

14. $\begin{array}{r} \frac{3}{4} \\ +\ \frac{1}{5} \\ \hline \end{array}$

15. $\begin{array}{r} \frac{2}{5} \\ +\ \frac{3}{8} \\ \hline \end{array}$

16. $\begin{array}{r} \frac{5}{6} \\ +\ \frac{1}{8} \\ \hline \end{array}$

17. $\begin{array}{r} \frac{5}{6} \\ +\ \frac{1}{10} \\ \hline \end{array}$

18. $\begin{array}{r} \frac{3}{8} \\ +\ \frac{3}{10} \\ \hline \end{array}$

19. $\begin{array}{r} \frac{5}{8} \\ +\ \frac{3}{10} \\ \hline \end{array}$

20. $\begin{array}{r} \frac{3}{4} \\ +\ \frac{1}{5} \\ \hline \end{array}$

Sums of Two or More Fractions

The sum of two or more fractions may be less than 1, exactly 1, or more than 1.

Examples:

- Less than 1

$$\begin{array}{rcl} \frac{1}{2} & = & \frac{2}{4} \\ + \frac{1}{4} & = & \frac{1}{4} \\ \hline & & \frac{3}{4} \end{array}$$

- Exactly 1

$$\begin{array}{r} \frac{1}{2} \\ + \frac{1}{2} \\ \hline \frac{2}{2} = 1 \end{array}$$

- More than 1

$$\frac{1}{4} + \frac{7}{8} =$$

$$\begin{array}{rcccl} \frac{1}{4} & \text{x} & \frac{2}{2} & = & \frac{2}{8} \\ + \frac{7}{8} & & & = & \frac{7}{8} \\ \hline & & & & \frac{9}{8} = 1\frac{1}{8} \end{array}$$

Practice

Complete the following showing your work. Write each answer in **simplest form**.

1. $\frac{1}{2} + \frac{2}{3}$

2. $\frac{1}{2} + \frac{3}{5}$

3. $\frac{1}{2} + \frac{5}{6}$

4. $\frac{1}{2} + \frac{5}{8}$

5. $\frac{1}{2} + \frac{7}{10}$

6. $\frac{1}{2} + \frac{11}{12}$

7. $\frac{1}{3} + \frac{3}{4}$

8. $\frac{1}{3} + \frac{4}{5}$

9. $\frac{1}{3} + \frac{5}{6}$

10. $\frac{1}{3} + \frac{7}{8}$

11. $\begin{array}{r} \frac{1}{3} \\ +\frac{9}{10} \\ \hline \end{array}$

12. $\begin{array}{r} \frac{3}{4} \\ +\frac{3}{5} \\ \hline \end{array}$

13. $\begin{array}{r} \frac{3}{4} \\ +\frac{5}{6} \\ \hline \end{array}$

14. $\begin{array}{r} \frac{3}{4} \\ +\frac{7}{8} \\ \hline \end{array}$

15. $\begin{array}{r} \frac{3}{5} \\ +\frac{5}{6} \\ \hline \end{array}$

16. $\begin{array}{r} \frac{4}{5} \\ +\frac{3}{8} \\ \hline \end{array}$

17. $\begin{array}{r} \frac{4}{5} \\ +\frac{7}{10} \\ \hline \end{array}$

18. $\begin{array}{r} \frac{5}{6} \\ +\frac{3}{8} \\ \hline \end{array}$

19. Choose one of the problems on the previous pages to fully explain what you did and how you did it. Get someone to read what you have written. If there are questions or suggestions for how it might be improved, consider a revision.

____

____

____

____

Read by ____________________ .

Revision, if necessary.

____

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____

____

20. At this point, how user friendly are fractions for you? Why?

____

____

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Practice

Use the list below to write the correct term for each definition on the line provided.

common denominator	**multiples**
common multiple	**simplest form**
difference	**sum**
greatest common factor (GCF)	**table (or chart)**
least common multiple (LCM)	

________________________ 1. the result of an addition

________________________ 2. the result of a subtraction

________________________ 3. the numbers that result from multiplying a given number by the set of whole numbers

________________________ 4. a common multiple of two or more denominators

________________________ 5. an orderly display of numerical information in rows and columns

________________________ 6. the largest of the common factors of two or more numbers

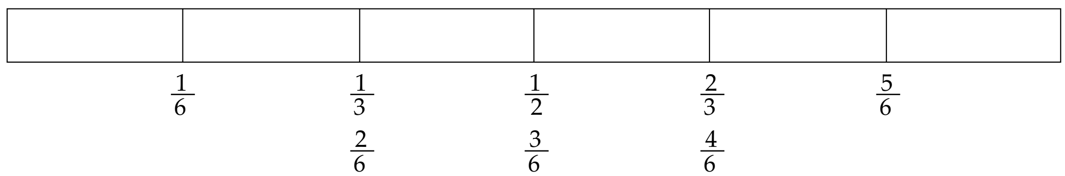
________________________ 7. the smallest of the common multiples of two or more numbers

________________________ 8. a number that is a multiple of two or more numbers

________________________ 9. a fraction whose numerator and denominator have no common factor greater than 1

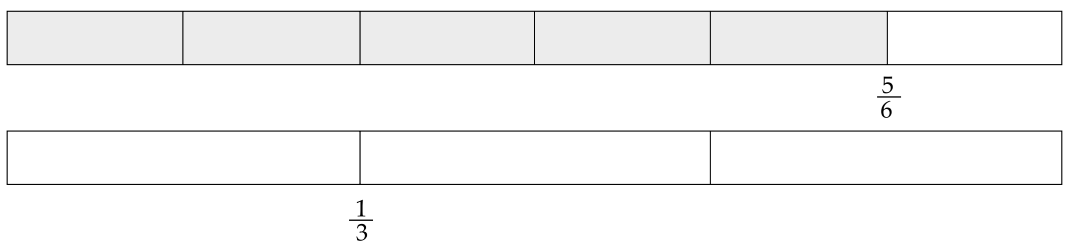
Subtracting Fractions

We can also use the candy bar models to subtract fractions.



Mike has $\frac{5}{6}$ candy bar and gives $\frac{1}{3}$ candy bar to Misty. **Please note that he is giving $\frac{1}{3}$ of a whole bar, *not* $\frac{1}{3}$ of his part of the bar!**

$\frac{5}{6}$ bar - $\frac{1}{3}$ bar =



We could start at $\frac{5}{6}$ on one bar. Then measure $\frac{1}{3}$ of a bar to the left because we are taking away $\frac{1}{3}$ bar from $\frac{5}{6}$ bar. Since we know that $\frac{1}{3}$ is the same as $\frac{2}{6}$, this is easily done. We would find a difference of $\frac{3}{6}$ or $\frac{1}{2}$.

Now, what if Mike has $\frac{5}{6}$ bar and gives Misty $\frac{2}{3}$ bar?

$\frac{5}{6}$ bar - $\frac{2}{3}$ bar =

We could start at $\frac{5}{6}$ and measure $\frac{2}{3}$ unit to the left. Since we know that $\frac{2}{3}$ is the same as $\frac{4}{6}$, it is easily done. We would find a difference of $\frac{1}{6}$.

We could also apply our understanding of those movements and do the following.

$$\begin{array}{rcccl} & \frac{5}{6} & \times & \frac{1}{1} & = \frac{5}{6} \\ - & \frac{1}{3} & \times & \frac{2}{2} & = \frac{2}{6} \\ \hline & & & & \frac{3}{6} \text{ or } \frac{1}{2} \end{array}$$

$$\begin{array}{rcccl} & \frac{5}{6} & \times & \frac{1}{1} & = \frac{5}{6} \\ - & \frac{2}{3} & \times & \frac{2}{2} & = \frac{4}{6} \\ \hline & & & & \frac{1}{6} \end{array}$$

We find the least common multiple of 6 and 3 and it is 6.

We rename each fraction so that each has a denominator of 6.

We can now reason that $\frac{5}{6}$ take away or minus $\frac{2}{6}$ is $\frac{3}{6}$. We can write $\frac{3}{6}$ in simplest form.

We can now reason that $\frac{5}{6}$ take away or minus $\frac{4}{6}$ is $\frac{1}{6}$.

Practice

Use the method of your choice to find the following **differences**. *Write each answer in* **simplest form**.

1. $\begin{array}{r} \frac{1}{2} \\ -\ \frac{1}{3} \\ \hline \end{array}$

2. $\begin{array}{r} \frac{2}{3} \\ -\ \frac{1}{2} \\ \hline \end{array}$

3. $\begin{array}{r} \frac{1}{2} \\ -\ \frac{1}{10} \\ \hline \end{array}$

4. $\begin{array}{r} \frac{3}{5} \\ -\ \frac{1}{2} \\ \hline \end{array}$

5. $\begin{array}{r} \frac{4}{5} \\ -\ \frac{1}{2} \\ \hline \end{array}$

6. $\begin{array}{r} \frac{7}{12} \\ -\ \frac{1}{4} \\ \hline \end{array}$

7. $\begin{array}{r} \frac{2}{3} \\ -\ \frac{5}{12} \\ \hline \end{array}$

8. $\begin{array}{r} \frac{5}{6} \\ -\ \frac{5}{12} \\ \hline \end{array}$

9. $\begin{array}{r} \frac{7}{8} \\ -\ \frac{3}{4} \\ \hline \end{array}$

10. $\begin{array}{r} \frac{5}{8} \\ -\ \frac{1}{2} \\ \hline \end{array}$

11. What was your method of choice? ____________________________

Why? ____

____

____

____

____

Does having a choice, in methods, make your work with fractions more user friendly? ____

Why or why not? ____

____

____

____

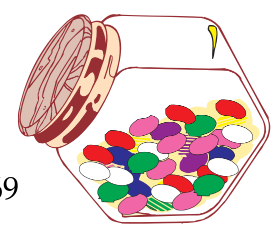
____

Lesson Three Purpose

- Associate verbal names, written word names, and standard numerals with decimals. (A.1.3.1)
- Understand relative size of decimals. (A.1.3.2)
- Understand that numbers can be represented in a variety of equivalent forms, including decimals. (A.1.3.4)
- Understand and explain the effects of multiplication on decimals. (A.3.3.1)
- Multiply whole numbers, decimals, and fractions, including mixed numbers, to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Use estimation strategies to predict results and check the reasonableness of results. (A.4.3.1)

Figuring Out Costs

Decimal Point Placement



Mike and Misty wanted a **chart** to show how much to charge a customer for a popular candy that sold for $1.69 per pound.

Mike remembers someone explaining a problem like this to him.

If 7 pounds of candy is purchased at $1.69 per pound, what is the total cost of the candy?

$1.69	(first factor)
x 7	(second factor)
$11.83	(product)
2 1	

A **decimal number** is any number written with a **decimal point** in the number. There are two **digits** to the right of the *decimal point* in the first factor (i.e., the cost of the candy, $1.69 per pound). The second factor has no decimal point. However, the decimal point is *understood* to be after the digit. That is a total of two digits to the right of the decimal point. In the answer, count two places from the right and place the decimal point. Your product should have two digits to the right of the decimal point.

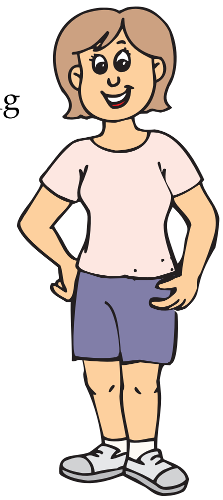
Practice

The following chart will only be helpful if **decimal points** *are placed correctly. Place decimal points on the chart below in the* **total cost per pound column**. *Use* **reasoning** *to determine the correct placement.*

Total Cost per Pound

Number of Pounds Purchased	Total Cost at $1.69 per Pound
1	$ 1 6 9
2	$ 3 3 8
3	$ 5 0 7
4	$ 6 7 6
5	$ 8 4 5
6	$ 1 0 1 4
7	$ 1 1 8 3
8	$ 1 3 5 2
9	$ 1 5 2 1
10	$ 1 6 9 0

Strategies to Produce Correct Results

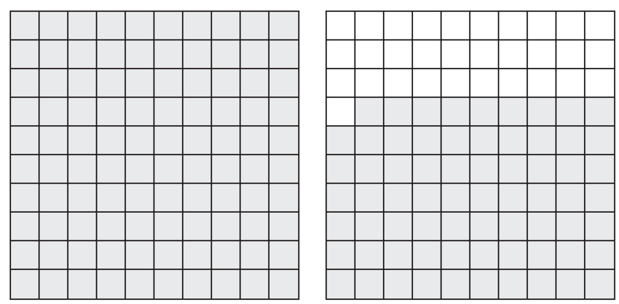


When she was young, Misty remembers someone explaining the problem to her like this.

You could write \$1.69 down 7 times and add.

$$
\begin{array}{r}
\$1.69 \\
\$1.69 \\
\$1.69 \\
\$1.69 \\
\$1.69 \\
\$1.69 \\
+\ \$1.69 \\
\hline
\$11.83
\end{array}
$$

She was also told that she could multiply.



1 or $\frac{100}{100}$ $\qquad$ $\frac{69}{100}$

Since

\$1.69 is the same as $1\frac{69}{100}$,

which is the same as $\frac{169}{100}$,

she could multiply

$\frac{169}{100} \times \frac{7}{1} = \frac{1183}{100} = 11.83$, and

the cost will be the same as when we wrote \$1.69 down 7 times and added \$11.83.

She also remembers being told this: First use **estimation** as a strategy to determine a *reasonable calculation*. Then place the decimal according to the estimate.

\$1.69 is closer to \$2 than it is to \$1.

$7 \times 2 = 14$

The cost should be less than \$14 but more than \$7.

If the product of 169 x 7 is 1183, then \$11.83 makes sense.

You are in a position to choose a way that is most user friendly to you as long as it produces correct results.

Practice

The following table provides information on the **cost** *of candies selling for $4.75 per pound and $5.25 per pound. Answer the following.*

1. Complete the table below.

Total Cost per Pound

Number of Pounds	Total Cost at $4.75 per Pound	Total Cost at $5.25 per Pound
1	$4.75	$5.25
2	$9.50	$10.50
3		
4		
5		
6		
7		
8		
9		
10		

2. If the cost of $4.75 is rounded to $5.00 for a quick estimate, the actual cost will be ____________ (<, =, >) the estimate.

3. If the cost of $5.25 is rounded to $5.00 for a quick estimate, the actual cost will be ____________ (<, =, >) the estimate.

4. Round \$4.75 to \$5.00 and \$5.25 to \$5.00. Then complete the following table of *estimated cost* to verify and check your responses to numbers 2 and 3 on the previous page.

Estimated Cost per Pound

Number of Pounds	Estimated Cost at \$4.75 per Pound	Estimated Cost at \$5.25 per Pound
1	\$5.00	\$5.00
2		
3		
4		
5		
6		
7		
8		
9		
10		

Practice

Challenge One

Mike and Misty have some challenges for you.

Sixteen multiplication problems can be written if each one of the **first factors** *in the chart is paired with each one of the* **second factors** *in the chart. A list of them has been started. Use the chart below the* **list of factors** *to continue until you have found all 16.*

First Factor	64	29	108	75
Second Factor	0.8	0.2	1.9	1.1

1. 64 x 0.8
2. 64 x 0.2
3. 64 x 1.9
4. 64 x 1.1
5. ______________________
6. ______________________
7. ______________________
8. ______________________
9. ______________________
10. ______________________
11. ______________________
12. ______________________
13. ______________________
14. ______________________
15. ______________________
16. ______________________

The products for the 16 problems are found in the following table.

Table of Products

51.2	23.2	15	82.5
205.2	12.8	55.1	121.6
70.4	5.8	21.6	86.4
118.8	142.5	31.9	60

How might you match the products with the problems without using a calculator or paper and pencil to actually do the multiplication?

Consider the following examples of reasoning. Fill in the blanks.

17. 64 x 0.8 is ____________ because
    - I know that 0.8 is less than but near 1, so the product must be less than but near 64.
    - I also know that 8 x 4 = 32, and the final digit in the product should be 2.
    - The product must therefore be 51.2.
18. 29 x 1.1 is ____________ because
    - I know that 1.1 is greater than but near 1, so the product must be greater than but near ____________ .
    - I also know that 9 x 1 = 9 so the final digit in the product should be ____________ .
    - The product must therefore be ____________ .

19. 75 x 0.2 is ____________ because

- I know that 0.2 is greater than 0 but near 0, so the product must be greater than but near 0.
- I also know that 2 x 5 = 10, so the final digit in the product should be zero. However, since this is a decimal number and the final digit is after the decimal point, it can be omitted.
- I know that 0.2 is the same as $\frac{2}{10}$ or $\frac{1}{5}$, and I know that $\frac{1}{5}$ of 75 is 15.
- The zero, I might expect, would make the product 15.0, and the 0 is not necessary.
- The product must therefore be ____________ .

20. 108 x 1.9 is ____________ because

- I know that 1.9 is less than but near 2, so the product should be less than twice 108 but near that amount.
- I also know that 9 x 8 = 72, so the final digit in the product should be 2.
- Since 108 and 1.9 are the two largest factors to be multiplied, the product should be the largest number on the Table of Products on the previous page.
- The product must therefore be ____________ .

Practice

Challenge Two

Use the same kind of **reasoning** *that you used in the previous practice. Determine the* **products** *for each of the 16 problems listed below that you found in that practice. Do* **not** *use a calculator or paper and pencil to multiply.*

Give at least **two reasons** *for your choice for each problem.*

After *you have completed the task, use a calculator or paper and pencil to* **check your answers**.

Table of Products

51.2	23.2	15	82.5
205.2	12.8	55.1	121.6
70.4	5.8	21.6	86.4
118.8	142.5	31.9	60

1. 64 x 0.8 = ____________

   Reasons: ____

   ____

2. 64 x 0.2 = ____________

   Reasons: ____

   ____

3. 64 x 1.9 = ____________

   Reasons: ____

   ____

4. 64 x 1.1 = ____________

Reasons: ____

____

5. 29 x 0.8 = ____________

Reasons: ____

____

6. 29 x 0.2 = ____________

Reasons: ____

____

7. 29 x 1.9 = ____________

Reasons: ____

____

8. 29 x 1.1 = ____________

Reasons: ____

____

9. 108 x 0.8 = ____________

Reasons: ____

____

10. 108 x 0.2 = ____________

Reasons: ____

____

11. 108 x 1.9 = ______________

Reasons: ____

____

12. 108 x 1.1 = ______________

Reasons: ____

____

13. 75 x 0.8 = ______________

Reasons: ____

____

14. 75 x 0.2 = ______________

Reasons: ____

____

15. 75 x 1.9 = ______________

Reasons: ____

____

16. 75 x 1.1 = ______________

Reasons: ____

____

Practice

Challenge Three

Mike thinks the task might be better if the **products** *were* **rearranged***. He wants you to list them from* **smallest to largest** *in the Table of Products below. Start in the* **upper left** *corner. Go* **across each row** *until the bottom right corner is reached to complete the rearrangement.*

Table of Products

5.8			21.6
23.2			
			205.2

Reflect: If you practice estimation and use other basic facts about numbers and operations with them, is your work with decimal numbers user friendly? Why or why not?

____

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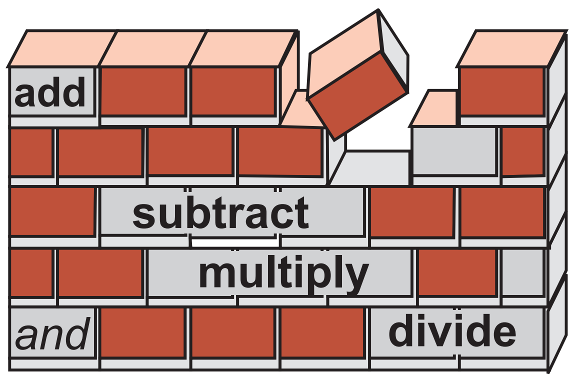
Lesson Four Purpose

- Associate verbal names, written word names, and standard numerals with fractions and decimals. (A.1.3.1)
- Understand concrete and symbolic representations of rational numbers in real-world situations. (A.1.3.3)
- Understand and explain the effects of addition, subtraction, multiplication and division on whole numbers, fractions, and decimals. (A.3.3.1)
- Select the appropriate operation to solve problems involving addition, subtraction, multiplication, and division of rational numbers. (A.3.3.2)

Problem Solving



The introduction to this unit talked about your need to know and be able to use accurate methods to add, subtract, multiply, and divide. These skills are also the foundations upon which problem solving is built.

You will use these skills as you solve the problems in this lesson. Keep in mind the following.

- A problem must be understood.
- Estimates are helpful.
- Knowing and using a variety of problem solving strategies will be helpful.
- Computational skill is necessary.
- Considering the reasonableness of the solution is important.

Consider this example.

> A customer in the candy store is purchasing 3 pounds of candy at $2.25 per pound and five items costing 10 cents each. He is paying with a $10 dollar bill. How much change will he receive if there is no tax?

Understanding the Problem

I know that the *total amount of the purchases* must be found. That total must be subtracted from $10.00 to determine the change.

Estimating a Solution

If $2.25 is **rounded** to $2.00, the estimated cost of 3 pounds would be $6.00. I know this is less than the actual cost since I rounded $2.25 down to $2.00.

Five items at 10 cents each would be 50 cents. A reasonable estimate for the total cost would be about $7.00. The change owed would be about $3.00.

Solving the problem

I review what I know.

cost of 3 pounds at $2.25 per pound = 3 x 2.25 = $6.75

cost of 5 items at 10 cents each = 5 x 0.10 = $0.50

total cost of purchases	$6.75
	+ $0.50
	$7.25
change owed to customer	$10.00
	- $ 7.25
	$ 2.75

Checking the Results

I ask myself—is the solution reasonable?

Yes, the total charged and the amount of change owed are supported by the estimates.

Practice

Answer the following showing your work **or** *explaining in words how you solved the problem.*

1. A customer in the candy store is purchasing 2 pounds of candy at \$3.00 per pound, $\frac{1}{2}$ pound of candy at \$4.50 per pound, and $\frac{1}{4}$ pound of candy at \$4.00 per pound. What is the **total cost of the purchases** if there is no tax?

   2 pounds at \$3.00 per pound
   $\frac{1}{2}$ pound at \$4.50 per pound
   $\frac{1}{4}$ pound at \$4.00 per pound

   Answer: ____________

2. A customer has \$20.00 and wants to purchase as many pounds as possible of the candy selling for \$2.50 per pound. What is the **maximum number of pounds** the customer can purchase?

   Answer: ____________

3. A customer has \$15.00 and wants four bags of candy marked \$3.50 each. How much money will the customer **have left** to use for other purchases?

   Answer: ____________

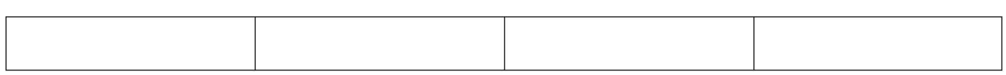
4. A customer buys $\frac{1}{2}$ pound of one candy, $\frac{1}{8}$ pound of another, and $\frac{1}{4}$ pound of another. What is the **total amount** of candy purchased?

Answer: ______________

5. A customer buys $\frac{3}{4}$ pound of peppermints and $\frac{3}{8}$ pound of chocolates. **How much greater** was the quantity of peppermints?

Answer: ______________

6. The candy store had a pan of fudge as shown in the diagram below. A customer purchased $\frac{1}{2}$ of $\frac{1}{4}$ of the pan. What **fractional part** of the pan did the customer purchase?



Answer: ______________

7. When Mike and Misty were in middle school, they went to a summer camp for two weeks. During camp, each camper had to take a turn helping in the kitchen. The cooks made meatballs for spaghetti sauce and used a recipe for 50 servings. There were 300 campers to feed. Rewrite the following recipe so that it serves 300 people rather than the 50 people it now serves.

Meatballs for 50 People

8 pounds of ground beef

$2\frac{1}{2}$ tablespoons of seasoning salt

$\frac{1}{2}$ teaspoon of pepper

8 eggs, beaten well

$1\frac{1}{4}$ cups of bread crumbs

2 tablespoons of Worcestershire sauce

Meatballs for 300 People

______________ pounds of ground beef

______________ tablespoons of seasoning salt

______________ teaspoons of pepper

______________ eggs, beaten well

______________ cups of bread crumbs

______________ tablespoons of Worcestershire sauce

You will use the information below to answer the following.

As Mike and Misty grew up, they liked to pretend that the candy M&Ms had been named for them since their first names begin with M. Since M&Ms originated in 1941 and their parents were not even born then, they knew it was **not** true. It is true, however, that they learned some math playing with M&Ms.

In 1995 the color blue was added to the colors in the packages of the plain variety of the candy. There was a plan for candy-color distribution. For every batch of 10 pieces of the candy, the plan is as follows.

Brown	Brown	Brown	Yellow	Yellow	Red	Red	Green	Orange	Blue

8. What fractional part of each batch of 10 pieces of candy is planned for each color?

   a. Brown ____________

   b. Yellow ____________

   c. Red ____________

   d. Green ____________

   e. Orange ____________

   f. Blue ____________

9. What fractional part of each batch of 50 pieces of candy is planned for each color?

   a. Brown ____________

   b. Yellow ____________

   c. Red ____________

   d. Green ____________

   e. Orange ____________

   f. Blue ____________

10. What fractional part of each batch of 100 pieces of candy is planned for each color?

   a. Brown ____________

   b. Yellow ____________

   c. Red ____________

   d. Green ____________

   e. Orange ____________

   f. Blue ____________

11. In the peanut variety, $\frac{3}{10}$ of the pieces of candy are blue. If you had 100 plain pieces of candy and 100 peanut pieces of candy, how many more peanut pieces of candy would be blue? ____________

12. In the peanut variety, $\frac{1}{10}$ of the pieces of candy are red. If you had 100 plain pieces of candy and 100 peanut pieces of candy, how many more plain pieces of candy would be red? ____________

13. In the peanut variety and in the almond variety, $\frac{2}{10}$ of the pieces of candy are brown. If you had 100 plain pieces of candy, 100 peanut pieces of candy, and 100 almond pieces of candy, how many brown pieces of candy would you have? ____________

14. In the Valentine's Day blend, $\frac{4}{10}$ are red, $\frac{4}{10}$ are pink, and $\frac{2}{10}$ are white. In a batch of 50 pieces of candy, how many would be red or white? ____________

15. In the peanut variety, the colors are planned as follows.

$\frac{1}{5}$ brown

$\frac{1}{10}$ red

$\frac{1}{10}$ green

$\frac{1}{10}$ orange

$\frac{3}{10}$ blue

_?_ yellow

What fractional part is yellow? ____________

16. In the almond variety, the colors are planned as follows.

$\frac{1}{5}$ brown

$\frac{1}{5}$ yellow

$\frac{1}{5}$ green

$\frac{1}{10}$ red

_?_ blue

What fractional part is blue? ____________

Equivalent Names for the Same Number

Study the following table for the Plain Variety Color Plan.

Plain Variety Color Plan

Color	Fractional Part	Fractional Part	Decimal Name for Fractional Part
Brown	$\frac{3}{10}$	$\frac{30}{100}$	0.30
Yellow	$\frac{2}{10}$	$\frac{20}{100}$	0.20
Red	$\frac{2}{10}$	$\frac{20}{100}$	0.20
Green	$\frac{1}{10}$	$\frac{10}{100}$	0.10
Orange	$\frac{1}{10}$	$\frac{10}{100}$	0.10
Blue	$\frac{1}{10}$	$\frac{10}{100}$	0.10

You can see that $\frac{3}{10} = \frac{30}{100} = 0.30$. These are *equivalent names for the same number*. They are correctly read as three-tenths, thirty-hundredths, and thirty-hundredths.

The sum of the fractional parts is $\frac{10}{10}$ or 1.

$$\frac{3}{10} + \frac{2}{10} + \frac{2}{10} + \frac{1}{10} + \frac{1}{10} + \frac{1}{10} = \frac{10}{10} \text{ or } 1$$

The sum of the fractional parts is $\frac{100}{100}$ or 1.

$$\frac{30}{100} + \frac{20}{100} + \frac{20}{100} + \frac{10}{100} + \frac{10}{100} + \frac{10}{100} = \frac{100}{100} \text{ or } 1$$

The sum of the decimal parts is 1.00.

$$0.30 + 0.20 + 0.20 + 0.10 + 0.10 + 0.10 = 1.00$$

Practice

Some errors have been made in the following table of the Peanut Variety Color Plan. In each row, **two** *entries are* **correct**, *but* **one** *is* **incorrect**. *Find the* **errors** *and* **correct** *them. Complete the statements below with the correct answers.*

Peanut Variety Color Plan

Color	Fractional Part	Fractional Part	Decimal Name for Fractional Part
Brown	$\frac{1}{5}$	$\frac{10}{100}$	0.20
Yellow	$\frac{2}{10}$	$\frac{2}{100}$	0.20
Red	$\frac{1}{10}$	$\frac{10}{100}$	0.01
Green	$\frac{2}{10}$	$\frac{10}{100}$	0.10
Orange	$\frac{1}{5}$	$\frac{10}{100}$	0.10
Blue	$\frac{3}{10}$	$\frac{30}{100}$	3.00

1. The brown ______________________ (fractional part/decimal name) should be ___________ .
2. The yellow ______________________ (fractional part/decimal name) should be ___________ .
3. The red ______________________ (fractional part/decimal name) should be ___________ .
4. The green ______________________ (fractional part/decimal name) should be ___________ .
5. The orange ______________________ (fractional part/decimal name) should be ___________ .
6. The blue ______________________ (fractional part/decimal name) should be ___________ .

Practice

Answer the following.

Mike and Misty think you are now ready to solve their most challenging problem for this unit. They think of it as the dessert! Take your time as you seek a solution. Enjoy each and every thing you do!

A customer comes into the store and says:

> "You may fill the following order for me. The cost of the candy must be more than $1.00 per pound and must cost less than $5.00 per pound. The number of pounds to be purchased is a two-digit whole number. You may use the digits 0, 1, 2, 3, and 4 one time each when writing the cost per pound and the number of pounds. You are to place the digits so that I spend the greatest amount possible."
>
> The clerk responds: "Give me one example so I'm sure I understand what you mean."
>
> Here's how the customer responded:
>
> - The candy could cost $1.20 per pound and I could be purchasing 34 pounds. For example: $1.20 x 34 = $40.80.
> - I used each of the digits once, the candy cost was more than $1.00 but less than $5.00, and I purchased 34 pounds. The total cost was $40.80.
> - However, that is not the greatest amount possible.

Mike found one solution to the problem, and Misty found a different one. Their solutions had different prices for the candy. They also had different amounts of the candy. However, they had the *same total amount* that the customer would spend.

1. You may use a calculator when finding your solution to this problem, but keep a written record of everything you do. How many examples did you try? ______________________________

   How are you sure you have found the greatest amount possible?

   ____

   ____

   ____

   ____

   ____

   ____

2. Reflect: As you solve problems involving fractions and decimal numbers, is your work with these numbers becoming more user friendly? Why or why not? ______________________________

   ____

   ____

   ____

   ____

   ____

Practice

Match each definition with the correct term. Write the letter on the line provided.

______ 1. a number that is to be divided by the divisor

______ 2. the result of a division

______ 3. any number representing some part of a whole; of the form $\frac{a}{b}$

______ 4. the numbers that result from multiplying a given number by the set of whole numbers

______ 5. a common multiple of two or more denominators

______ 6. the use of rounding and/or other strategies to determine a reasonably accurate approximation without calculating an exact answer

______ 7. the dot dividing a decimal number's whole part from its fractional part

______ 8. any number written with a decimal point in the number

______ 9. a number approximated to a specified place

______ 10. any one of the 10 symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9

A. common denominator

B. decimal number

C. decimal point

D. digit

E. dividend

F. estimation

G. fraction

H. multiples

I. quotient

J. rounded number

Unit 2: Measurement

This unit emphasizes how estimation and measuring are used to solve problems.

Unit Focus

Number Sense, Concepts, and Operations

- Understand the structure of number systems other than the decimal system. (A.2.3.2)
- Select the appropriate operation to solve problems involving addition, subtraction, multiplication, and division of rational numbers. (A.3.3.2)
- Add, subtract, multiply, and divide whole numbers, decimals, and fractions to solve real-world problems, using appropriate methods of computing, such as mental mathematics. (A.3.3.3)
- Use estimation strategies to predict results and to check reasonableness of results. (A.4.3.1)

Measurement

- Use concrete and graphic models to derive formulas for finding perimeter and area. (B.1.3.1)
- Use concrete and graphic models to derive formulas for finding rates. (B.1.3.2)
- Construct, interpret, and use scale drawings to solve real-word problems. (B.1.3.4)
- Use direct (measured) and indirect (not measured) measures to compare a given characteristic in either metric or customary units. (B.2.3.1)
- Solve problems involving units of measure and convert answers to a larger or smaller unit within either the metric or customary system. (B.2.3.2)
- Solve real-world and mathematical problems involving estimates of measurements. (B.3.3.1)

- Select appropriate units of measurement. (B.4.3.1)
- Select and use appropriate instruments and techniques to measure quantities. (B.4.3.2)

Algebraic Thinking

- Describe a wide variety of patterns, relationships, and functions through models. (D.1.3.1)

Vocabulary

Study the vocabulary words and definitions below.

area (*A*) .... the inside region of a two-dimensional figure measured in square units
Example: A rectangle with sides of four units by six units contains 24 square units or has an area of 24 square units.

base (*b*) .... the line or plane upon which a figure is thought of as resting

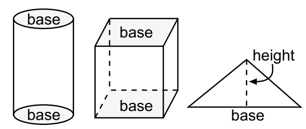


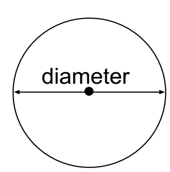
chart .... see *table*

data .... information in the form of numbers gathered for statistical purposes

decimal number .... any number written with a decimal point in the number
Example: A decimal number falls between two whole numbers, such as 1.5 falls between 1 and 2. Decimal numbers smaller than 1 are sometimes called decimal fractions, such as five-tenths is written 0.5.

decrease .... to make less

diameter (*d*) .... a line segment from any point on the circle passing through the center to another point on the circle

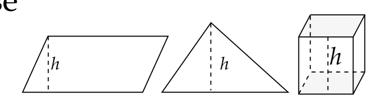


difference .... the result of subtraction
Example: In 16 - 9 = 7, 7 is the difference.

digit .... any one of the 10 symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9

estimation .... the use of rounding and/or other strategies to determine a reasonably accurate approximation without calculating an exact answer

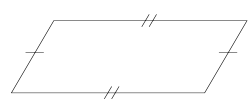
height (*h*) .... a line segment extending from the vertex or apex (highest point) of a figure to its base and forming a right angle with the base or basal plane



increase .... to make greater

length (*l*) .... a one-dimensional measure that is the measurable property of line segments

parallelogram .... a quadrilateral with two pairs of parallel sides



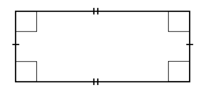
pattern (relationship) .... a predictable or prescribed sequence of numbers, objects, etc.; also called a *relation* or *relationship*; may be described or presented using manipulatives, tables, graphics (pictures or drawings), or algebraic rules (functions)
Example: 2, 5, 8, 11...is a pattern. Each number in this sequence is three more than the preceding number. Any number in this sequence can be described by the algebraic rule, $3n - 1$, using the set of counting numbers for n.

perimeter (*P*) .... the length of the boundary around a figure; the distance around a polygon

positive numbers .... numbers greater than zero

quotient .... the result of a division
Example: In 42 ÷ 7 = 6, 6 is the quotient.

rectangle .... a parallelogram with four right angles



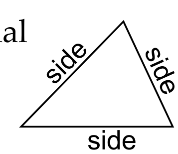
relationship (relation) .... see *pattern*

rounded number .... a number approximated to a specified place
Example: A commonly used rule to round a number is as follows.

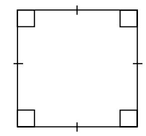
- If the digit in the first place after the specified place is 5 or more, *round up* by adding 1 to the digit in the specified place ($\underline{4}61$ rounded to the nearest hundred is 500).
- If the digit in the first place after the specified place is less than 5, *round down* by *not* changing the digit in the specified place ($\underline{4}41$ rounded to the nearest hundred is 400).

scale model .... a model or drawing based on a ratio of the dimensions for the model and the actual object it represents
Example: a map

side .... the edge of a two-dimensional geometric figure
Example: A triangle has three sides.



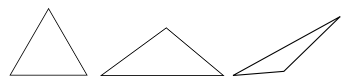
square .... a rectangle with four sides the same length



square units .... units for measuring area; the measure of the amount of an area that covers a surface

table (or chart) .... an orderly display of numerical information in rows and columns

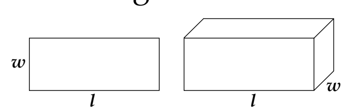
triangle .... a polygon with three sides



unit (of length) .... a precisely fixed quantity used to measure measurement in inches, feet, yards, and miles; centimeters, meters, and kilometers

weight .... measures that represent the force that attracts an object to the center of Earth; in the customary system, the basic unit of weight is the pound

width (*w*) .... a one-dimensional measure of something side to side



Unit 2: Measurement

Introduction

There is a book called *Spaghetti and Meatballs for All* by Marilyn Burns. The story is about a couple getting ready for a family reunion. Thirty-two family members will be coming. The couple sets up 8 tables with 4 chairs at each table.

The couple sets up 8 tables with 4 chairs at each table.

Family members arrive a few at a time. The family members begin rearranging the tables. At this, the host couple becomes alarmed. They know they have plenty of spaghetti and meatballs for all. But they also know there will not be seats for all with the changes being made in table arrangements.

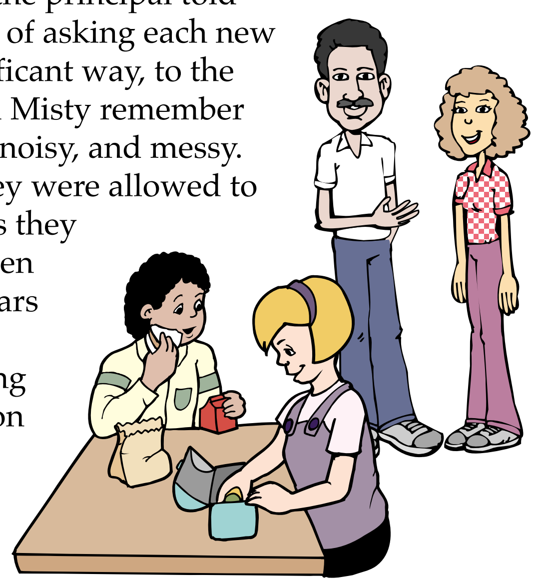
This unit will help you understand what the couple knew when they set up for the reunion. It will also help you understand what the guests did not think about when they began rearranging the tables.

Lesson One Purpose

- Select the appropriate operation to solve problems involving addition, subtraction, multiplication, and division of rational numbers. (A.3.3.2)
- Add, subtract, multiply, and divide whole numbers, decimals, and fractions to solve real-world problems, using appropriate methods of computing, such as mental mathematics. (A.3.3.3)
- Use concrete and graphic models to derive formulas for finding perimeter and area. (B.1.3.1)
- Construct, interpret, and use scale drawings to solve real-world problems. (B.1.3.4)
- Use direct (measured) and indirect (not measured) measures to compare a given characteristic in either metric or customary units. (B.2.3.1)
- Describe a wide variety of patterns, relationships, and functions through models. (D.1.3.1)

Choosing Arrangements

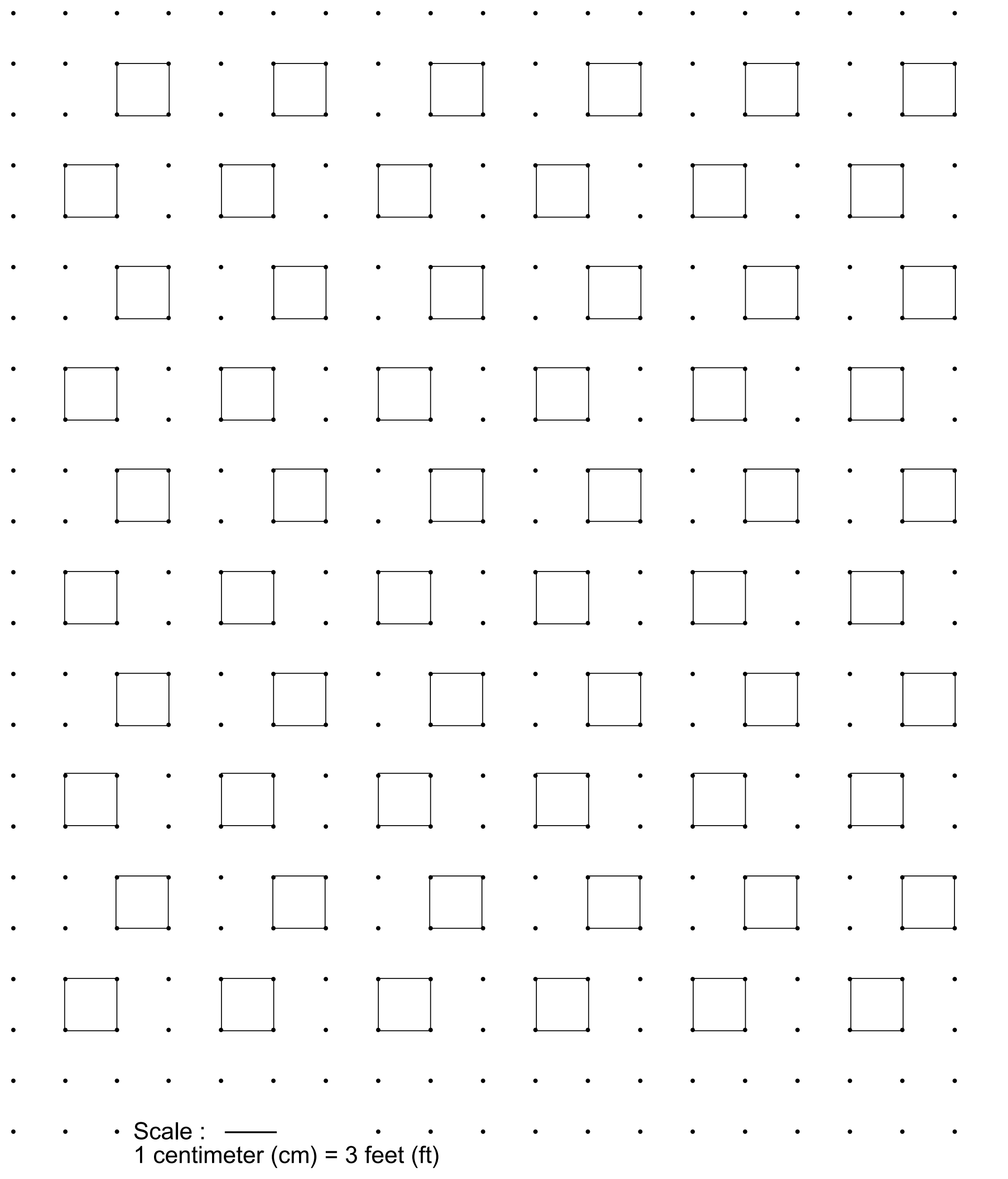
When Misty and Mike were hired, the principal told them that the school has a tradition of asking each new teacher to contribute, in some significant way, to the total school environment. Mike and Misty remember their school cafeterias as being big, noisy, and messy. They remember that some years they were allowed to choose a place to sit and other years they were not. They remember years when their teachers ate with them and years when the teachers ate in a teacher's lounge. They don't remember having any pictures or interesting posters on the walls. They decide that if they work together and involve their students, the cafeteria might be a good choice for their project.



The cafeteria now has 60 square tables with 4 chairs at each table. The tables are not large enough to seat more than 4, so students are not allowed to move a fifth or sixth chair to their table to accommodate friends. The tables are so close together, it is hard to get around in the cafeteria. Each grade level goes to lunch at a different time, and the grade levels have from 195 to 215 students. The school has 6^{th}, 7^{th}, and 8^{th} grades. See the **scale model** or drawing of the table arrangements on the next page.

Mike and Misty would like to arrange the tables so that more than 4 people could be accommodated at some tables.

- They go to the cafeteria and move two tables together. They find that 6 people could be seated with this arrangement, 2 on each side and 1 on each end.
- They add a third table and find that 8 people could be seated, 3 on each side and 1 on each end.
- They quickly see that when they put two tables together, 6 people can be seated instead of the 8 when the two tables are separate. They lose 2 seats.
- Misty is a bit hot and tired and tells Mike that making a *scale drawing* on square dot paper would be easier than moving tables. If each square represents a table and 1 student can sit at a side of each table, what could be easier?
- Mike reminds Misty that he is a "hands on" person. If he can't move tables and chairs, then he wants to use some square tiles to *model* the problem.
- Misty and Mike head back to Mike's classroom. He gets 60 tiles out for himself and some square dot paper out for Misty. They begin their work together.


Scale :
1 centimeter (cm) = 3 feet (ft)

Practice

Answer the following. Use the **square dot paper** *on the following page as requested.*

1. If 60 tables are arranged separately and each seats 4 students, the cafeteria now seats ____________ students. Draw one table on square dot paper. Underneath your square, write 1 table, 4 people.

2. If two tables are arranged together, 6 students can now be seated. Draw 2 tables placed end-to-end and underneath this arrangement, write 2 tables, 6 people.

3. If three tables are placed together, end-to-end, ____________ students can now be seated. Illustrate and label this as you did for the previous questions.

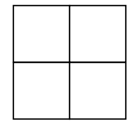
4. If four tables are placed together, end-to-end, ____________ students can now be seated. Illustrate and label this as you did for the previous questions.

5. If two tables are placed together, end-to-end, and two more are placed by those side-to-side, a square arrangement of four tables results. This would seat ____________ students. Illustrate and label this as you did for the previous questions.

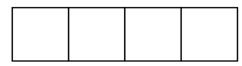
Practice

Answer the following.

Mike and Misty decide that the square arrangement of four tables puts so much distance between the students, they would have to speak pretty loudly to be heard. They also see that fewer students are seated with this arrangement than with four tables end-to-end.



Mike and Misty also decide that if more than four tables are placed together, end-to-end, conversation becomes difficult except for the students sitting nearest you. They decide no more than four tables should be placed together.



1. If Mike and Misty arranged all 60 of the tables in sets of two tables, end-to-end, there would be __________ sets. Six students could sit at each of these sets for a total of __________ .

2. If Mike and Misty arranged all 60 of the tables in sets of three tables, end-to-end, there would be __________ sets. Eight students could sit at each of these sets for a total of __________ .

3. If Mike and Misty arranged all 60 of the tables in sets of four tables, end-to-end, there would be __________ sets. Ten students could sit at each of these sets for a total of __________ .

4. Mike and Misty realize that they must seat at least 215 students, so there will have to be some single tables with seats for 4. They decide to organize their work in a **chart** or **table** to try to find as many possibilities as possible. Complete the chart they have started on the following page.

Seating for Groups of 4 Students and 6 Students

Number and Type of Table Arrangements and Number of Seats for People	Number and Type of Table Arrangements and Number of Seats for People	Total Number of People in Column 1 and 2
60 tables placed separately 60 x 4 = 240 people	0	240
58 tables placed separately 58 x 4 = 232 people	1 set of two tables end-to-end 1 x 6 = 6 people	238
56 tables placed separately 56 x 4 = 224 people	2 sets of two tables end-to-end ______ = ______ people	______
54 tables placed separately 54 x 4 = 216 people	3 sets of two tables end-to-end ______ = ______ people	______
52 tables placed separately ______ = ______ people	4 sets of two tables end-to-end ______ = ______ people	______
50 tables placed separately ______ = ______ people	5 sets of two tables end-to-end ______ = ______ people	______
48 tables placed separately ______ = ______ people	6 sets of two tables end-to-end ______ = ______ people	______
46 tables placed separately ______ = ______ people	7 sets of two tables end-to-end ______ = ______ people	226
44 tables placed separately ______ = 176 people	8 sets of two tables end-to-end ______ = ______ people	______
42 tables placed separately ______ = ______ people	9 sets of two tables end-to-end ______ = ______ people	______
40 tables placed separately ______ = ______ people	10 sets of two tables end-to-end ______ = ______ people	______
38 tables placed separately ______ = ______ people	11 sets of two tables end-to-end ______ = ______ people	______
36 tables placed separately 36 x 4 = ______ people	12 sets of two tables end-to-end ______ = ______ people	______
34 tables placed separately 34 x 4 = 136 people	13 sets of two tables end-to-end ______ = ______ people	214

Since they must be able to seat at least 215 people, the last entry in the chart above is not helpful nor would additional entries be helpful.

The chart "Seating for Groups of 4 Students and 6 Students" on the previous page will be useful to Misty and Mike if the number of students in a grade level increases. The chart would help them with rearrangements to meet needs of additional students. They can also see that if they chose to have tables to seat 4 or 6, they could have a *maximum* (greatest number) of 12 arrangements to seat 6, and all other tables would need to seat 4.

5. Misty's and Mike's work continues as they look at placing three tables end-to-end. Complete the following chart.

Seating for Groups of 4 Students and 8 Students

Number and Type of Table Arrangements and Number of Seats for People	Number and Type of Table Arrangements and Number of Seats for People	Total Number of People in Column 1 and 2
60 tables placed separately 60 x 4 = 240 people	0	240
57 tables placed separately 57 x 4 = 228 people	1 set of three tables end-to-end 1 x 8 = 8 people	______
54 tables placed separately 54 x 4 = 216 people	2 sets of three tables end-to-end __________ = ______ people	______
51 tables placed separately __________ = ______ people	3 sets of three tables end-to-end __________ = ______ people	______
48 tables placed separately __________ = ______ people	4 sets of three tables end-to-end __________ = ______ people	______
45 tables placed separately __________ = ______ people	5 sets of three tables end-to-end __________ = ______ people	______
42 tables placed separately __________ = ______ people	6 sets of three tables end-to-end __________ = ______ people	______
39 tables placed separately 39 x 4 = 156 people	7 sets of three tables end-to-end __________ = 56 people	212

Since seating must accommodate at least 215, they don't need to continue the chart.

6. If Misty and Mike decided to use only tables for 4 or tables for 8, (arranged end-to-end) they could have a maximum of ____________ arrangements for 8, and all other tables would have to be for 4.

7. They now want to consider the four tables, end-to-end, to seat 10. Complete the following chart.

Seating for Groups of 4 Students and 10 Students

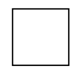
Number and Type of Table Arrangements and Number of Seats for People	Number and Type of Table Arrangements and Number of Seats for People	Total Number of People in Column 1 and 2
60 tables placed separately 60 x 4 = 240 people	0	240
56 tables placed separately ________ = ______ people	1 set of four tables end-to-end 1 x 10 = 10 people	234
52 tables placed separately ________ = ______ people	2 sets of four tables end-to-end ________ = ______ people	______
48 tables placed separately ________ = ______ people	3 sets of four tables end-to-end ________ = ______ people	______
44 tables placed separately 44 x 4 = 176 people	4 sets of four tables end-to-end ________ = 40 people	216
40 tables placed separately 40 x 4 = 160 people	5 sets of four tables end-to-end ________ = 50 people	210

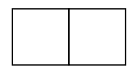
We have again reached a stopping point in the chart if we must seat at least 215 people.

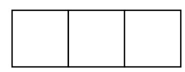
Mike and Misty notice their charts get shorter as the number of tables being placed together end-to-end increase. They can see that if they tried five tables or six tables, few students would have an option of sitting at tables for more than 4.

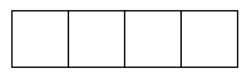
8. If Misty and Mike decided to use only tables for 4 or tables for 10, they could have a maximum of ____________ arrangements for 10, and all other tables would have to be for 4.

They now think of a mixture of arrangements of tables end-to-end, for 4, 6, 8, and 10.

Each arrangement for 4 requires 1 table. 

Each arrangement for 6 requires 2 tables. 

Each arrangement for 8 requires 3 tables. 

Each arrangement for 10 requires 4 tables. 

If they had one arrangement each for 6, 8, and 10, the number of tables required would be 2 tables + 3 tables + 4 tables = 9 tables.

Those arrangements would seat 6 + 8 + 10 or 24 people in total. They would have 51 tables for 4 seating 204 people. This would make a total of 228 people.

9. If they had 2 arrangements each for 6, 8, and 10, the number of tables required would be 2 (2 + 3 + 4) = ____________ tables.

10. That would seat 2 (6 + 8 + 10) = ____________ people.

The remaining 42 tables for 4 would seat ____________ for a total of ____________ people.

11. How many seats would be provided if the 60 tables were set up as follows?

43 tables placed separately = ____________

4 tables placed in 2 sets of two = ____________

9 tables placed in 3 sets of three = ____________

4 tables in 1 set of four = ____________

= ____________ total seats

12. Can you find another way to combine arrangements for 4, 6, 8, and 10 students seating as close to 215 as possible? ________________

____

____

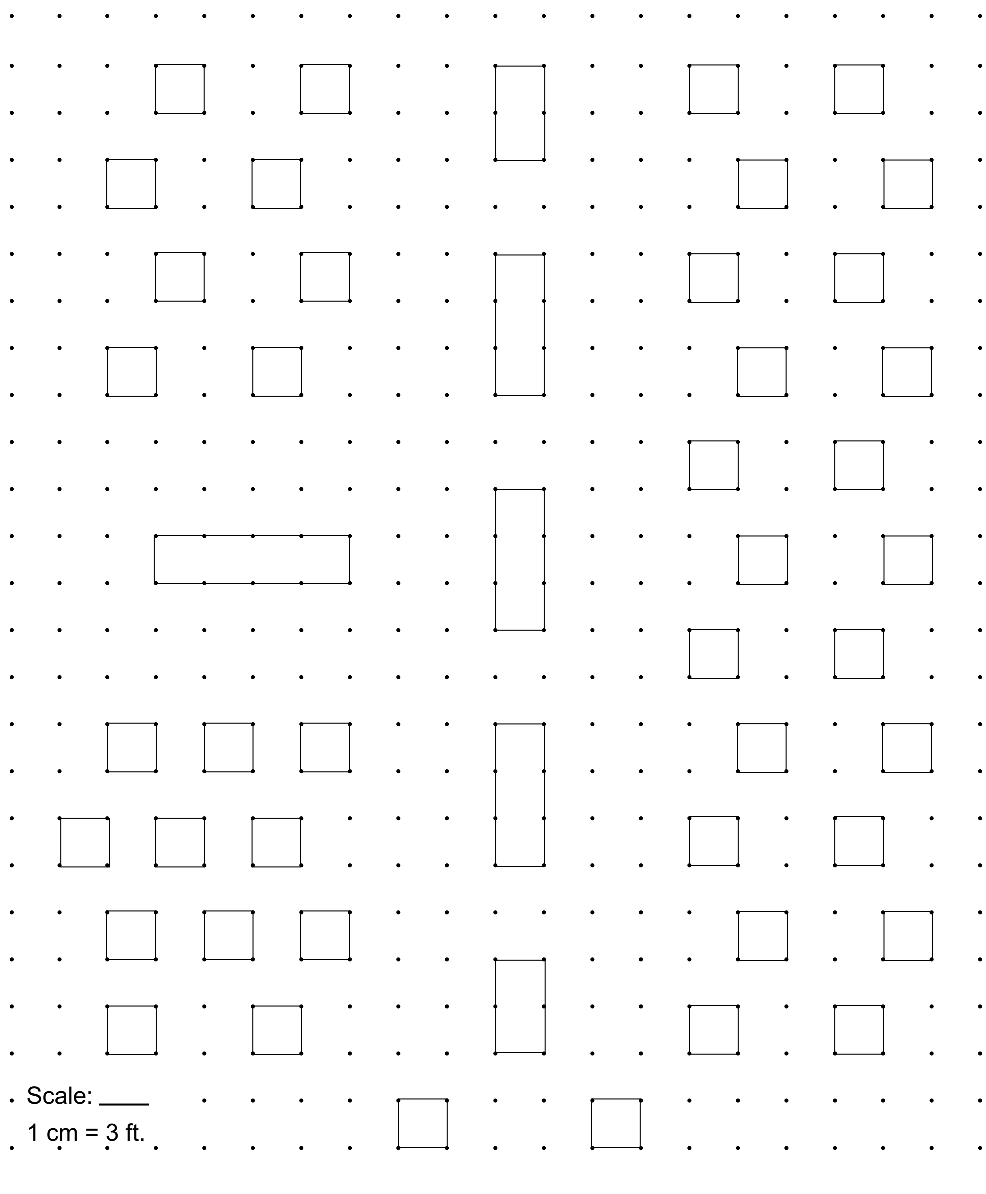
____

____

____

13. Using the square dot paper on the following page, show how you would arrange the tables using a total of 60 tables to seat as close to 215 people as possible. The cafeteria should be a maximum of 20 units wide and 27 units long.

14. Study the scale drawing on the square dot paper on the next page. On your own paper, write a summary explaining the table arrangement plan as shown on that scale drawing. Include in it the number of tables for 4, 6, 8, and 10 students and the total number of students the arrangement accommodates.



15. If each **unit (of length)** on the square dot paper represents 3 feet in the cafeteria, the **width (*w*)** of the cafeteria is ____________ feet and the **length (*l*)** is ____________ feet. Each table has a **side** measure of ____________ feet and an **area (*A*)** of ____________ square feet.

(**Remember:** The area (A) of a rectangle equals length (l) times width (w), $A = lw$. The area of a square equals one side (s) squared, s^2. Also remember that **square units** are the measure of an area that covers a surface.)

Practice

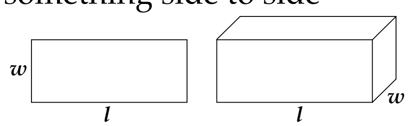
Match each definition with the correct term. Write the letter on the line provided.

______ 1. a model or drawing based on a ratio of the dimensions for the model and the actual object it represents

______ 2. an orderly display of numerical information in rows and columns

______ 3. a precisely fixed quantity used to measure measurement in inches, feet, yards, and miles; centimeters, meters, and kilometers

______ 4. a one-dimensional measure of something side to side



______ 5. a one-dimensional measure that is the measurable property of line segments

______ 6. the inside region of a two-dimensional figure measured in square units

______ 7. units for measuring area; the measure of the amount of an area that covers a surface

______ 8. the edge of a two-dimensional geometric figure

A. area (A)

B. length (l)

C. scale model

D. side

E. square units

F. table

G. unit (of length)

H. width (w)

Practice

Answer the following.

Using the **data** in our charts will help us in problem solving. Finding **patterns** or **relationships** in data is also helpful. Use your charts from the previous practice to complete the following statements.

1. In the chart for seating for 4 students and 6 students:

   a. As the number of sets of two tables **increases** by 1, the number of tables placed separately **decreases** by ____________ .

   b. As the number of sets of two tables increases by 1, the total number of seats for people decreases by ____________ .

   c. If 20 sets of two tables were arranged, the number of seats available would decrease by ____________ .

   d. The cafeteria now seats 240 people, and it must seat a *minimum* (least number) of 215 with any arrangement created. The reduction in seats can be a *maximum* (greatest number) of ____________ .

   e. If we eliminate 2 seats each time we put two tables together, then the maximum number of times we could do this and *not* exceed the maximum reduction is ____________ .

2. In the chart for seating for 4 students and 8 students:

   a. As the number of sets of three tables increases by 1, the number of tables placed separately decreases by ____________ .

   b. As the number of sets of three tables increases by 1, the total number of seats for people decreases by ____________ .

   c. If 10 sets of three tables were arranged, the number of seats available would decrease by ____________ .

   d. The cafeteria now seats 240 people, and it must seat a minimum of 215 with any arrangement created. The reduction in seats can be a maximum of ____________ .

   e. If we eliminate 4 seats each time we put three tables together, then the maximum number of times we could do this and *not* exceed the maximum reduction is ____________ .

3. In the chart for seating for 4 students and 10 students:

   a. As the number of sets of four tables increases by 1, the number of tables placed separately decreases by ____________ .

   b. As the number of sets of four tables increases by 1, the total number of seats for people decreases by ____________ .

c. If 10 sets of four tables were arranged, the number of seats available would decrease by ____________ .

d. The cafeteria now seats 240 people, and it must seat a minimum of 215 with any arrangement created. The reduction in seats can be a maximum of ____________ .

e. If we eliminate 6 seats each time we put four tables together, then the maximum number of times we could do this and *not* exceed the maximum reduction is ____________ .

4. If we eliminate 2 seats when placing two tables together, 4 seats when placing three tables together, and 6 seats when placing four tables together:

   a. ____________ seats are eliminated when we use 9 of the 60 tables to make
   one arrangement for 6,
   one for 8, and
   one for 10.
   The total number of seats under this plan would be

   (51 x 4) + (1 x 6) + (1 x 8) + (1 x 10) = ____________ .

b. ____________ seats are eliminated when we use 18 of the 60 tables to make

two arrangements for 6,

two for 8, and

two for 10.

The total number of seats under this plan would be

(42 x 4) + (2 x 6) + (2 x 8) + (2 x 10) = ____________ .

c. ____________ seats are eliminated when we use 17 of the 60 tables to make

three arrangements for 10,

one for 8, and

one for 6.

The total number of seats under this plan would be

(43 x 4) + (3 x 10) + (1 x 8) + (1 x 6) = ____________ .

d. ____________ seats are eliminated when we use 21 of the 60 tables to make

seven arrangements for 6,

one for 8, and

one for 10.

The total number of seats under this plan would be

(39 x 4) + (7 x 6) + (1 x 8) + (1 x 10) = ____________ .

e. The sum of the number of seats eliminated and the total number of seats under the plan for part *a* of this question is ____________ ; for part *b* is ____________ ; for part *c* is ____________ ; and for part *d* is ____________ .

f. In part *a* of this question, 9 of the 60 tables were used for arrangements which left ____________ tables to be placed separately seating 4 people each.

g. In part *b* of this question, 18 of the 60 tables were used for arrangements which left ____________ tables to be placed separately seating 4 people each.

h. In part *c* of this question, 17 of the 60 tables were used for arrangements which left ____________ tables to be placed separately seating 4 people each.

i. In part *d* of this question, 21 of the 60 tables were used for arrangements which left ____________ tables to be placed separately seating 4 people each.

Practice

Using what you have learned in this unit, **create a plan** *for the cafeteria that is* **different** *from the one you created in the practice on pages 95-102. Tell which plan was* **easier** *for you to create and* **why**. *Include a* **scale drawing** *of your plan using the square dot paper below.*

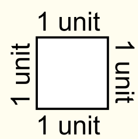
Practice

Answer the following.

The cafeteria currently has 60 tables, placed separately, and 240 places for students to sit.

If both the length and width of an individual table is 1 unit, then the area of a table is 1 square unit, and the **perimeter (*P*)** of the table is 4 units. These *dimensions* help to describe the table. This is a **square**, 1 unit on each side or a 1 by 1 square.

Perimeter is the distance around a figure.
Perimeter (P) = 2 length (l) + 2 width (w)
$P = 2l + 2w$
So for this square:
$P = 2(l) + 2(l)$
$P = 2(1) + 2(1)$
$P = 4$ units



Area measures the inside of a figure in square units.
Area (A) = length (l) x width (w)
$A = lw$
So for this square:
$A = 1 \times 1$
$A = 1$ square unit

In this case, the number of people the table can seat and the number of units in the *perimeter* of the table are both 4.

The following chart organizes our table arrangements.

1. Complete the following chart.

One to Four Tables and Seating Arrangements

Dimensions of Table Arrangements	Number of Square Tables Required for Arrangement or the Area	Number of People Arrangement Can Seat or the Perimeter
1 by 1	1 table or 1 square unit	4 people or 4 units
1 by 2	____ tables or ____ square units	____ people or ____ units
1 by 3	____ tables or ____ square units	____ people or ____ units
1 by 4	____ tables or ____ square units	____ people or ____ units

2. The following chart represents a number of ways to arrange the 60 tables into one large **rectangular** arrangement that Mike and Misty are now considering. The area will always be 60 because all of the tables are being used. Find the number of seats for people or the *perimeter* for each of them.

Sixty Tables and Seating Arrangements

Dimensions of Table Arrangements	Number of Square Tables Required for Arrangement or the Area	Number of People Arrangement Can Seat or the Perimeter
1 by 60	60 tables or 60 square unit	122 people or 122 units
2 by 30	_______ tables or _______ square units	_______ people or _______ units
3 by 20	_______ tables or _______ square units	_______ people or _______ units
4 by 15	_______ tables or _______ square units	_______ people or _______ units
5 by 12	_______ tables or _______ square units	_______ people or _______ units
6 by 10	_______ tables or _______ square units	_______ people or _______ units

3. Study the chart on the previous page to find one or more relationships between the dimensions of the rectangular arrangement of tables and the area. Write an explanation about the relationships you find.

____

____

____

____

____

____

4. Study the chart on the previous page to find one or more relationships between the dimensions of the rectangular arrangement of tables and the perimeter of the arrangement. Write an explanation about the relationships you find.

____

____

____

____

____

____

Practice

Use the list below to write the correct term for each definition on the line provided.

data	**perimeter (*P*)**
decrease	**rectangle**
increase	**square**
pattern	

____________________ 1. a parallelogram with four right angles

____________________ 2. a rectangle with four sides the same length

____________________ 3. the length of the boundary around a figure; the distance around a polygon

____________________ 4. to make greater

____________________ 5. to make less

____________________ 6. a predictable or prescribed sequence of numbers, objects, etc.; also called a *relation* or *relationship*

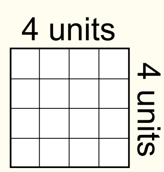
____________________ 7. information in the form of numbers gathered for statistical purposes

Lesson Two Purpose

- Use concrete and graphic models to derive formulas for finding perimeter and area. (B.1.3.1)
- Use direct (measured) and indirect (not measured) measures to compare a given characteristic in either metric or customary units. (B.2.3.1)
- Describe a wide variety of patterns, relationships, and functions through models. (D.1.3.1)

Finding Perimeter and Area

Mike and Misty remember being confused when they found answers for the *area* and *perimeter* of a square with a side measure of 4 units. The perimeter was 16 units, and the area was 16 square units.



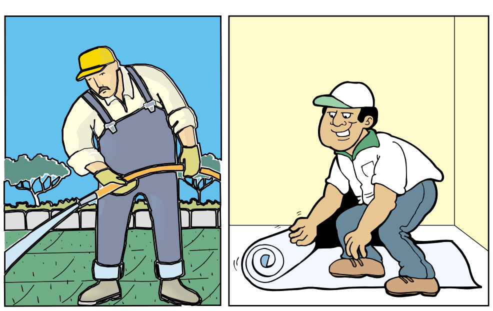
perimeter = 16 units
area = 16 square units

Perimeter

Mike and Misty suggest that you think of perimeter as the length of a piece of string required to go around the figure or the length of the fence required to go around or enclose the area.

Area

The area is the number of square units required to cover the enclosed space. Examples of this might be the amount of squares of sod to cover the back yard or the number of square units of carpet to cover the floor. As you write the number of units for your answers for perimeter and the number of square units for your answers for area, think of examples to help visualize what you are writing.



Examples of area might be the amount of squares of sod to cover the back yard or the number of square units of carpet to cover the floor.

Practice

Answer the following.

A series of squares drawn on square dot paper are provided on the following page. Use them to complete the following chart for you to complete. An analysis of your work will follow.

You may want to use tiles, or squares of paper, to make models of the squares that are drawn on the square dot paper on the following page.

Do the following for each square to complete the chart below.

- Determine the length of a side of the square.
- Find the perimeter of the square.
- Find the area of the square.

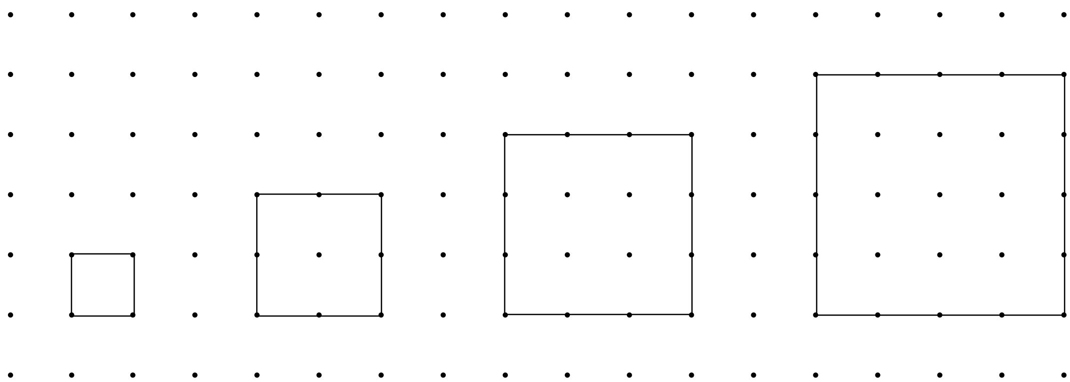
1. Complete the following chart.

Perimeter and Area of Squares

Side Measure of Square	Perimeter of Square	Area of Square
1 unit	4 units	1 square unit
_______ units	_______ units	_______ square units
_______ units	_______ units	_______ square units
_______ units	_______ units	_______ square units

2. I found the perimeter by ____ .

3. I found the area by ____ .



4. If I wanted to determine the perimeter of a square with a side measure of 10 units, I would ________________________________

____ .

The perimeter would be ____________ units.

5. If I wanted to determine the area of a square with a side measure of 10 units, I would ____

____.

The area would be ____________ square units.

6. When the side measure was increased from 1 unit to 2 units, the side measure doubled. What effect did that have on the perimeter?

____

____

7. When the side measure was increased from 2 units to 4 units, the side measure doubled. What effect did that have on the perimeter?

____

____

8. When the side measure was increased from 1 unit to 3 units, the side measure tripled. What effect did that have on the perimeter?

____

____

9. When the side measure was increased from 1 unit to 2 units, the side measure doubled. What effect did that have on the area?

____

____

10. When the side measure was increased from 2 units to 4 units, the side measure doubled. What effect did that have on the area?

____

____

11. When the side measure was increased from 1 unit to 3 units, the side measure tripled. What effect did that have on the area?

____

____

12. A square with a side measure of 3 units has a perimeter of 12 units and an area of 9 square units. If the side measure is tripled, the perimeter would be __________ units and the area would be __________ square units.

Practice

Answer the following.

A series of rectangles drawn on square dot paper are provided on the following page. Use them to complete the following table. An analysis of your work will follow.

You can put tiles or squares of paper on your desk to make the rectangles drawn on the square dot paper on the following page.

Do the following for each rectangle to complete the chart below.

- Determine the length and width of the rectangle.
- Find the perimeter of the rectangle.
- Find the area of the rectangle.

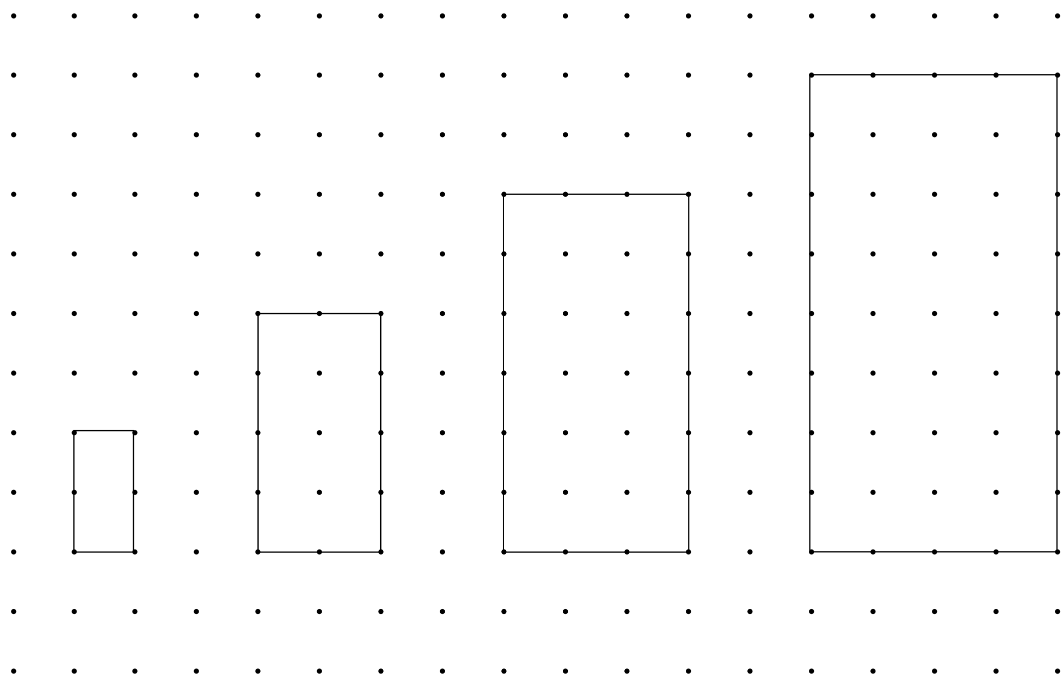
1. Complete the following chart.

Perimeter and Area of Rectangles

Length of Rectangle	Width of Rectangle	Perimeter of Rectangle	Area of Rectangle
1 unit	2 units	6 units	2 square units
______ units	______ units	______ units	______ square units
______ units	______ units	______ units	______ square units
______ units	______ units	______ units	______ square units

2. I found the perimeter by ________________________________ .

3. I found the area by ____________________________________ .



4. If I wanted to determine the perimeter of a rectangle with a length of 10 units and a width of 20 units, I would ______________________

   ____ .

   The perimeter would be ____________ units.

5. If I wanted to determine the area of a rectangle with a length of 10 units and a width of 20 units, I would _________________________

   ____ .

   The area would be ____________ units.

6. When the length was increased from 1 unit to 2 units and the width was increased from 2 units to 4 units, they doubled. How did this affect the perimeter? ____

   ____

7. When the length was increased from 2 units to 4 units and the width was increased from 4 units to 8 units, they doubled. How did this affect the perimeter? ____

____

8. When the length was increased from 1 unit to 3 units and the width was increased from 2 units to 6 units, they tripled. How did this affect the perimeter? ____

____

9. When the length was increased from 1 unit to 2 units and the width was increased from 2 units to 4 units, they doubled. How did this affect the area? ____

____

10. When the length was increased from 2 units to 4 units and the width was increased from 4 units to 8 units, they doubled. How did this affect the area? ____

____

11. When the length was increased from 1 unit to 3 units and the width was increased from 2 units to 6 units, they tripled. How did this affect the area? ____

____

12. If a length of 3 units was tripled to 9 units and the width of 6 units was tripled to 18 units, I would expect the perimeter to be ____________ units and the area to be ____________ square units.

Practice

Answer the following.

1. Using tiles, squares of paper, square dot paper for drawing, or some other method, determine the area and perimeter of the following rectangles in the chart below.

(**Remember:** A square is also a rectangle.)

Perimeter and Area

Length	Width	Perimeter	Area
4 units	5 units	18 units	20 square units
6 units	6 units	____ units	____ square units
7 units	7 units	____ units	____ square units
8 units	8 units	____ units	____ square units
1 unit	3 units	____ units	____ square units
2 units	6 units	____ units	____ square units
3 units	9 units	____ units	____ square units
4 units	12 units	____ units	____ square units

2. Draw pictures on the square dot paper on the following page and then write a paragraph to explain how the perimeter of the 8 by 8 square and the 4 by 12 rectangle can be the same but their areas can be different.

Explanation: ____

____

____

____

____

____

____

____

Practice

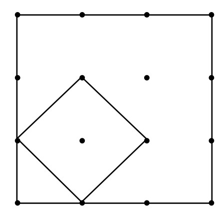
Answer the following.

Mike and Misty remember competing on math teams and having questions like the one that follows. They are anxious to see how you like it.

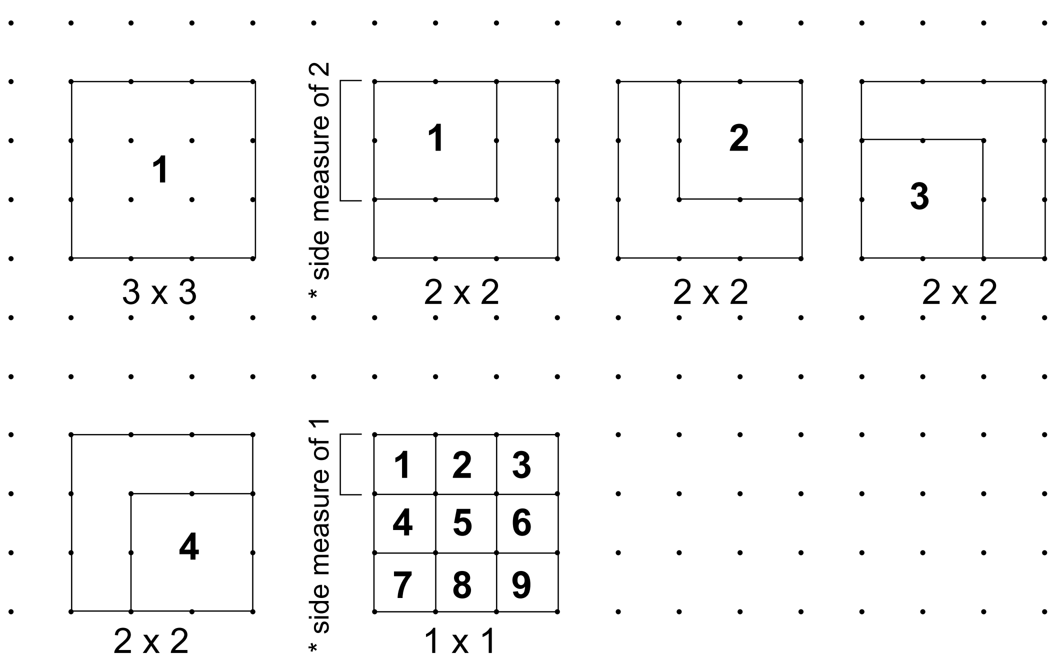
1. How many squares can be found in the set of squares below? Complete the table on the following page. Do not include the "tilted squares" in your count. (The sides of a tilted square are not parallel to the sides of the original square.) Consider only squares with horizontal and vertical squares.

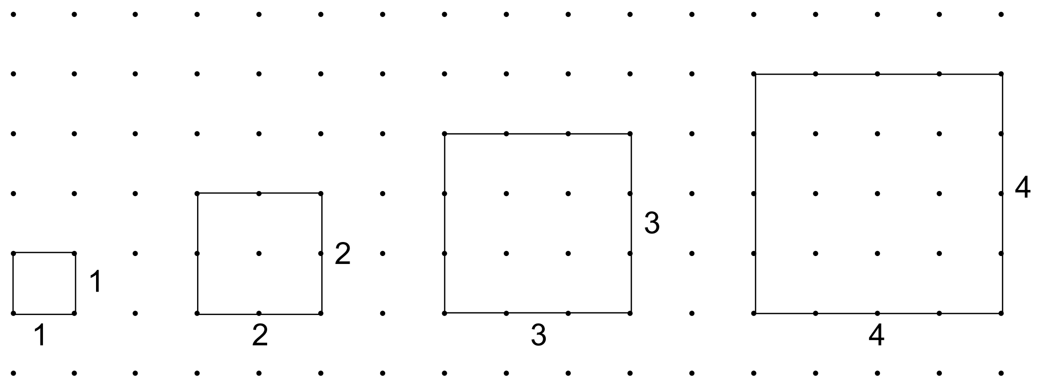
The 3 by 3 square is illustrated for you.

- You can see 1 square that is 3 by 3.
- You can see 4 squares that are 2 by 2.
- You can see 9 squares that are 1 by 1.
- That is a total of 14 squares.



Do not include the "tilted squares."





Number of Squares

Side Measure	Number of 1 by 1 Squares	Number of 2 by 2 Squares	Number of 3 by 3 Squares	Number of 4 by 4 Squares	Total Number of Squares
1 unit	______	______	______	______	______
2 units	______	______	______	______	______
3 units	9	4	1	0	14
4 units	______	______	______	______	______
Total	______	______	______	______	______

2. Look for patterns in your table to find relationships. Use the table to help organize your thinking and work. Write an explanation about the patterns you found and their relationships.

Explanation: ____

____

____

____

____

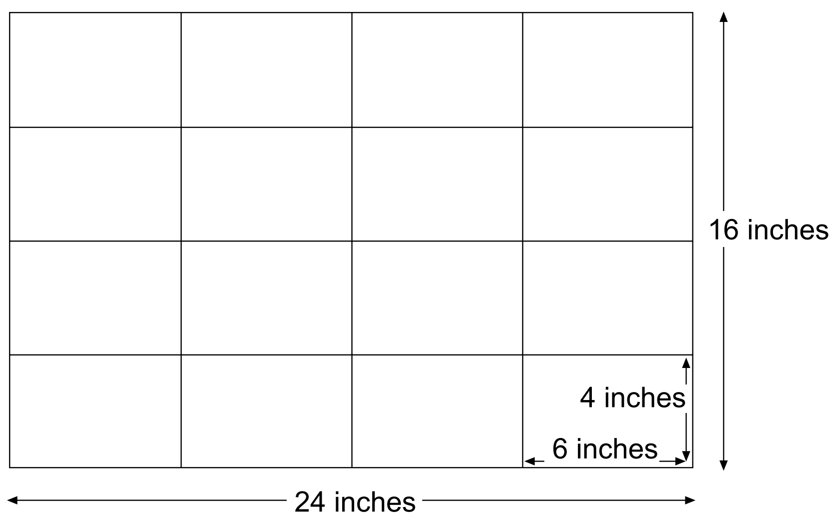
____

Practice

Answer the following.

Misty and Mike's favorite cafeteria meal was pizza. The pizzas were made in large rectangular pans and cut into rectangular pieces. They would like for you to explore a variety of ways the pizza might be cut.

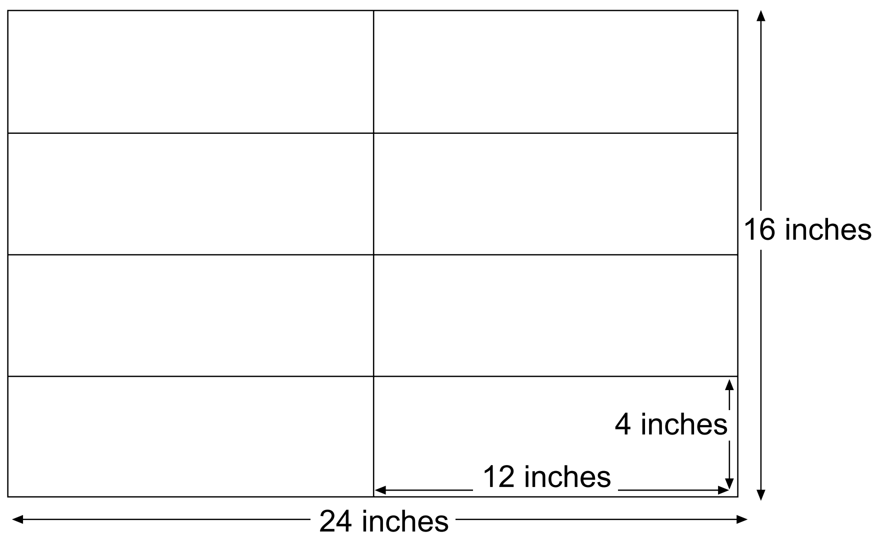
- The pans in the school cafeteria are 16 inches wide and 24 inches long.
- The pizza slices are rectangular in shape with a width of 4 inches and a length of 6 inches.
- Each pan yields 16 slices of pizza.



1. The area of the 16-inch by 24-inch pan is ____________ square inches.

2. The area of a 4-inch by 6-inch slice is ____________ square inches.

3. Draw a diagonal in each rectangle to show how each of the following rectangular pieces could be divided into two triangular (**triangle-shaped**) pieces. Remember that the pan is 16 inches by 24 inches. Each rectangle is now 4 inches by 12 inches.



4. The area of each of the 8 rectangular pieces in question 3 is

    ____________ square inches. When each rectangle was divided into two equal parts, each of which was a *triangle*, the area became one-half the original amount or ____________ square inches.

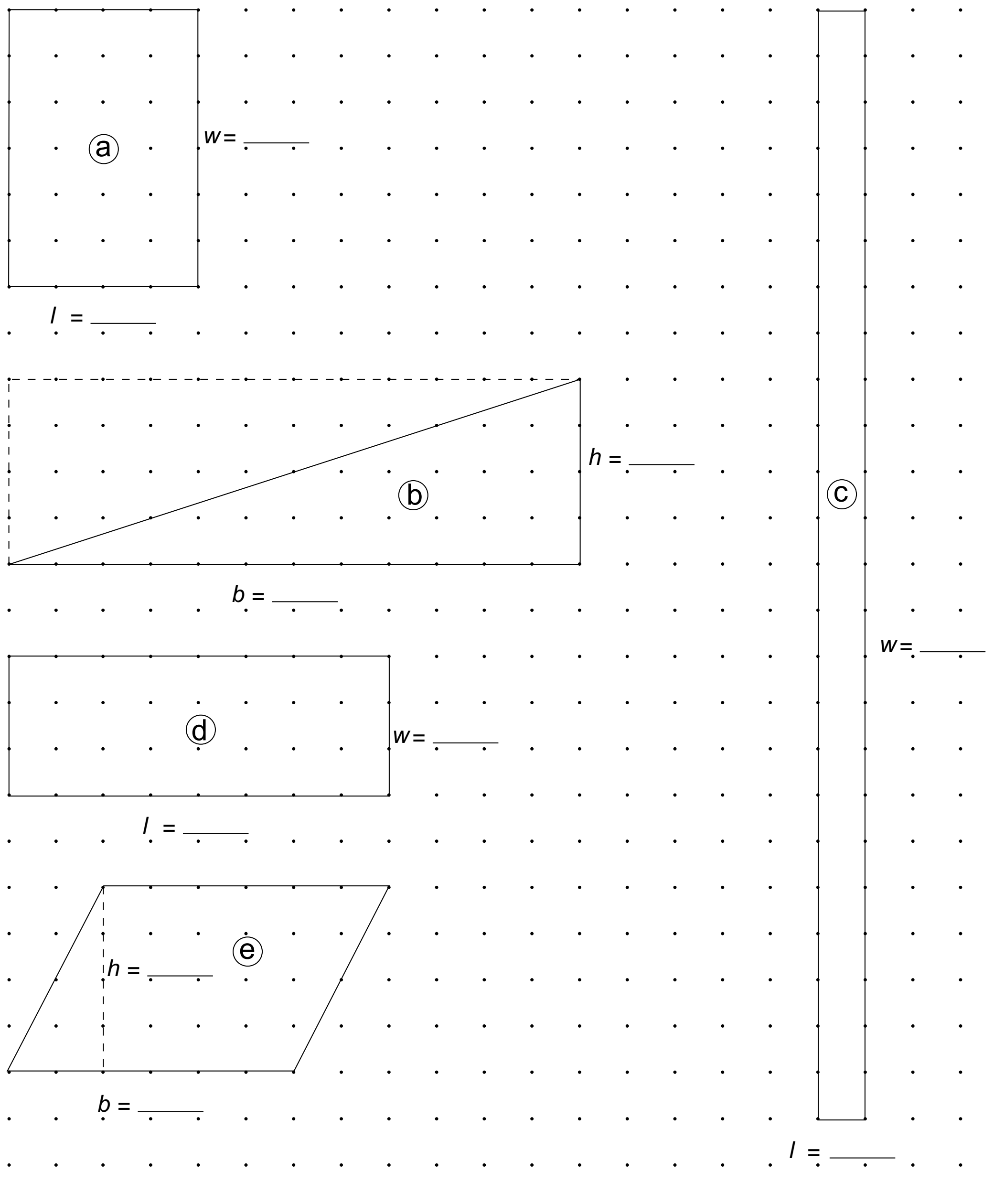
Refer to the illustrations of pizza slices on the following pages to answer the following. **Note:** *or represent 1 unit,* but *the distance along a diagonal is greater than 1 unit.*

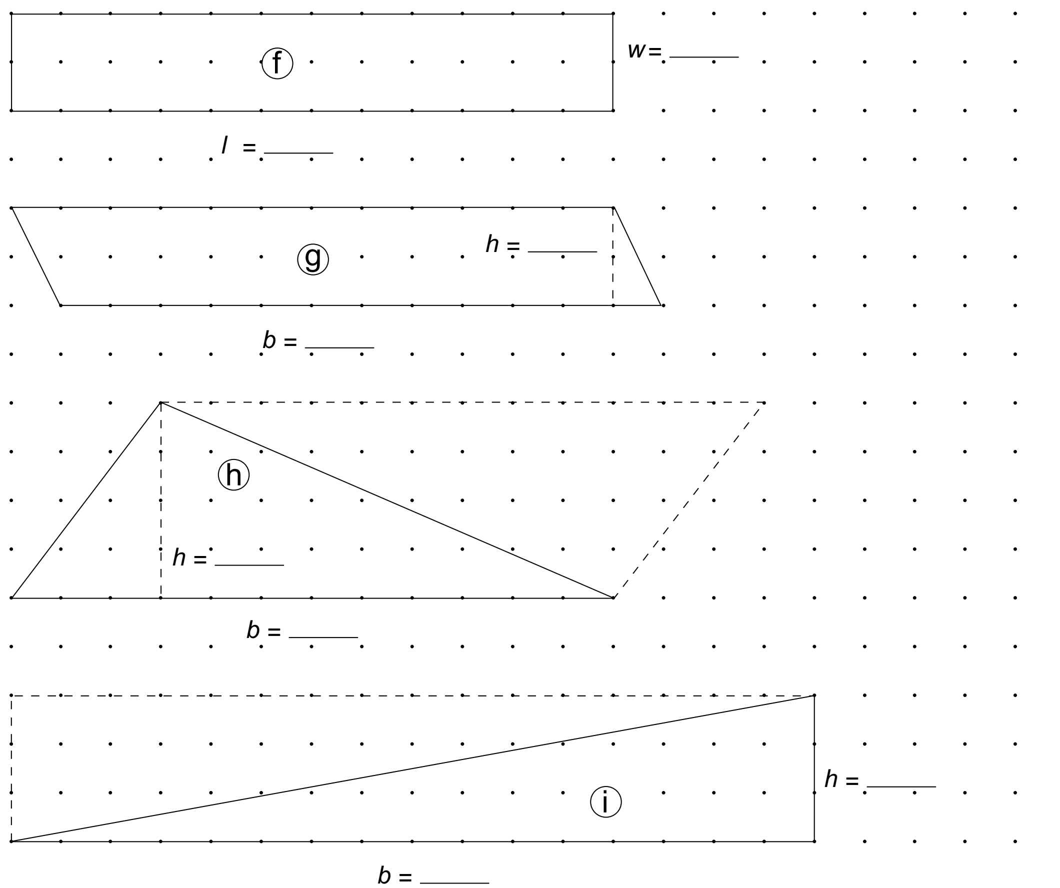
5. Slice *a* is rectangular, and its dimensions are ____________ by

    ____________ . The area is ____________ square units.

6. Slice *b* is a right triangle with a **base (*b*)** of ____________ units and a **height (*h*)** of ____________ units. The area is ____________ square units.

The slice is also one-half of a rectangle. The area of the rectangle is ____________ square units, and the area of the right triangle is one-half of that amount or ____________ square units.





7. Slice *c* is a rectangle, and its dimensions are ____________ by ____________ . The area is ____________ square units.

8. Slice *d* is rectangular, and its dimensions are ____________ by ____________ . The area is ____________ square units.

9. Slice *e* is a **parallelogram** with a *base* of ____________ units and a height of ____________ units. The area is ____________ square units.

10. Slice *f* is a rectangle, and its dimensions are ____________ by ____________ . The area is ____________ square units.

11. Slice *g* is a *parallelogram* with a base of ____________ units and a height of ____________ units. The area is ____________ units.

12. Slice *h* is a triangle with a base of ____________ units and a height of ____________ units. The area is ____________ square units. The slice is also one-half of a parallelogram with a base of ____________ units and a height of ____________ units. The area of the parallelogram is ____________ square units and the area of the triangle is one-half as much or ____________ square units.

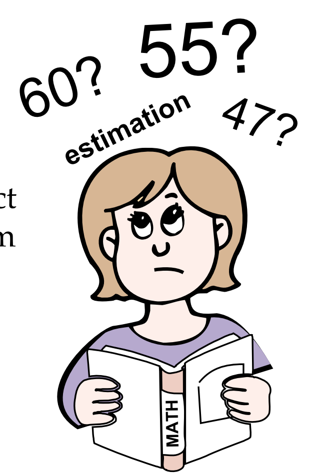
13. Slice *i* is a triangle with a base of ____________ units and a height of ____________ units. The area is ____________ square units. The slice is also one-half of a rectangle with dimensions of ____________ by ____________ . The area of the rectangle is ____________ square units and the area of the triangle is one-half as much or ____________ square units.

Lesson Three Purpose

- Understand the structure of number systems other than the decimal system. (A.2.3.2)
- Add, subtract, multiply, and divide whole numbers and decimals to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Use estimation strategies to predict results and to check the reasonableness of results. (A.4.3.1)
- Use concrete and graphic models to derive formulas for finding perimeter and area. (B.1.3.1)
- Use concrete and graphic models to derive formulas for finding rates. (B.1.3.2)
- Use direct (measured) and indirect (not measured) measures to compare a given characteristic in either metric or customary units. (B.2.3.1)
- Solve problems involving units of measure and convert answers to a larger or smaller unit within either the metric or customary system. (B.2.3.2)
- Solve real-world and mathematical problems involving estimates of measurements. (B.3.3.1)
- Select appropriate units of measurement. (B.4.3.1)
- Select and use appropriate instruments and techniques to measure quantities. (B.4.3.2)

Estimating before Solving

Using **rounded numbers** is one way to determine a reasonable approximation without calculating an exact answer. A quick **estimation** of the answer to a problem can be done by rounding numbers before they are added, subtracted, multiplied, or divided.



Practice

Answer the following.

In the following set of problems, an estimate will be made and then the problem will be solved.

The world's largest meatpacker, located in Lexington, Nebraska, handles one head of cattle every 11 seconds. At this rate, how many cattle would be handled in an 8-hour period? **Round your answers to the nearest whole number.**

1. A reasonable estimate would be ____________ . I used the following strategy to make my estimate.

2. To determine a more exact answer, consider the following strategy.

We know	1 minute	=	60	seconds			
	1 hour	=	60	minutes =		3,600	seconds
	8 hours	=	480	minutes =		28,000	seconds

If you divide 28,800 seconds by 11 seconds and round the **quotient** to the nearest whole number, the answer is 2,618. So about 2,618 head of cattle can be handled in an 8-hour period. How does your estimate compare to this?

$$11\overline{)28{,}800}\quad = 2{,}618$$

____

____

____

3. Note that the answer is rounded to the nearest whole number. What impact would it have on your final answer if you rounded the number of cattle per minute instead of using the **decimal number** *quotient* produced when you divided using paper and pencil or your calculator? Solve and explain.

Answer: ____________

Explanation: ____

____

____

____

____

____

Practice

Answer the following.

In a month with five Mondays, a school declares the fifth Monday a holiday. If school begins on Monday, September 4, what is the date of the first fifth Monday?

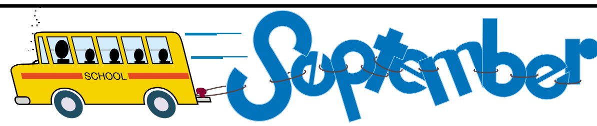
You need to know the number of days in each month. Mike and Misty learned a rhyme when they were children that helps them.

> Thirty days hath September,
>
> April, June, and November.
>
> All the rest have 31,
>
> except the second month alone.
>
> It has 28 days so fine,
>
> 'til Leap Year brings it 29.

1. If September 4 is Monday, then there are ____________ Mondays in September since it is a month with 30 days.

2. An estimate might be ____________ (early, mid-, late) October. The exact date is only found by solving the problem.

3. Complete the following calendars for reference in solving the problem.



Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
					1	2
3	4					
	11					
	18					
	25					

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
1	2					
	9					
	16					
	23					
	30					

4. What pattern do you see in the dates for Mondays in September?

____

____

____

____

What patterns do you see in the dates for Mondays in October?

____

____

____

5. Is it necessary to fill in all of the dates on the calendar to solve the problem? ____________

   Why or why not? ____

   ____

   ____

   ____

6. The solution is ________________________ .

Practice

Answer the following.

What is the **positive difference**, in inches, of the greatest cabin height and the least cabin height of the following midsize business jets?

Business Jet Cabin Heights

Business Jet	Cabin Height
Learjet 31A	4 feet 5 inches
Learjet 60	5 feet 8 inches
Cessna Citation V	4 feet 7 inches
Gulfstream V	6 feet 2 inches

1. The greatest height is ________________ , and the least height is ________________ . To find the positive difference, the smaller is subtracted from the larger. An estimate would be ________________ .

2. When solving the problem, it might help to review the following example.

   Since you know that one foot equals 12 inches, you may chose to solve this by renaming 6 feet 2 inches to 5 feet ______ inches and then subtract.

6 feet 2 inches =	5	feet	______	inches
– 4 feet 5 inches =	– 4	feet	5	inches
	______	feet	______	inches

or

You may choose to solve this by renaming 6 feet 2 inches to ______ inches *and* 4 feet 2 inches to ______ inches and then subtract.

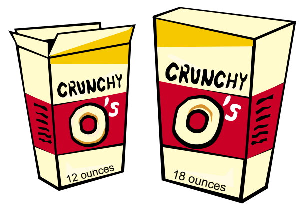
6 feet 2 inches =	______	inches
– 4 feet 5 inches =	– ______	inches
	______	inches

3. Solve the problem. ________________________

Practice

Read the item above each section to answer the following in that section.

The price of a 12-ounce box of cereal is $2.88, and an 18-ounce box of the same cereal sells for $3.78. How much do you save in cents per ounce with the larger box?



1. My estimate for price per ounce for the 12-ounce box is ________ .

   The strategy I used for my estimate was ______________________

   ____

   ____ .

2. My estimate for the price per ounce for the 18-ounce box is

   ____________ .

   The strategy I used for my estimate was ______________________

   ____

   ____ .

3. The positive difference in my estimates for the per ounce cost of the two boxes is ____________ .

4. Solve the problem. ____________

   Show all your work.

Our capital city, Washington, D.C., has a land area of 68 square miles and a population of nearly 570,000. What is the number of people per square mile? Let's first estimate an answer and then solve the problem.

5. The population appears to be rounded. Round the 68 to the nearest ten, which would be ____________ .

6. The number of **digits** in the whole number portion of the quotient is expected to be ____________ .

7. An estimate would be ____________ .

8. Solve the problem. ____________
   Show all your work.

Practice

Read each **item** *above each section to answer the following in that section.*

A foyer in a building under construction will have a tile floor. The tiles are 18 inches wide and 30 inches long. The foyer is 15 feet wide and 25 feet long. How many tiles will be needed to cover the floor?

1. The foyer is 15 by 25 feet. The product of

   10 times 20 is ____________ (rounding both down),

   20 times 30 is ____________ (rounding both up), and

   20 times 20 is 400 (rounding one up and one down since both have units digits of 5).

   An estimated area of ____________ square feet seems reasonable.

2. The same kind of thinking could be done with 18 inches and 30 inches. If there are 12 inches in one foot, then 18 inches could be rounded up to ____________ feet, and 30 inches could be rounded down to ____________ feet. A reasonable estimate for the area of a tile would be ____________ square feet.

3. If the area of the room is divided by the area of a tile, the number of tiles needed is approximately ____________ .

4. Use a visual approach for another way to produce an estimate. If 18 inches can be converted to ____________ feet, then 2 of the tiles would have a width of ____________ feet. It would take ____________ sets of 2 to cover the width of the foyer measuring 15 feet.

5. The same kind of visualization can be applied to the length of the tile and the length of the foyer. A reasonable estimate would be ____________ tiles across the length of the room.

6. If there are ____________ rows of tiles with ____________ in each row, then approximately ____________ tiles are needed to cover the floor of the foyer.

7. Solve the problem. ____________
   Show all your work.

A local middle school has six class periods, each 50 minutes in length; one homeroom class 20 minutes in length; one lunch period 25 minutes in length; and 5 minutes for changing classes between any two periods except lunch. What is the total length of the school day?

8. Choose your own strategies to provide an estimate and a precise answer. Give your responses in hours and minutes. Explain how you arrived at your estimate and how you solved the problem.

   Estimate: ____________ Precise Answer: ____________

   Explanation: ____

   ____

   Show all your work.

A flooring company sells hardwood flooring at a price of $7.25 per square foot, including installation. If a room is 12 feet wide and 15 feet long, what is the cost of putting a hardwood floor in this room?

9. Choose your own strategies to provide an estimate and a precise answer. Explain how you arrived at your estimate and your precise answer.

   Estimate: ____________ Precise Answer: ____________

   Explanation: ____

   ____

   Show all your work.

Practice

Use the list below to write the correct term for each definition on the line provided.

base (*b*)	**estimation**	**quotient**
decimal number	**height (*h*)**	**rounded number**
difference	**parallelogram**	**triangle**
digit	**positive numbers**	

____________________ 1. a polygon with three sides

____________________ 2. a quadrilateral with two pairs of parallel sides

____________________ 3. a line segment extending from the vertex or apex (highest point) of a figure to its base and forming a right angle with the base or basal plane

____________________ 4. the line or plane upon which a figure is thought of as resting

____________________ 5. the use of rounding and/or other strategies to determine a reasonably accurate approximation without calculating an exact answer

____________________ 6. a number approximated to a specified place

____________________ 7. any number written with a decimal point in the number

____________________ 8. the result of a division

____________________ 9. the result of subtraction

____________________ 10. numbers greater than zero

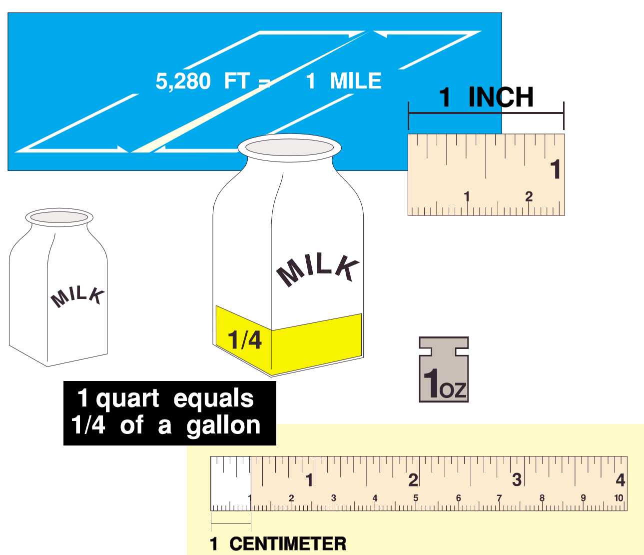
____________________ 11. any one of the 10 symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9

Lesson Four Purpose

- Understand the structure of number systems other than the decimal system. (A.2.3.2)
- Use concrete and graphic models to derive formulas for finding perimeter and area. (B.1.3.1)
- Use direct (measured) and indirect (not measured) measures to compare a given characteristic in either metric or customary units. (B.2.3.1)
- Solve real-world and mathematical problems involving estimates of measurements. (B.3.3.1)

Measurement and Estimation

In this lesson, knowledge of measurement and estimation strategies will be used to solve problems.



Practice

Complete the following.

Ask someone to carefully trace your hand on the next page. Then record the following in the table and on the lines below.

- Estimate the length of your longest finger and shortest finger, other than your thumb, in centimeters and in inches.
- Using a ruler or tape measure, find the actual length of your longest and shortest fingers in centimeters and inches.

Length in Centimeters and Inches

	Estimated Length in Centimeters	Estimated Length in Inches	Actual Length in Centimeters	Actual Length in Inches
Shortest Finger				
Longest Finger				
Palm of Hand				

Tracing of hand

Practice

Answer the following.

1. On a large piece of paper (a large brown paper bag from the grocery store works well), draw the following:
   - 1 square centimeter
   - 1 square inch
   - 1 square foot

2. Find one or more items used in everyday life having an area of the following:

   a. 1 square centimeter: ________________________________

   b. 1 square inch: ________________________________

   c. 1 square foot: ________________________________

3. If a round pizza, having a **diameter (*d*)** of 12 inches, and an approximate area of 113 square inches is priced at $8.99, what is the cost per square inch? Round to the nearest cent. ___________

4. If the pizza from question 3 is cut into 8 slices, each having an area of approximately 14 square inches, what is the cost per slice? Round to the nearest cent. ___________

5. The quotient of $8.99 and 8 is ___________ . The product of the cost per square inch and 14 is ___________ . These ___________ (are, are not) approximately the same.

Practice

Answer the following.

Mike and Misty found three boxes of cereal in the pantry that were the same size. Their lengths, widths, and heights were all the same. They have placed other data from the boxes in the table below.

Three Cereal Boxes

	Weight of Box with Contents	**Serving Size**	**Number of Servings per Box**	**Carbohydrates per Serving**
Cereal A	12 ounces	1 cup	12	24 grams
Cereal B	9 ounces	$1\frac{1}{3}$ cups	8	26 grams
Cereal C	19 ounces	24 biscuits	9	48 grams

1. Why do you think the three boxes are the same size but one has more than twice the **weight** of another?

____

____

____

____

____

2. Mike's father has diabetes and is allowed a maximum of 45 grams of carbohydrates per meal. If he especially likes Cereal A and uses skim milk with 13 grams of carbohydrates per cup, he could have, at most, ____________ ($1\frac{1}{4}$, $1\frac{1}{3}$, $1\frac{1}{2}$, $1\frac{2}{3}$, $1\frac{3}{4}$) cups of cereal with one cup of milk at breakfast.

3. One serving of Cereal A is expected to weigh ____________________ (more than, less than, approximately) one ounce.

4. One serving of Cereal B is expected to weigh ____________________ (more than, less than, approximately) one ounce.

5. One serving of Cereal C is expected to weigh ____________________ (more than, less than, approximately) one ounce.

Practice

Answer the following.

Estimate and then find the cost of installing a hardwood floor costing $7.25 per square foot in your classroom.

1. Estimated length of classroom: ____________ feet
2. Estimated width of classroom: ____________ feet
3. Estimated area of classroom: ____________ feet
4. Estimated cost of hardwood floor for classroom: $ ____________
5. Actual length of classroom: ____________ feet
6. Actual width of classroom: ____________ feet
7. Area of classroom: ____________ square feet
8. Cost of hardwood floor for classroom: $ ____________
9. Make a scale drawing of your classroom. Be sure to include the scale you choose to use. You may use unlined paper or the square dot paper on the following page.

10. Find an advertisement for carpet sold in square yards and attach it to this paper. If a square yard is 3 feet long and 3 feet wide, there are ____________ square feet in a square yard. What would be the cost of carpet for the classroom based on the price in your advertisement?

Explain how you solved the problem. ______________________________

____

____

____

____

____

____

____

____

Practice

Use the list below to write the correct term for each definition on the line provided.

area (*A*)	**perimeter (*P*)**	**square**
diameter (*d*)	**rectangle**	**square units**
estimation	**rounded number**	**weight**
height (*h*)	**scale model**	**width (*w*)**
length (*l*)		

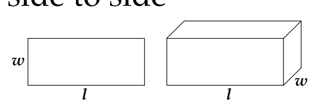
____________________ 1. a line segment from any point on the circle passing through the center to another point on the circle

____________________ 2. measures that represent the force that attracts an object to the center of Earth

____________________ 3. units for measuring area; the measure of the amount of an area that covers a surface

____________________ 4. a one-dimensional measure that is the measurable property of line segments

____________________ 5. a one-dimensional measure of something side to side



____________________ 6. a model or drawing based on a ratio of the dimensions for the model and the actual object it represents

____________________ 7. a parallelogram with four right angles

____________________ 8. the use of rounding and/or other strategies to determine a reasonably accurate approximation without calculating an exact answer

____________________ 9. a number approximated to the nearest specified place

____________________ 10. the length of the boundary around a figure; the distance around a polygon

____________________ 11. the inside region of a two-dimensional figure measured in square units

____________________ 12. a rectangle with four sides the same length

____________________ 13. a line segment extending from the vertex or apex (highest point) of a figure to its base and forming a right angle with the base or basal plane

Unit 3: Geometry

This unit emphasizes angles and their measures. You should have a protractor or angle ruler available for your use.

Unit Focus

Number Sense, Concepts, and Operations

- Select the appropriate operation to solve problems involving addition, subtraction, multiplication, and division of rational numbers. (A.3.3.2)
- Add and subtract whole numbers to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)

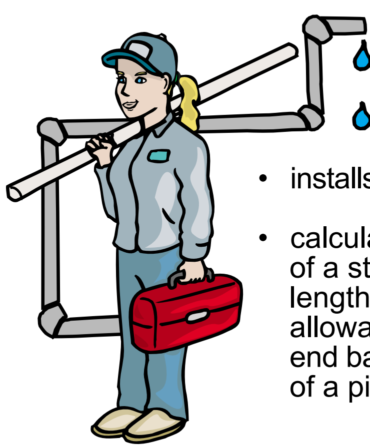
Measurement

- Use concrete and graphic models to derive formulas for finding perimeter and area. (B.1.3.1)
- Use concrete and graphic models to derive formulas for finding angle measures. (B.1.3.2)
- Understand and describe how the change of a figure in such dimensions as length, width, or height affects its other measurements such as perimeter and area. (B.1.3.3)
- Use direct (measured) and indirect (not measured) measures to compare a given characteristic in either metric or customary units. (B.2.3.1)

Geometry and Spatial Relations

- Understand the basic properties of, and relationships pertaining to, regular and irregular geometric shapes in two and three dimensions. (C.1.3.1)

- Understand the geometric concepts of symmetry, reflections, congruency, perpendicularity, parallelism, and transformations, including flips, slides, and turns. (C.2.3.1)
- Represent and apply geometric properties and relationships to solve real-world and mathematical problems. (C.3.3.1)
- Identify and plot ordered pairs in all four quadrants of a rectangular coordinate system (graph). (C.3.3.2)



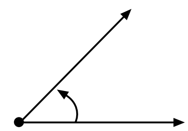
Plumber

- installs pipes for water lines
- calculates the total length of a straight pipe's visible length and the fitting allowance needed at each end based on the diameter of a pipe

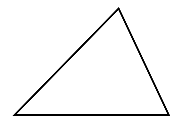
Vocabulary

Study the vocabulary words and definitions below.

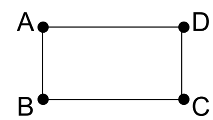
acute angle .... an angle with a measure of less than 90°



acute triangle .... a triangle with three acute angles

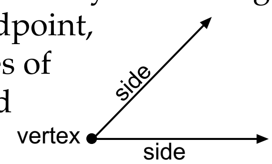


adjacent sides .... sides that are next to each other and share a common vertex



Sides *AB* and *AD* are adjacent. They share the common vertex *A*.

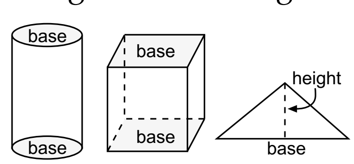
angle .... the shape made by two rays extending from a common endpoint, the vertex; measures of angles are described in degrees (°)



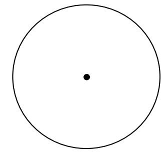
area (*A*) .... the inside region of a two-dimensional figure measured in square units
Example: A rectangle with sides of four units by six units contains 24 square units or has an area of 24 square units.

axes (of a graph) .... the horizontal and vertical number lines used in a rectangular graph or coordinate grid system as a fixed reference for determining the position of a point; (singular: *axis*)

base (*b*) .... the line or plane upon which a figure is thought of as resting

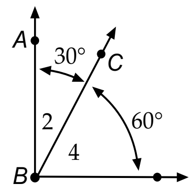


center of circle .... the point from which all points on the circle are the same distance



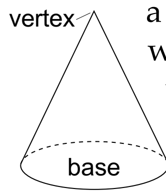
circle .... the set of all points in a plane that are all the same distance from a given point called the center

complementary angles .... two angles, the sum of which is exactly 90°



m∠ABC + m∠CBD = 90°
complementary angles

cone .... a three-dimensional figure with one circular base in which a curved surface connects the base to the vertex

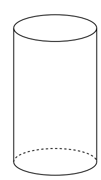


congruent (≅) .... figures or objects that are the same shape and the same size

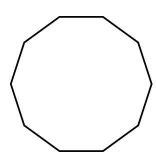
coordinate grid or system .... network of evenly spaced, parallel horizontal and vertical lines especially designed for locating points, displaying data, or drawing maps

coordinates .... numbers that correspond to points on a graph in the form (x, y)

cylinder .... a three-dimensional figure with two parallel congruent circular bases

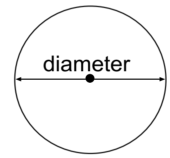


decagon .... a polygon with 10 sides

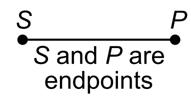


degree (°) .... common unit used in measuring angles

diameter (*d*) .... a line segment from any point on the circle passing through the center to another point on the circle

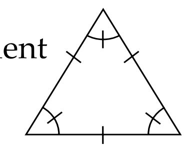


endpoint .... either of two points marking the end of a line segment

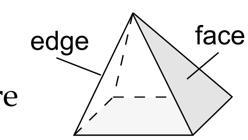


equidistant .... equally distant

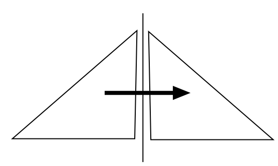
equilateral triangle .... a triangle with three congruent sides and three congruent angles



face .... one of the plane surfaces bounding a three-dimensional figure



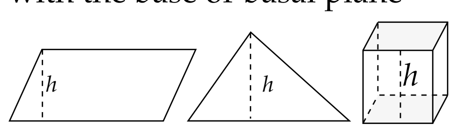
flip .... a transformation that produces the mirror image of a geometric figure; also called a *reflection*



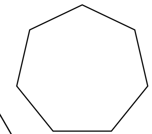
formula .... a way of expressing a relationship using variables or symbols that represent numbers

grid .... a network of evenly spaced, parallel horizontal and vertical lines

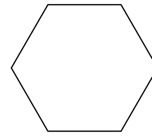
height (*h*) .... a line segment extending from the vertex or *apex* (highest point) of a figure to its base and forming a right angle with the base or basal plane



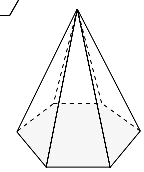
heptagon .... a polygon with seven sides



hexagon .... a polygon with six sides



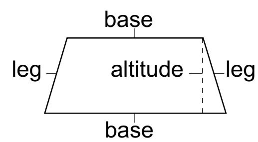
hexagonal pyramid .... a pyramid with a hexagonal base and six faces that are triangular



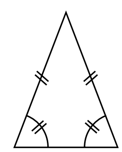
horizontal .... parallel to or in the same plane of the horizon

intersection .... the point at which two lines meet

isosceles trapezoid .... a trapezoid with congruent legs and two pairs of congruent base angles

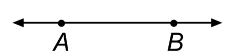


isosceles triangle .... a triangle with at least two congruent sides and two congruent angles

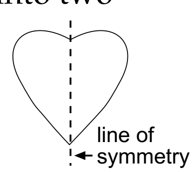


length (*l*) .... a one-dimensional measure that is the measurable property of line segments

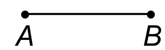
line .... a straight line that is endless in length



line of symmetry .... a line that divides a figure into two congruent halves that are mirror images of each other



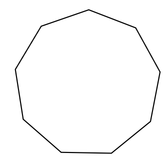
line segment .... a portion of a line that has a defined beginning and end



Example: The line segment *AB* is between point *A* and point *B* and includes point *A* and point *B*.

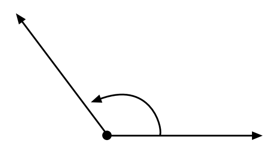
line symmetry .... see *symmetry*

nonagon .... a polygon with nine sides

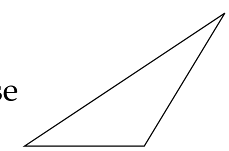


number line .... a line on which numbers can be written or visualized

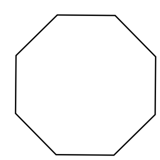
obtuse angle .... an angle with a measure of more than 90° but less than 180°



obtuse triangle .... a triangle with one obtuse angle



octagon .... a polygon with eight sides



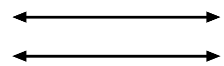
opposite sides .... sides that are directly across from each other

ordered pair .... the location of a single point on a rectangular coordinate system where the digits represent the position relative to the x-axis and y-axis
Example: (x, y) or (3, 4)

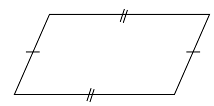
origin .... the intersection of the x-axis and y-axis in a coordinate plane, described by the ordered pair (0,0)

parallel (||) .... being an equal distance at every point so as to never intersect

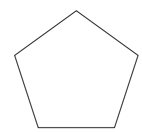
parallel lines .... two lines in the same plane that never meet; also, lines with equal slopes



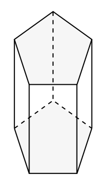
parallelogram .... a quadrilateral with two pairs of parallel sides



pentagon .... a polygon with five sides



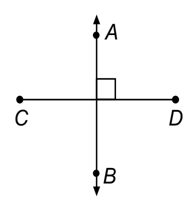
pentagonal prism .... prism with pentagonal bases



perimeter (*P*) .... the length of the boundary around a figure; the distance around a polygon

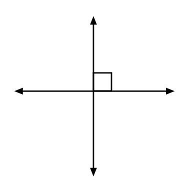
perpendicular (⊥) .... forming a right angle

perpendicular bisector (of a segment) .... a line that divides a line segment in half and meets the segment at right angles



$\overleftrightarrow{AB}$ is a perpendicular bisector of $\overline{CD}$

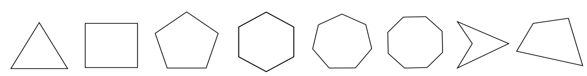
perpendicular lines .... two lines that intersect to form right angles



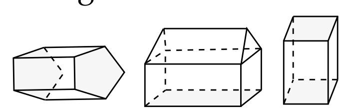
plane .... an undefined, two-dimensional (no depth) geometric surface that has no boundaries specified; a flat surface

point .... a location in space that has no length or width

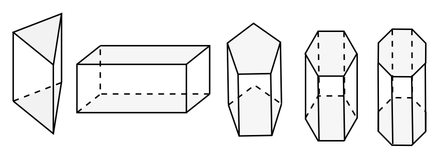
polygon .... a closed plane figure whose sides are straight lines and do not cross
Example: triangle (3 sides), quadrilateral (4 sides), pentagon (5 sides), hexagon (6 sides), heptagon (7 sides), octagon (8 sides); concave, convex



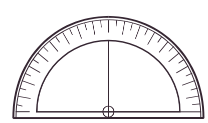
polyhedron .... a three-dimensional figure in which all surfaces are polygons



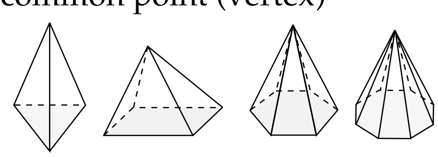
prism .... a three-dimensional figure (polyhedron) with congruent, polygonal bases and lateral faces that are all parallelograms



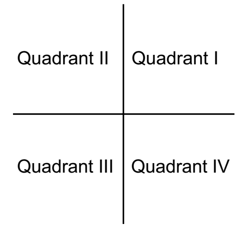
protractor .... an instrument used for measuring and drawing angles



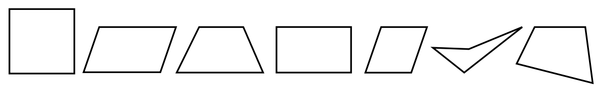
pyramid .... a three-dimensional figure (polyhedron) with a single base that is a polygon and whose faces are triangles and meet at a common point (vertex)



quadrant .... any of four regions formed by the axes in a rectangular coordinate system

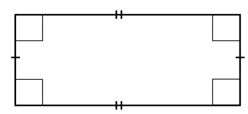


quadrilateral .... polygon with four sides
Example: square, parallelogram, trapezoid, rectangle, rhombus, concave quadrilateral, convex quadrilateral

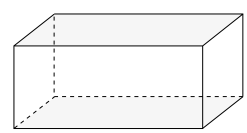


ray .... a portion of a line that begins at a point and goes on forever in one direction

rectangle .... a parallelogram with four right angles



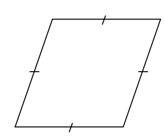
rectangular prism .... a six-sided prism whose faces are all rectangular
Example: a brick



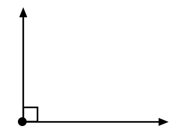
reflection .... see *flip*

reflectional symmetry .... when a figure has at least one line which splits the image in half, such that each half is the mirror image or reflection of the other; also called *line symmetry* or *mirror symmetry*

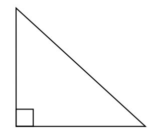
rhombus .... a parallelogram with four congruent sides



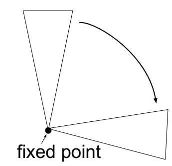
right angle .... an angle whose measure is exactly 90°



right triangle .... a triangle with one right angle

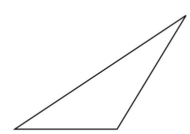


rotation .... a transformation of a figure by *turning* it about a center point or axis; also called a *turn*
Example: The amount of rotation is usually expressed in the number of degrees, such as a 90° rotation.

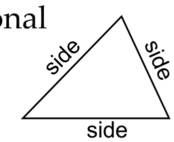


rotational symmetry .... when a figure can be turned less than 360 degrees about its center point to a position that appears the same as the original position; also called *turn symmetry*

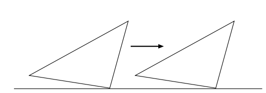
scalene triangle .... a triangle with no congruent sides



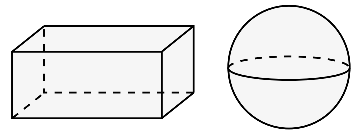
side .... the edge of a two-dimensional geometric figure
Example: A triangle has three sides.



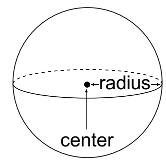
slide .... to move along in constant contact with the surface in a vertical, horizontal, or diagonal direction; also called a *translation*



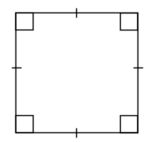
solid figures .... three-dimensional figures that completely enclose a portion of space
Example: rectangular solid and a sphere



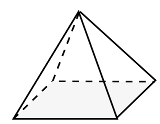
sphere .... a three-dimensional figure in which all points on the surface are the same distance from the center



square .... a rectangle with four sides the same length

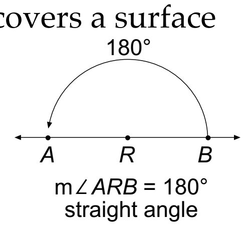


square pyramid .... a pyramid with a square base and four faces that are triangular

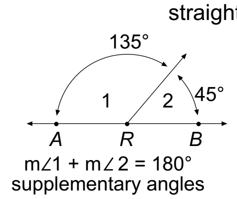


square units .... units for measuring area; the measure of the amount of area that covers a surface

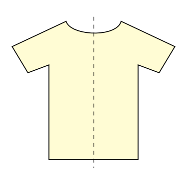
straight angle .... an angle whose measure is exactly 180°



supplementary angles .... two angles, the sum of which is exactly 180°



symmetry .... when a line can be drawn through the center of a figure such that the two halves are congruent; also called *line symmetry*



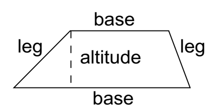
three-dimensional (3-dimensional) .... existing in three dimensions; having length, width, and height

transformation .... an operation on a geometric figure by which another image is created
Example: Common transformations include flips, slides, and turns.

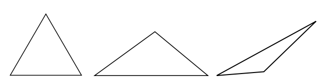
translation .... see *slide*

translational symmetry .... when a figure can slide on a plane (or flat surface) without turning or flipping and with opposite sides staying congruent

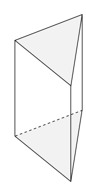
trapezoid .... a quadrilateral with just one pair of opposite sides parallel



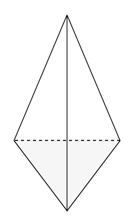
triangle .... a polygon with three sides; the sum of the measures of the angles is 180°



triangular prism .... a prism with triangular bases and lateral faces that are rectangular

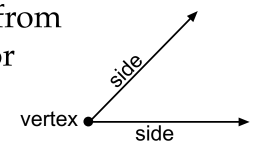


triangular pyramid .... a pyramid with a triangular base and three faces that are triangular



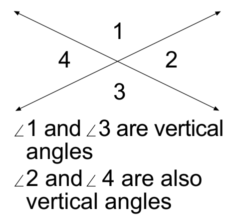
turn .... see *rotation*

vertex .... the common endpoint from which two rays begin or the point where two lines intersect; the point on a triangle or pyramid opposite to and farthest from the base; (plural: *vertices*); vertices are named clockwise or counterclockwise



vertical .... at right angles to the horizon; straight up and down

vertical angles .... the opposite angles formed when two lines intersect



***x*-axis** .... the horizontal (↔) axis on a coordinate plane

***x*-coordinate** .... the first number of an ordered pair

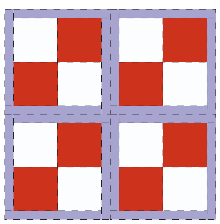
***y*-axis** .... the vertical (↕) axis on a coordinate plane

***y*-coordinate** .... the second number of an ordered pair

Unit 3: Geometry

Introduction

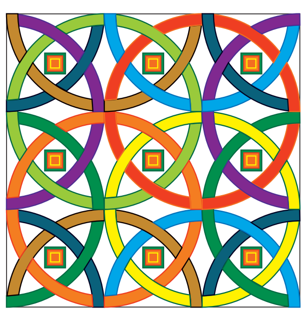
As you study this unit, look for examples of geometric shapes around you. Keep paper and pencil handy so you can make sketches and notes of your observations. In addition to signs on streets and roads, look for geometry in architecture, fabrics, gift wrap, quilts, floor coverings, wall paper, flag designs, commercial logos, or other examples of art. Share your findings often with the class, as well as with friends and family. If a camera is available for your use, you might enjoy making a display of pictures you take.



quilt

architecture

floor coverings



art

fabric

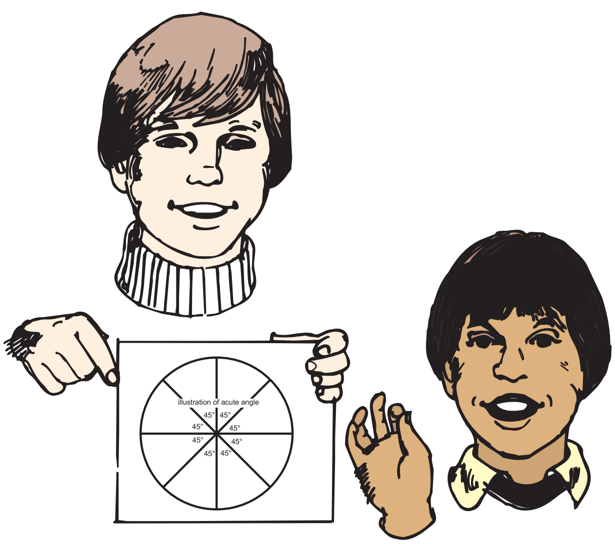
Lesson One Purpose

- Adds and subtracts whole numbers to solve real-world problems, using appropriate methods of computing, such as mental math, paper and pencil, and calculator. (A.3.3.3)
- Uses concrete and graphic models to derive formulas for finding angle measures. (B.1.3.2)
- Understands the basic properties of, and relationships pertaining to, regular and irregular geometric shapes in two dimensions. (C.1.3.1)

Creating Graphic Models

In this lesson you will be drawing graphic models. Your models will help you "see" certain geometric relationships.

(**Remember:** Each new vocabulary word is bolded and defined on the vocabulary pages.)



Practice

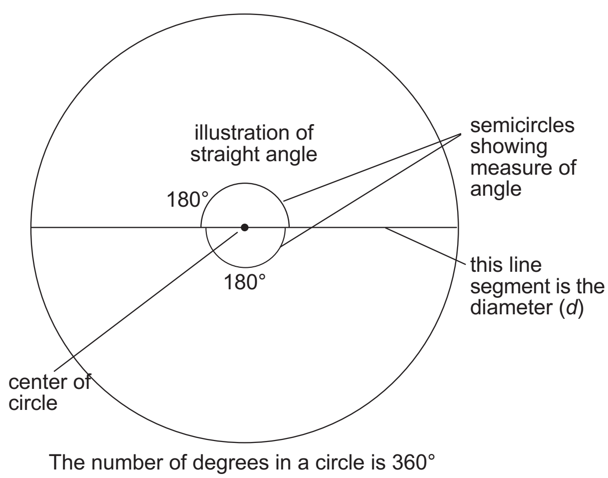
Complete the following.

1. Use scissors to cut out the four **circles** on the following page. (Your teacher may ask you to use your own paper to trace the circles or use a compass to draw the circles and then cut them out.) The number of **degrees (°)** in a circle is 360.

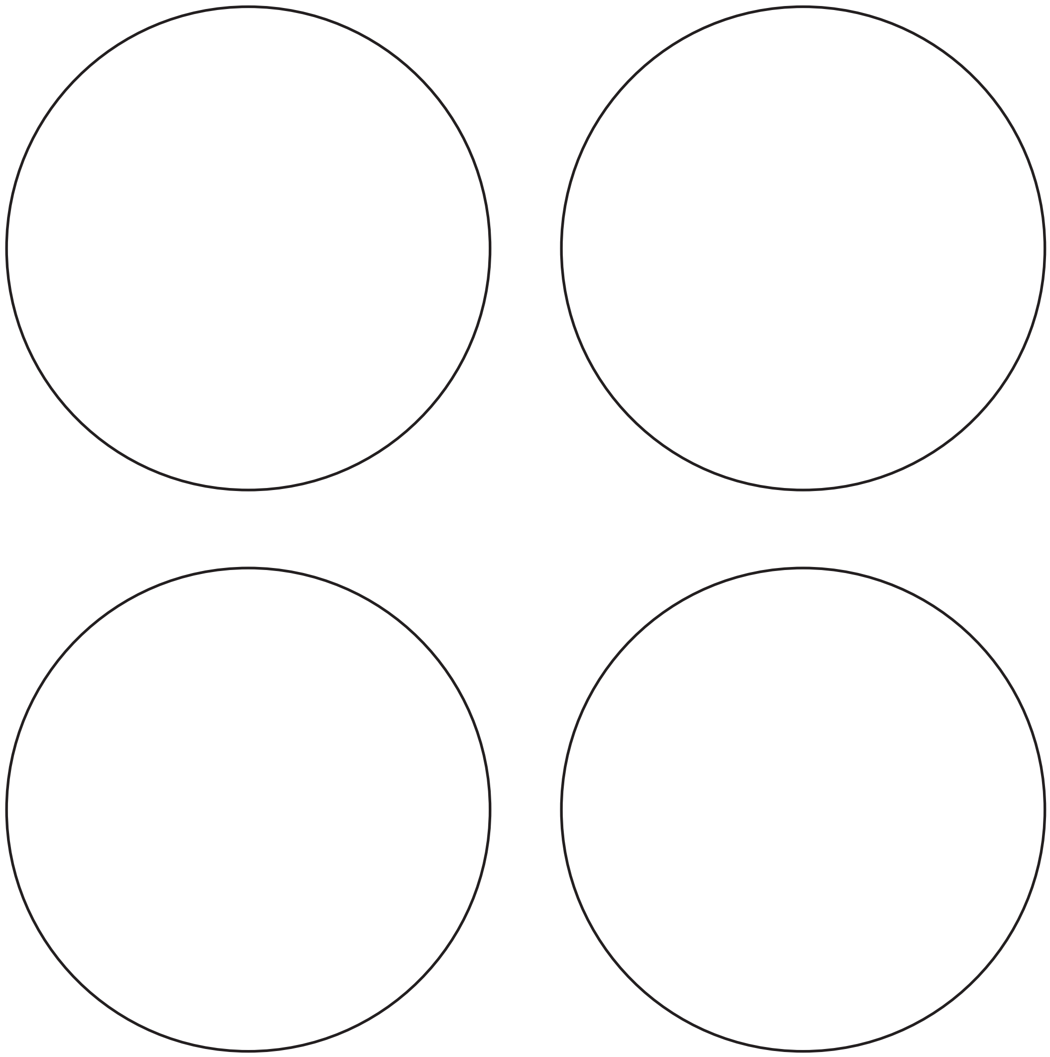
2. Fold one of the circles in half, making a sharp crease. Unfold the circle and use a straight edge to draw a **line segment** along the crease.

   The circle now has 180 degrees above the fold and 180 degrees below the fold. A **straight angle** has 180 degrees, so the line segment you drew represents a *straight angle*. (The line segment is also a **diameter (*d*)** of the circle.)

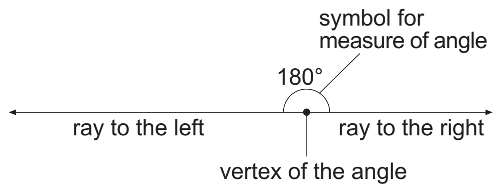
   Record the measure of 180 degrees above the line segment and draw a small *semicircle*, or exactly half a circle, at the center of the line segment showing the **angle (∠)**. Use the symbol ° to indicate degrees. On this circle, write: "illustration of straight angle." Your circle should look similar to the one pictured below.



Circle 1



The center of the line segment was used to place of the **point** for the **vertex** in this instance because we are using a circle. (If you drew a **line** on a piece of paper to illustrate a straight angle, the chosen point could be any point on the line.) The **ray** (•⟶) to the left of the point would be one side of the angle, and the ray to the right would be the other side of the angle. That point is called the *vertex of the angle*.

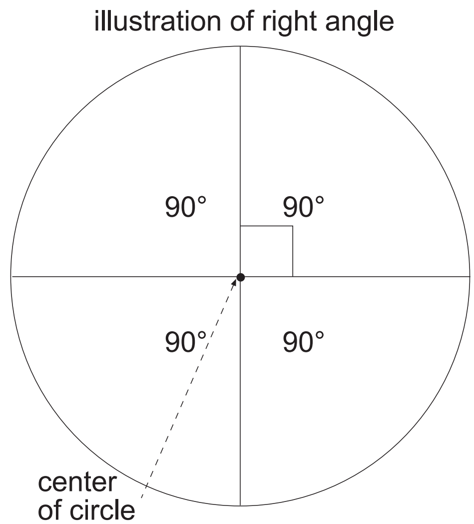


straight angle

3. Fold the second circle in half and then fold it in half again, making sharp creases. Unfold the circle and note the four **congruent (≅)** parts. Each *congruent* part will be the same shape and the same size. Using your straight edge, draw a line segment along each of the creases.

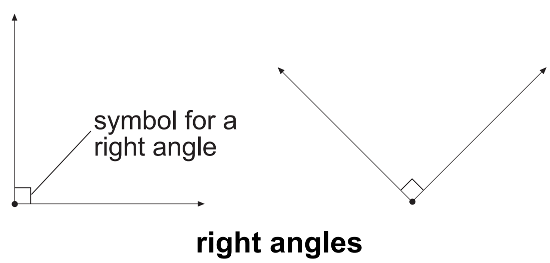
   The point at which the two line segments meet is called the **intersection** and should be the **center of the circle**. The line segments should be **perpendicular (⊥)** to each other, forming four **right angles**. The symbol for *right angle* is a small square. Place one at the point of *intersection* for one of the angles.

We have divided each of the two straight angles in half so the measure of each right angle is one-half of 180 degrees or 90 degrees. Indicate this on each of your four right angles. On this circle, write: "illustration of right angle." Your circle should look similar to the one pictured below.



Circle 2

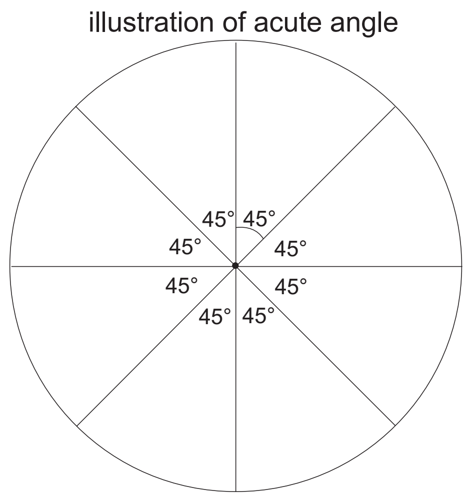
A right angle can also be illustrated with two *rays* with a common **endpoint** that are *perpendicular* to each other.



The right angle is a good benchmark for estimating size of angles. The corner of a standard piece of paper fits perfectly in a right angle, and the measure is 90 degrees.

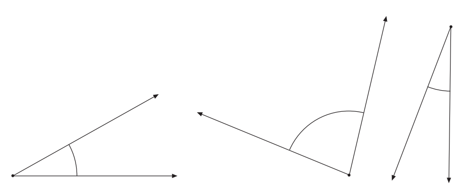
4. Fold the third circle in half, in half again, and in half again, making sharp creases each time. Unfold the circle and draw line segments along each crease.

You should see eight congruent parts as well as eight congruent angles. Since each of these angles is one-half of a right angle, each should have measure of 45 degrees. Mark each angle accordingly. An angle with more than 0 degrees and less than 90 degrees is called an **acute angle**. On this circle, write: "illustration of acute angle." Your circle should look similar to the one pictured below.



Circle 3

Acute angles are often illustrated as shown below.



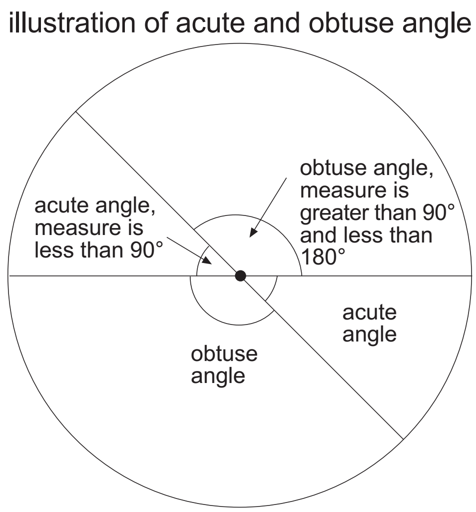
acute angles

If the angle is smaller than a right angle, it is an acute angle with a measure greater than 0 degrees and less than 90 degrees.

5. Fold the fourth circle in half. On the next fold, be sure that it is *not* halved. Our goal is to divide each straight angle into two angles so that one is *larger* than the other.

   Unfold the circle and draw line segments along each crease. Two of the angles formed should be *larger* than a right angle, and two should be *smaller* than a right angle.

   On the *two* that are *larger than a right angle*, write: "**obtuse angle**, measure is greater than 90 degrees and less than 180 degrees." On one of the *two* that are *smaller than a right angle*, write: "*acute angle*, measure is less than 90 degrees." On this circle, write: "illustration of acute and obtuse angles." Your circle should look similar to the one pictured below.



Circle 4

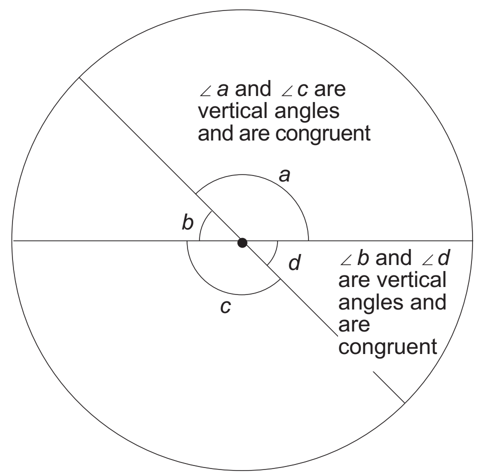
6. Continue working with the fourth circle. Notice that the two obtuse angles are opposite each other and that the two acute angles are opposite each other. The four angles are formed by two intersecting line segments.

   When two lines intersect, angles *opposite* each other are called **vertical angles**, and their *measures are the same*.

   - The two obtuse angles formed by folding are congruent in your fourth circle.

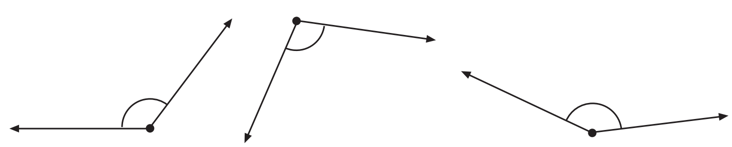
   - The two acute angles formed by folding are congruent in your fourth circle.

Label your four angles *a*, *b*, *c*, and *d* so that *a* and *c* are opposite each other and *b* and *d* are opposite each other. On your circle, write: "angle (∠) *a* and angle *c* are vertical angles and are congruent." And write "angle *b* and angle *d* are vertical angles and are congruent." Use the symbol ∠ to indicate an angle. Your circle should look similar to the one pictured below.



Circle 4 continued

If an angle is larger than a right angle, it is an obtuse angle with a measure greater than 90 degrees and less than 180 degrees.



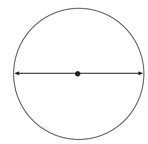
obtuse angles

Practice

Use the list below to write the correct term for each definition on the line provided.

angle	**diameter (*d*)**
circle	**line segment**
degree (°)	**vertex**

____________________ 1. a line segment from any point on the circle passing through the center to another point on the circle

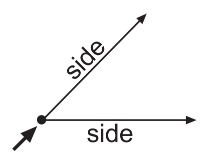


____________________ 2. common unit used in measuring angles

____________________ 3. the set of all points in a plane that are all the same distance from a given point called the center

____________________ 4. a portion of a line that has a defined beginning and end

____________________ 5. the common endpoint from which two rays begin or the point where two lines intersect



____________________ 6. the shape made by two rays extending from a common endpoint, the vertex

Practice

Match each definition with the correct term. Write the letter on the line provided.

______ 1. either of two points marking the end of a line segment

______ 2. forming a right angle

______ 3. the point at which two lines meet

______ 4. figures or objects that are the same shape and the same size

______ 5. a portion of a line that begins at a point and goes on forever in one direction

______ 6. the point from which all points on the circle are the same distance

A. center of circle

B. congruent (≅)

C. endpoint

D. intersection

E. perpendicular (⊥)

F. ray

Practice

Match each definition with the correct term. Write the letter on the line provided.

_______ 1. an angle whose measure is exactly 180°

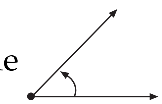
_______ 2. the opposite angles formed when two lines intersect

_______ 3. an angle with a measure of less than 90° and more than 0°

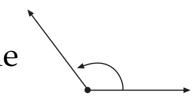
_______ 4. an angle whose measure is exactly 90°

_______ 5. an angle with a measure of more than 90° but less than 180°

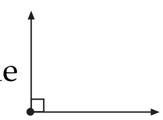
A. acute angle



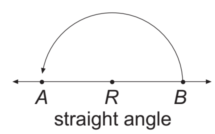
B. obtuse angle



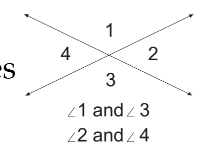
C. right angle



D. straight angle

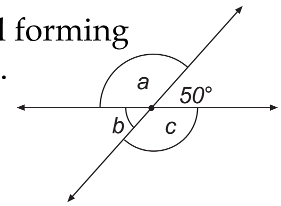


E. vertical angles



Finding Angle Measures

Look at the illustration of the two lines intersecting and forming four angles. The measure of only one angle is provided.



What We Know

- We know that the 50 degree angle is *congruent* to angle *b* because they are *vertical angles*. Therefore, the measure of the angle *b* is 50 degrees.

- We know that angle *a* can be paired with the 50 degree angle to form a pair of **supplementary angles**.

  Together the *supplementary angles form a straight angle*. Therefore, the sum of the measures of the pair of supplementary angles is 180 degrees.

  If we subtract 50 degrees from 180 degrees, then the measure of angle *a* is 130 degrees.

  We can also see that it is an *obtuse angle*. Therefore, it must have a measure greater than 90 degrees but less than 180 degrees.

- The measure of angle *c* can be determined in several ways.

  Since angle *a* and *c* are *vertical angles*, they are *congruent*.

  Angle *b* and *c*, or angle *c* and the angle (illustrated above) with a measure of 50 degrees, form supplementary angles. Therefore, we can find the measure of angle *c*.

  All four angles have a total measure of 360 degrees. We can subtract the three measures known to get the measures of angle *c*.

  We know the following.

  measure (m) of angle ($\angle$) $a = 130$ degrees (°) or m $\angle a = 130°$

  measure (m) of angle ($\angle$) $b = 50$ degrees or m $\angle b = 50°$

  measure (m) of angle ($\angle$) $c = 130°$ or m $\angle c = 130°$

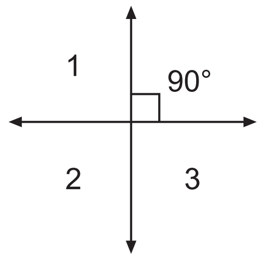
Practice

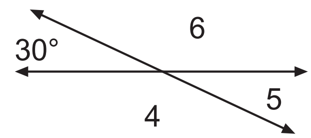
Complete the following.

In the illustrations on the following page, five pairs of intersecting lines are formed. The measure of one angle is provided for each pair of intersecting lines.

If you know that

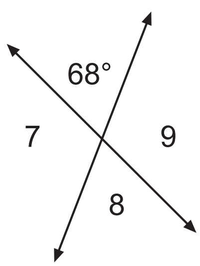
- the sum of the four angles is 360 degrees and that
- vertical angles are *congruent* ($\cong$)
- you should be able to determine the measure of each of the other three angles for the pair of intersecting lines.

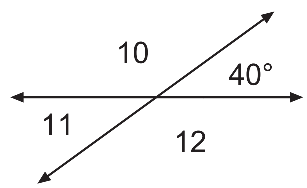




1. m∠1 = ________ degrees
2. m∠2 = ________ degrees
3. m∠3 = ________ degrees

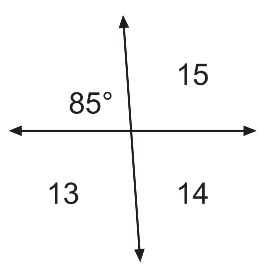
4. m∠4 = ________ degrees
5. m∠5 = ________ degrees
6. m∠6 = ________ degrees





7. m∠7 = ________ degrees
8. m∠8 = ________ degrees
9. m∠9 = ________ degrees

10. m∠10 = ________ degrees
11. m∠11 = ________ degrees
12. m∠12 = ________ degrees



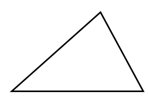
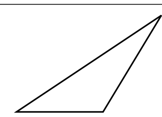
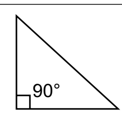
13. m∠13 = ________ degrees
14. m∠14 = ________ degrees
15. m∠15 = ________ degrees

Classifying Triangles by Angles

Triangles are **polygons** that have three **sides**. They are classified in two different ways. Triangles are classified either by the measure of their angles or the measure of their sides.

Triangles classified by their angles are called **right triangles**, **acute triangles**, or **obtuse triangles**.

(**Remember:** No matter how a triangle is classified, the sum of the measures of the angles in a triangle is 180 degrees.)

name	example	description
acute triangle		**all angles < 90°**
obtuse triangle		**one angle > 90°**
right triangle	 Right angles are marked with ⊾ .	**one angle = 90°**

< means less than   > means greater than

Practice

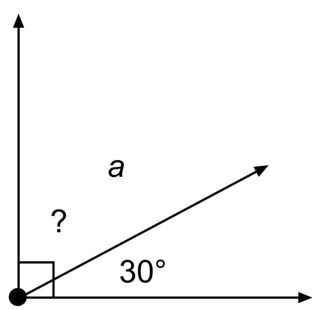
Complete the following.

In the previous practices, angles were illustrated and categorized as acute, right, obtuse, or straight. Vertical angles were also introduced. In this practice, we'll include **complementary angles**, supplementary angles, and the sum of the measures of the angles in a triangle.

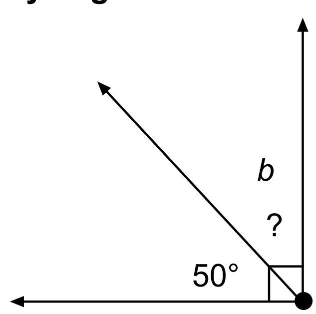
The following four right angles have been divided into two acute angles. The pair of acute angles are called *complementary angles* because the sum of their measures is 90 degrees.

The measure of one of the acute angles is given. Provide the measure of the other angle without using a **protractor** or *angle ruler*.

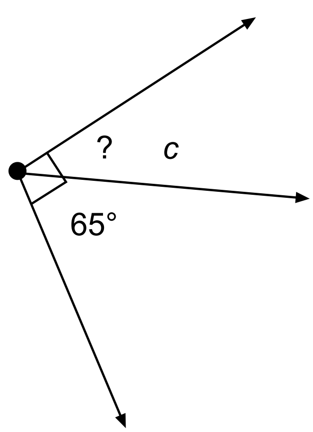
complementary angles



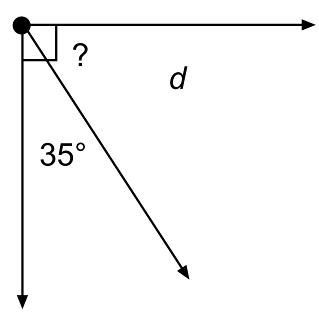
1. m∠ *a* = ________ degrees



2. m∠ *b* = ________ degrees



3. m∠ *c* = ________ degrees

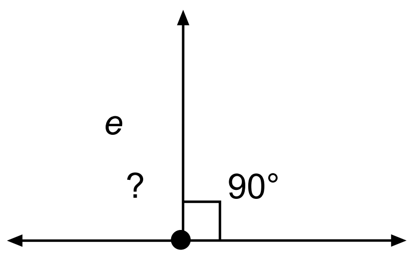


4. m∠ *d* = ________ degrees

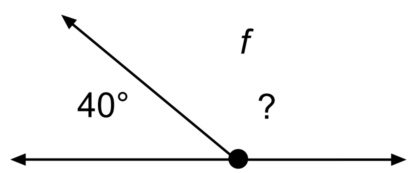
The following four straight angles have been divided into two angles. The pair of angles are called *supplementary angles* because the sum of their measures is 180 degrees.

The measure of one of the angles is given. Provide the measure of the other angle without using a protractor or angle ruler.

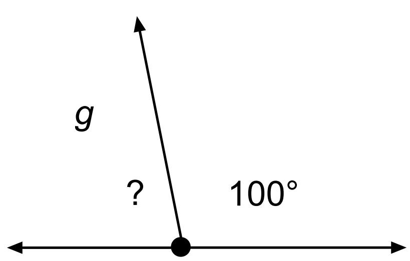
supplementary angles



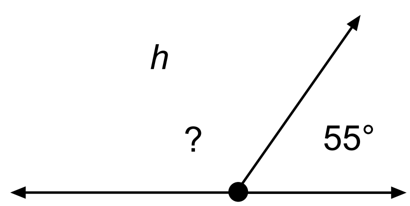
1. m∠*e* = ________ degrees



2. m∠*f* = ________ degrees



3. m∠*g* = ________ degrees

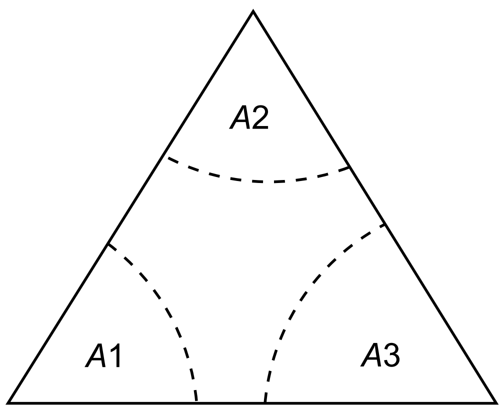


4. m∠*h* = ________ degrees

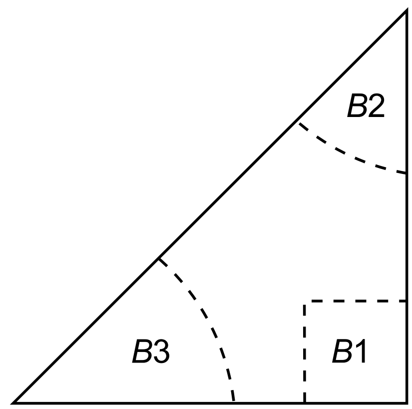
Practice

Complete the following.

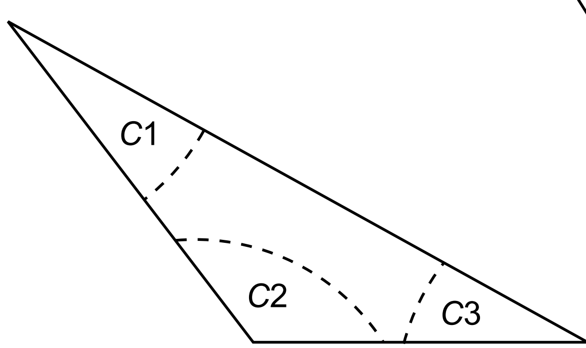
1. Using scissors, cut out each of the four triangles below. (Your teacher may ask you to use your own paper to trace the triangles, and then cut them out.) The angles in triangle *A* are labeled *A1*, *A2*, and *A3*. The angles on triangle *B* are labeled *B1*, *B2*, *B3*, etc.



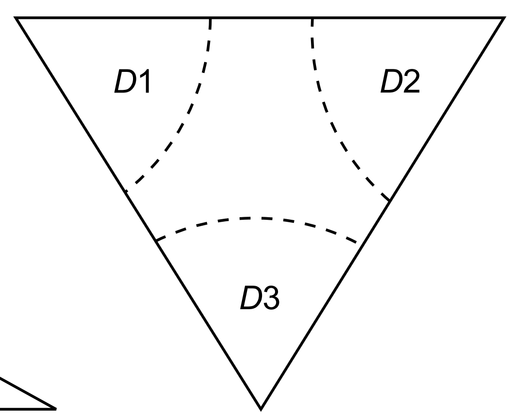
triangle *A*



triangle *B*

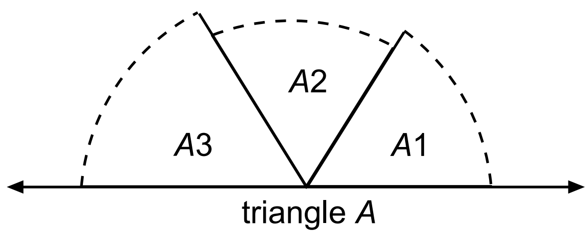


triangle *C*



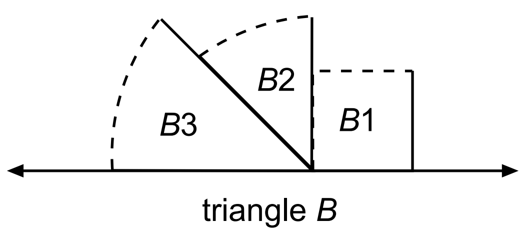
triangle *D*

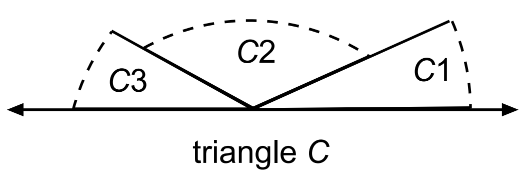
2. Using scissors, cut the three angles off of triangle *A* along the dotted lines. Fit the angle pieces along the segment drawn below as shown.

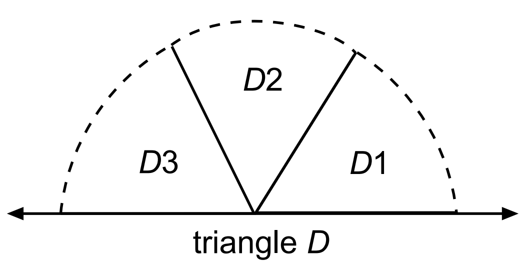


3. The segment represents a straight angle and we can see that the three angles of the triangle *A* have a sum of 180 because they fill the straight angle.

   Continue the same procedure for triangles *B*, *C*, and *D*.

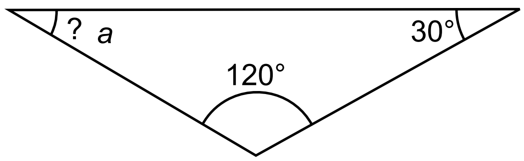




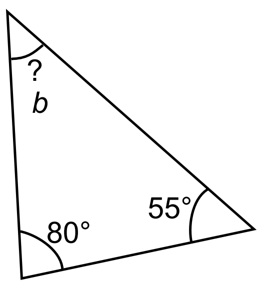


In each triangle pictured below, the measures of two of the angles are provided. Give the measure of the third angle without using a protractor or angle ruler.

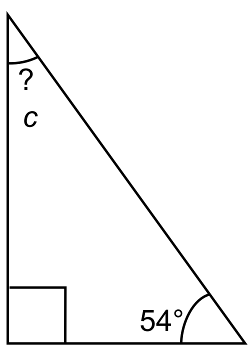
(**Remember:** The sum of the measures of the three angles in *any* triangle is 180 degrees.)



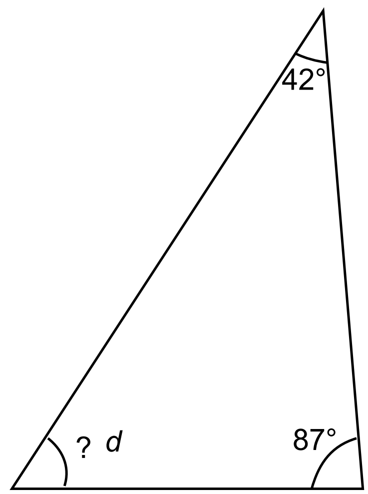
4. m∠ *a* = ______ degrees



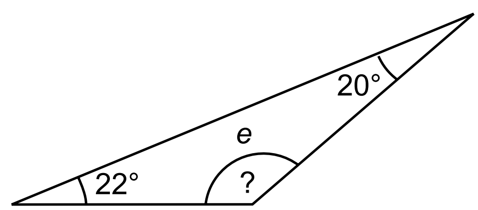
5. m∠ *b* = ______ degrees



6. m∠ *c* = ______ degrees



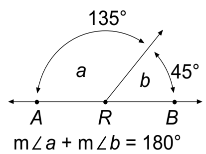
7. m∠ *d* = ______ degrees

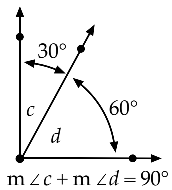


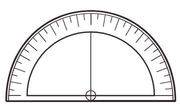
8. m∠ *e* = ______ degrees

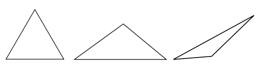
Practice

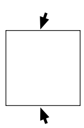
Match each graphic with the correct term. Write the letter on the line provided.

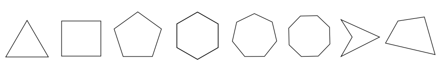
______ 1. 

______ 2. 

______ 3. 

______ 4. 

______ 5. 

______ 6. 

A. complementary angles

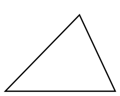
B. polygons

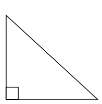
C. protractor

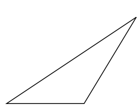
D. sides

E. supplementary angles

F. triangles

______ 7. 

______ 8. 

______ 9. 

A. acute triangle

B. obtuse triangle

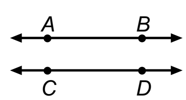
C. right triangle

Lesson Two Purpose

- Select the appropriate operation to solve problems involving addition, subtraction, multiplication, and division of rational numbers. (A.3.3.2)
- Use concrete and graphic models to derive formulas for finding perimeter and area. (B.1.3.1)
- Understand and describe how the change of a figure in such dimensions as length, width, or height affects its other measurements such as perimeter and area. (B.1.3.3)
- Use direct (measured) and indirect (not measured) measures to compare a given characteristic in either metric or customary units. (B.2.3.1)
- Understand the basic properties of, and relationships pertaining to, regular and irregular geometric shapes in two dimensions. (C.1.3.1)
- Understand the geometric concepts of congruency, perpendicularity, and parallelism. (C.2.3.1)

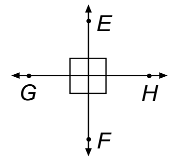
Parallel and Perpendicular Lines

Parallel lines never meet.



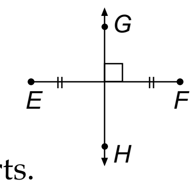
- Lines *AB* and *CD* will never *intersect* or meet at one point.
- They are *parallel lines*.
- Line *AB* is **parallel (||)** to line *CD*.
- $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

Perpendicular lines form right angles.



- They form right angles where they meet.
- Segments of *perpendicular lines* can also be perpendicular.
- Line *EF* is *perpendicular* (⊥) to line *GH*.
- $\overleftrightarrow{EF} \perp \overleftrightarrow{GH}$

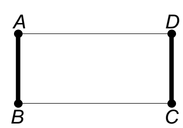
A perpendicular bisector divides a line segment in half.



- It meets the segment at right angles.
- Line *GH* is perpendicular (⊥) to line segment *EF*.
- Line *GH* splits line segment *EF* into two congruent parts.
- Line *GH* is the *perpendicular bisector* of line segment *EF*.
- $\overleftrightarrow{GH} \perp \overline{EF}$

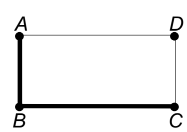
Opposite and Adjacent Sides

Opposite sides are *directly across* from each other.



- Side *AB* is *opposite* or directly across from side *CD*.
- The opposite sides do *not* share a vertex.

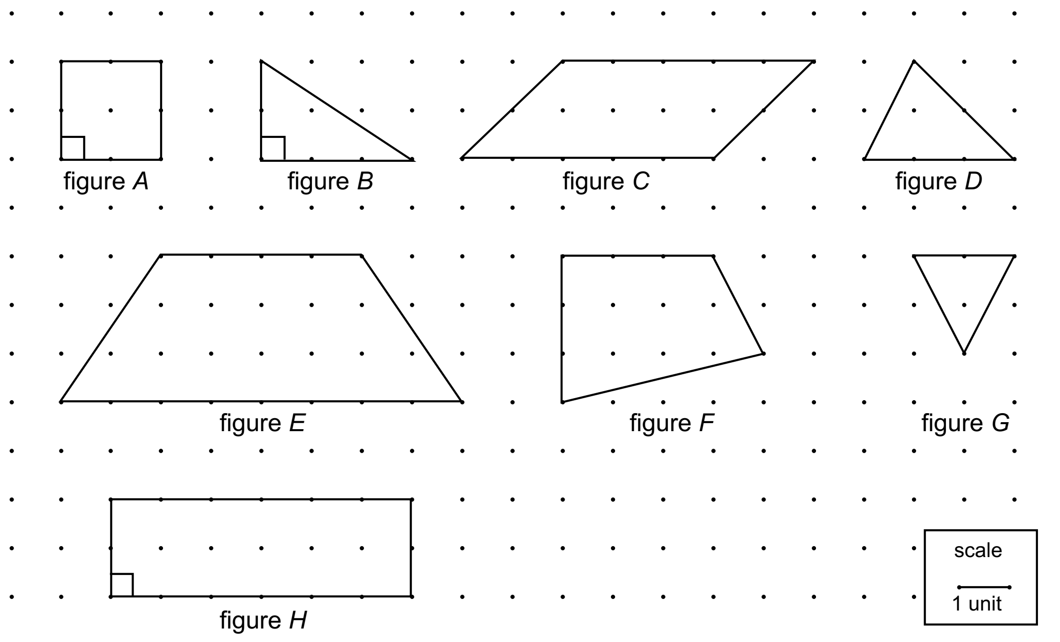
Adjacent sides are *next to* each other and share a common *vertex*.



- Side *AB* is *adjacent* or next to side *BC*.
- These adjacent sides share vertex *B*.

Practice

Complete the following.



Mike drew Figures *A-H* pictured above. Misty created descriptions for each one. One description applies to one and only one figure. For each description, indicate which of Mike's shapes Misty is describing.

____________________ 1. This figure has four sides. One pair of *opposite sides* is parallel. No *adjacent sides* are perpendicular. One pair of opposite sides is the same **length (*l*)**.

____________________ 2. This figure has four sides. Opposite sides are the same length and are parallel. Adjacent sides are perpendicular. The **area (*A*)** of this figure is 12 **square units**.

____________________ 3. This figure has three sides, and one side is perpendicular to another. The area of this figure is the same as the area of another of the figures provided.

_____________________ 4. This figure has four sides. One pair of the adjacent sides is perpendicular and the same length. Opposite sides are *not* parallel.

_____________________ 5. This figure has four sides. Opposite sides are parallel. Adjacent sides are perpendicular. All sides are equal in length.

_____________________ 6. This figure has three sides, and two of them are the same length. The area of the figure is 2 square units.

_____________________ 7. This figure has two pairs of opposite sides that are parallel. No adjacent sides are perpendicular.

_____________________ 8. This figure has no two sides of the same length, and no two sides are perpendicular. The area of this figure is the same as the area of another figure.

Practice

Match each definition with the correct term. Write the letter on the line provided.

______	1. sides that are *directly across* from each other	A. adjacent sides
______	2. sides that are *next to* each other and share a common vertex	B. congruent (≅)
______	3. two lines in the same plane that *never* meet	C. opposite sides
______	4. two lines that intersect to form right angles	D. parallel lines
______	5. figures or objects that are the *same shape* and the *same size*	E. perpendicular lines

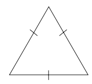
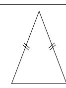
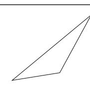
---

______	6. being an equal distance at every point so as to never intersect	A. area (A)
______	7. a line that divides a line segment in half and meets the segment at right angles	B. length (l)
______	8. a one-dimensional measure that is the measurable property of line segments	C. parallel (||)
______	9. units for measuring area; the measure of the amount of area that covers a surface	D. perpendicular bisector
______	10. the inside region of a two-dimensional figure measured in square units	E. square units

Classifying Triangles by Sides

As you learned in the previous lesson, triangles have three sides. They are classified in two different ways. Triangles are classified either by the measure of their angles or their sides. Triangles classified by their *sides* are called **equilateral triangles**, **isosceles triangles**, or **scalene triangles**—based on their number of congruent sides.

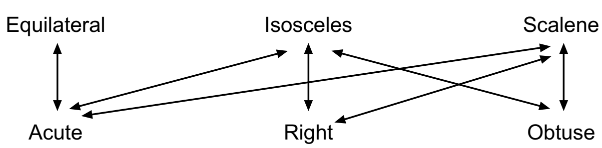
Triangles Classified by Their Sides

name	example	description
equilateral triangle	 **Sides marked the same way are congruent.***	**all sides congruent**
isosceles triangle		**two sides congruent**
scalene triangle		**no sides congruent**

* Congruent means they are exactly the same size.

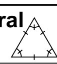
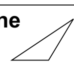
Overlapping Classifications of Triangles

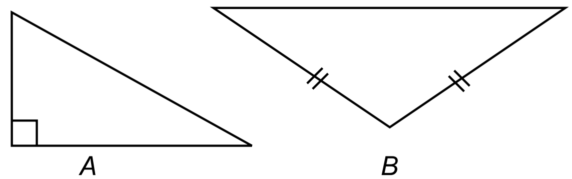
All triangles can be classified by their sides (equilateral, isosceles, scalene), by their angles (acute, right, obtuse), or both. This means that every triangle can be described in more than one way. However, no matter how a triangle is classified, the sum of the measures of the angles in a triangle is 180 degrees.



For example, to read the diagram above, a right triangle *may be* either isosceles or scalene, but *is never* acute, equilateral, or obtuse. Here is another way to express the same information.

Triangles

Classification	**Acute** < 90°	**Right** = 90°	**Obtuse** > 90° and <180°
Equilateral 	✓		
Isosceles 	✓	✓	✓
Scalene 	✓	✓	✓

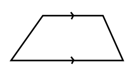
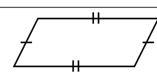
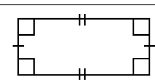
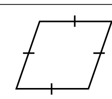
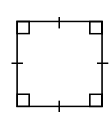


Triangle *A* is a right triangle. It is also a scalene triangle.
Triangle *B* is an obtuse triangle. It is also an isosceles triangle.

Classifying Quadrilaterals

Quadrilaterals are polygons that have four sides. They are classified according to their properties by special names.

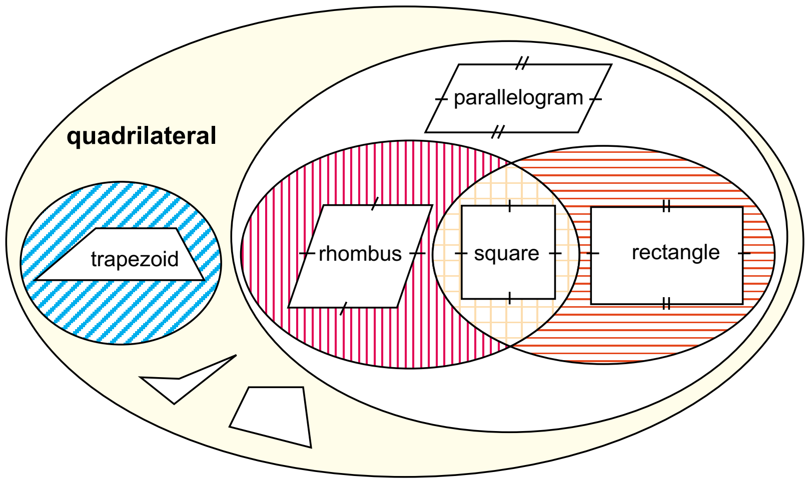
Quadrilaterals

name	example	description
trapezoid	 Sides marked the same are parallel.	exactly one pair of parallel sides
parallelogram	 Sides marked the same way are congruent.	opposite sides the same length and parallel
rectangle	 Angles marked with square corners are right angles.	parallelogram with four right angles
rhombus		parallelogram with four congruent sides
square		rectangle with four equal sides

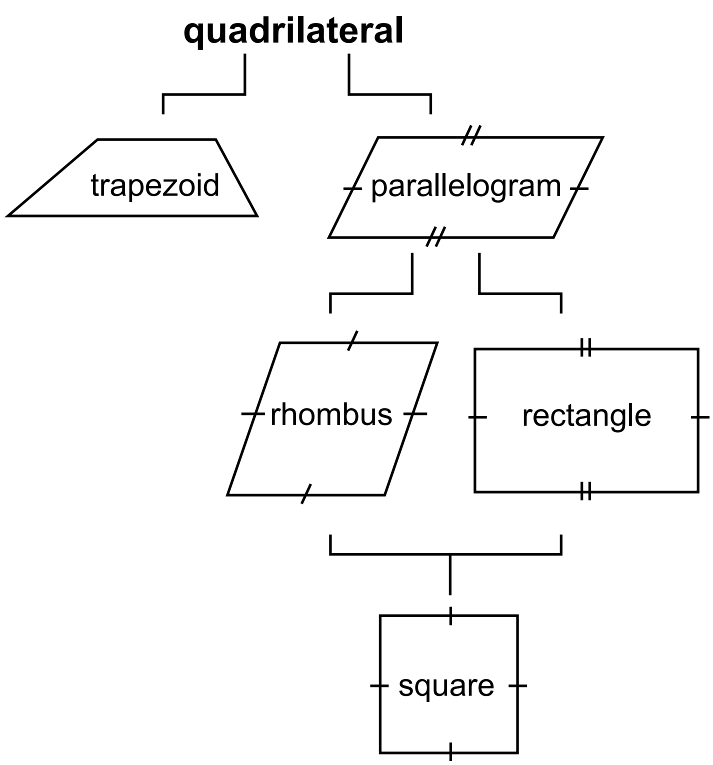
Overlapping Classifications of Quadrilaterals

Many quadrilaterals can be described using more than one name. The Venn diagram on the following page shows the relationships among different quadrilaterals.

Relationships among Different Quadrilaterals



Read the Venn diagram above or the graphic organizer below to see the relationships among different quadrilaterals. Note that a rhombus also *may be* a square, but a rhombus *is never* a trapezoid.



Practice

Complete the following.

Mike liked Misty's descriptions, but he also wants to be sure his students can name the figures, as well as match them to a description of characteristics. Remember that a *square* is also a *rectangle*, a *parallelogram*, a *rhombus*, and a *quadrilateral*, but the **best** name for it is a *square*. Choose the **best** name for each figure on page 195.

Match each **term** with the correct **figure** on page 195. Write the letter of the figure on the line provided.

1. scalene triangle ____________
2. scalene right triangle ____________
3. isosceles triangle ____________
4. quadrilateral ____________
5. trapezoid ____________
6. parallelogram ____________
7. rectangle ____________
8. square ____________

Practice

Complete the following.

Misty is ready to take this lesson a step further. She'd like her students to place all shapes into a set according to whatever characteristic she names. For each characteristic provided, indicate which shapes could be placed in a set having that characteristic from page 195.

Match each **description** with the correct **figures** on page 195. Write the letters on the line provided.

______________________ 1. All of these shapes are triangles.

______________________ 2. All of these shapes are quadrilaterals.

______________________ 3. All of these shapes are parallelograms.

______________________ 4. All of these shapes are rectangles.

______________________ 5. All of these shapes have at least one right angle.

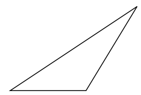
______________________ 6. All of these shapes have at least two congruent sides.

______________________ 7. All of these shapes have *no* two congruent sides.

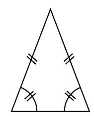
Practice

Match each **definition and illustration** *with the* **most** *correct term. Write the letter on the line provided.*

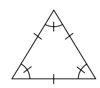
______ 1. a triangle with no congruent sides



______ 2. a triangle with *at least* two congruent sides and two congruent angles



______ 3. a triangle with three congruent sides and three congruent angles

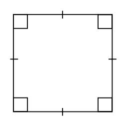


A. equilateral triangle

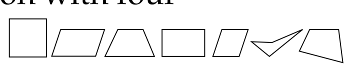
B. isosceles triangle

C. scalene triangle

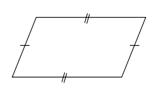
______ 4. a rectangle with four sides the same length



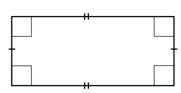
______ 5. polygon with four sides



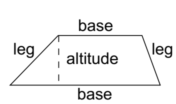
______ 6. a quadrilateral with two pairs of parallel sides



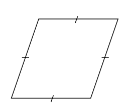
______ 7. a parallelogram with four right angles



______ 8. a quadrilateral with just one pair of opposite sides parallel



______ 9. a parallelogram with four congruent sides



A. parallelogram

B. quadrilateral

C. rectangle

D. rhombus

E. square

F. trapezoid

Practice

Complete the following statements with the correct answer. Refer to the **Mathematics Reference Sheet** *in Appendix A of this book for* **formulas** *for finding* **area**.

1. In a rectangle, opposite sides are ______________________ (parallel or perpendicular).

2. In a scalene triangle, the side opposite the largest angle is the ______________________ (shortest or longest) side.

3. In a rectangle, adjacent sides are ______________________ (parallel or perpendicular).

4. Since the sum of the measures of the angles in any triangle is 180 degrees, and the measures of the angles in an equilateral triangle are all the same, the measure of each angle in an equilateral triangle is __________ degrees.

5. If the **perimeter (*P*)** of an equilateral triangle is 21 units, the measure of each side is __________ units.

   (**Remember:** *Perimeter* is the *distance around* a figure.)

6. If the perimeter of an isosceles triangle is 21 units and one of the two congruent sides measures 8 units, the length of the **base (*b*)**, or third side, is __________ units.

7. In a parallelogram, the lengths of a pair of opposite sides are ______________________ (congruent, not congruent).

8. Adjacent sides in a parallelogram are ______________________ (sometimes, always, never) perpendicular.

9. In an **isosceles trapezoid**, the lengths of the pair of opposite sides that are *not* parallel are ______________________ (congruent, *not* congruent).

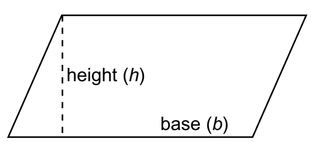
10. Since each angle in a rectangle is 90 degrees, the sum of the measures of the angles in a rectangle is __________degrees.

11. If the perimeter of a rectangle is 20 units and the width is 4 units, then the length is __________ units.

12. If the perimeter of a square is 20 square units, then the length of each side is __________ units.

13. If the area (A) of a parallelogram is 16 square units and the base (b) is 8 units, then the **height (h)** is __________ units.

(**Remember:** The *area of a parallelogram* is *base* times *height*. $A = bh$.)



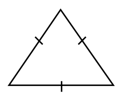
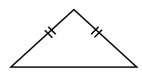
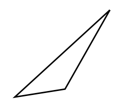
parallelogram

Classifying Polygons

A polygon is a closed plane figure formed by *line segments* called *sides*. Most polygons are named for the number of sides and the number of angles they have.

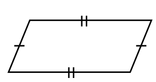
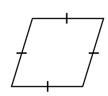
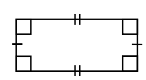
Triangles are polygons that have three sides. Some triangles have special names. (Also see page 198.)

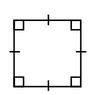
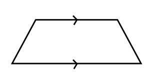
Triangles

		
equilateral triangle	**isosceles triangle**	**scalene triangle**
has three congruent sides	has at least two congruent sides	has no congruent sides

Quadrilaterals are polygons that have four sides. Some quadrilaterals have special names. (Also see page 199.)

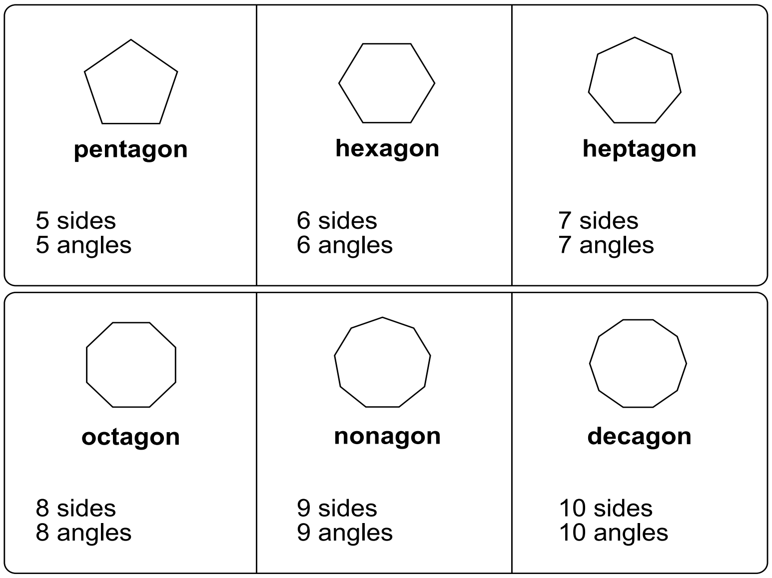
Quadrilaterals

		
parallelogram	**rhombus**	**rectangle**
a quadrilateral whose opposite sides are parallel and congruent	a parallelogram with four congruent sides	a parallelogram with four right angles

	
square	**trapezoid**
a rectangle with four congruent sides	a quadrilateral with two and only two sides parallel

These polygons are also named for the number of sides and angles they have.

Polygons



Polygons

Name of Polygon	Prefix	Meaning of Prefix	Number of Sides and Angles
triangle	tri	three	3
quadrilateral	quadri	four	4
pentagon	penta	five	5
hexagon	hexa	six	6
heptagon	hepta	seven	7
octagon	octa	eight	8
nonagon	nona	nine	9
decagon	deca	ten	10

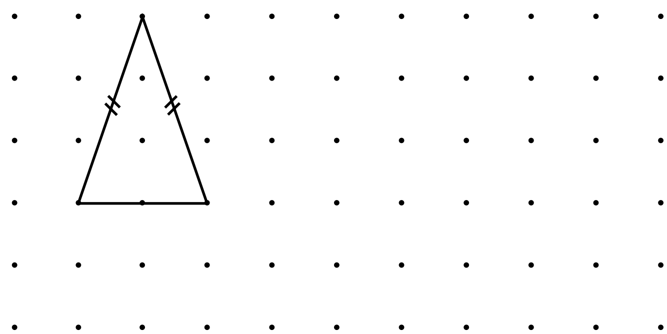
Practice

Use square dot paper provided below and on the following page (or your teacher may give you isometric dot paper or unlined paper) to **sketch an illustration** *of each shape listed below. You may use a straight edge, ruler, protractor, and/or angle ruler.* **Label** *each sketch.*

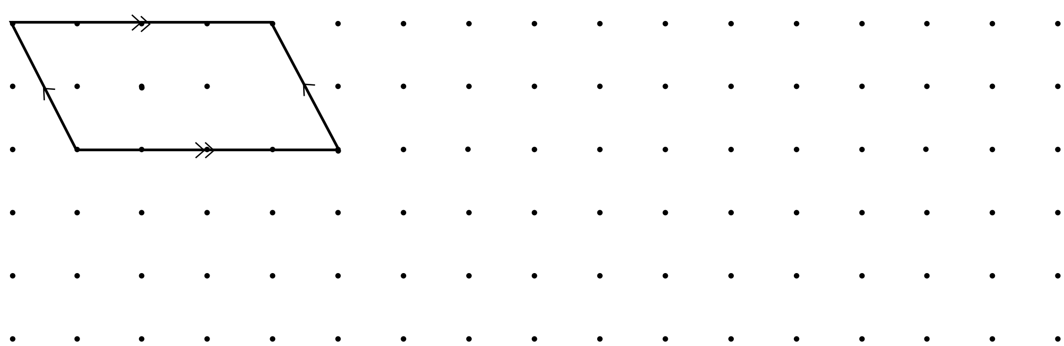
1. polygon
2. triangle
3. quadrilateral
4. pentagon
5. hexagon
6. heptagon
7. octagon
8. nonagon
9. decagon
10. scalene triangle
11. isosceles triangle
12. equilateral triangle
13. parallelogram
14. rectangle
15. square

Hint: Note the markings on drawings of the shapes in this practice. You will see these markings in the future, and it will be important for you to know how to interpret them.

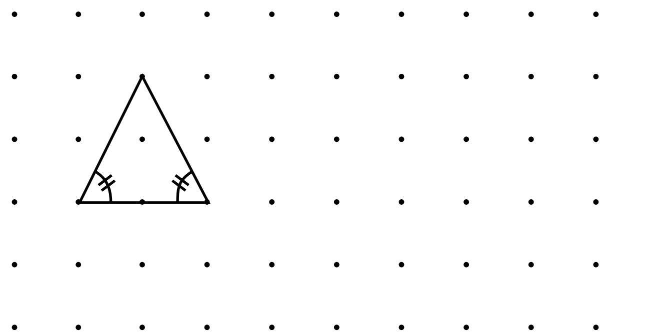
16. Sketch another isosceles triangle, and place two marks on each of the two congruent sides to indicate they are congruent. Your triangle should look somewhat like this.



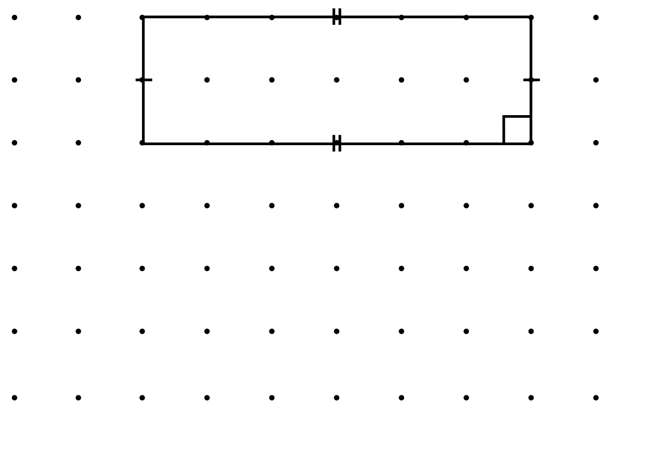
17. Sketch another parallelogram that is *not* a rectangle. Place one arrow on each side of one pair of opposite sides and two arrows on each side of the other pair of opposite sides to indicate that opposite sides are parallel. Your parallelogram should be somewhat like this.



18. Sketch another isosceles triangle, and place two marks on each of the two angles that are congruent to each other. Your triangle should look somewhat like this.

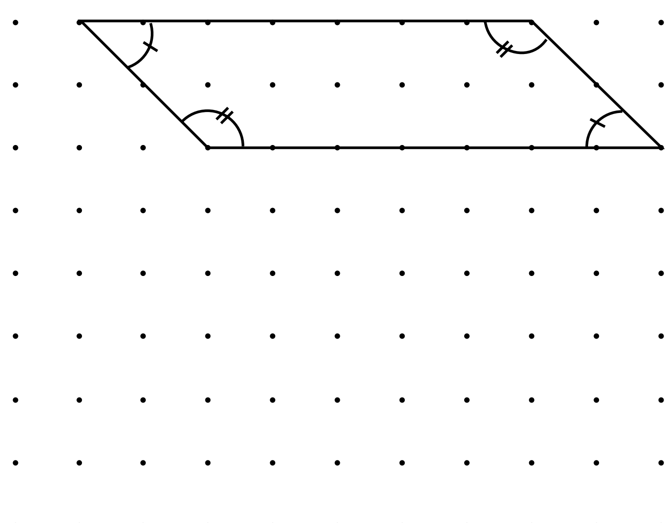


19. Draw a rectangle that is *not* a square. Place one mark on each side of one pair of opposite sides that are congruent, and place two marks on each side of the other pair of opposite sides that are congruent. Remember to also indicate that adjacent sides are perpendicular by showing a right angle. Your rectangle should look somewhat like this.



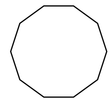
Hint: You may want to verify this congruency by either using a protractor or angle ruler to measure; by cutting two congruent parallelograms out and turning one to compare; by folding one over another; or in another way you might choose.

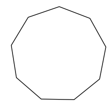
20. Draw another parallelogram that is *not* a rectangle. One pair of opposite angles will be acute angles as well as congruent to each other. The other pair of opposite angles will be obtuse angles as well as congruent to each other. We can indicate this by marking one pair with one mark and the other pair with two marks. It makes no difference which pair has one mark. Include this on your sketch. Your parallelogram should look somewhat like this.

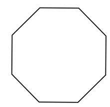


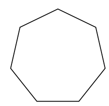
Practice

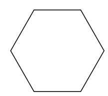
Match each definition with the correct term. Write the letter on the line provided.

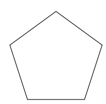
______ 1. a polygon with ten sides  A. decagon

______ 2. a polygon with nine sides  B. heptagon

______ 3. a polygon with eight sides  C. hexagon

______ 4. a polygon with seven sides  D. nonagon

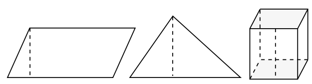
______ 5. a polygon with six sides  E. octagon

______ 6. a polygon with five sides  F. pentagon

Practice

Match each definition with the correct term. Write the letter on the line provided.

_______ 1. a line segment extending from the vertex or *apex* (highest point) of a figure to its base and forming a right angle with the base or basal plane



_______ 2. the line or plane upon which a figure is thought of as resting

_______ 3. the length of the boundary around a figure; the distance around a polygon

_______ 4. a way of expressing a relationship using variables or symbols that represent numbers

_______ 5. a trapezoid with congruent legs and two pairs of congruent base angles

A. base (b)

B. formula

C. height (h)

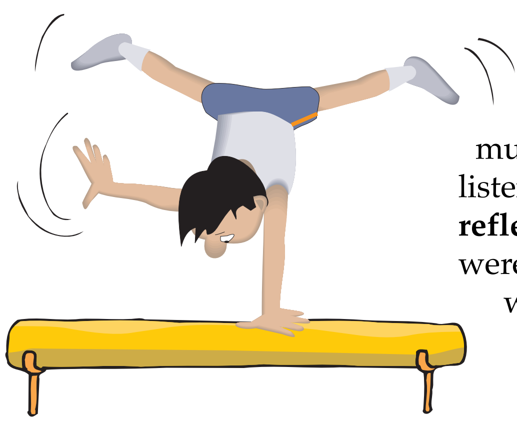
D. isosceles trapezoid

E. perimeter (P)

Lesson Three Purpose

- Understand the geometric concepts of symmetry, reflections, and transformations, including flips, slides, and turns. (C.2.3.1)
- Represent and apply geometric properties and relationships to solve real-world and mathematical problems. (C.3.3.1)
- Identify and plot ordered pairs in all four quadrants of a rectangular coordinate system (graph). (C.3.3.2)

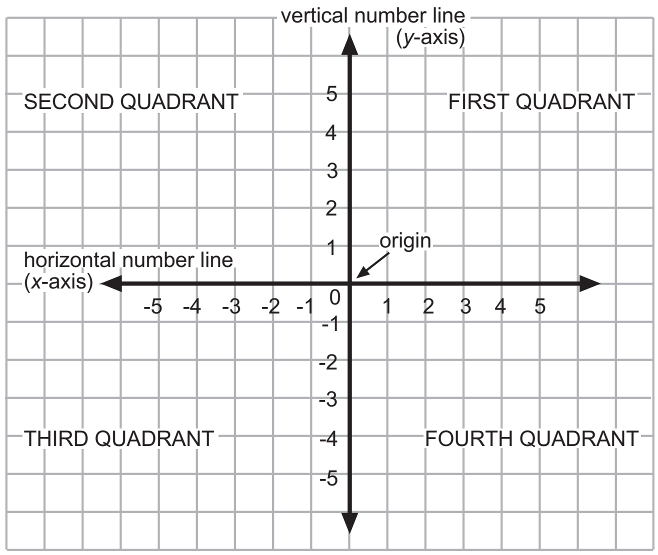
Moving Shapes on a Flat Surface



Mike and Misty enjoy gymnastics. When they were first introduced to the terms **flips**, **slides**, and **turns** in mathematics, they thought they must be in a gymnastics session listening to their coach. The words, **reflections**, **translations**, and **rotations** were used, and they thought, "Oops, we're back in math class!" We will learn more about these terms and **transformations**, the ways geometric figures can be changed or *transformed* in this lesson.

A Coordinate Grid or System

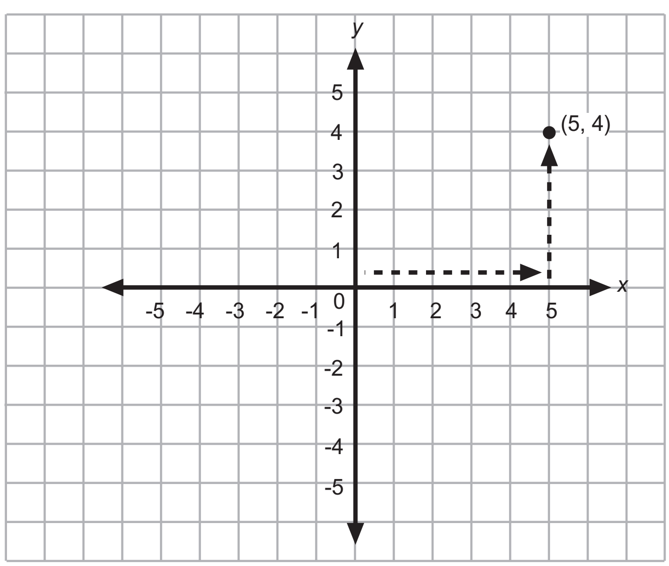
Let's first review what we know about a **coordinate grid or system**. The **grid** on the following page is called a *coordinate grid or system*. It has a **horizontal (⟷) number line** (*x*-**axis**) and a **vertical (↕) number line** (*y*-**axis**). These two number lines or **axes of a graph** *intersect* or meet at the **origin**. The **coordinates** of the origin are (0, 0.) The axes form four regions or **quadrants**. However, the origin and the axes are not in any quadrant.



coordinate grid or system

To locate **ordered pairs** or *coordinates* such as (5, 4) on a coordinate system, do the following.

- start at the origin (0, 0) of the grid
- locate the first number of the ordered pair or the ***x*-coordinate** on the *x*-axis (⟷)
- then move parallel (||) to the *y*-axis and locate the second number of the ordered pair or the ***y*-coordinate** on the y-axis (↕) and draw a point



Practice

Match each definition with the correct term. Write the letter on the line provided.

______ 1. network of evenly spaced, parallel horizontal and vertical lines especially designed for locating points, displaying data, or drawing maps

______ 2. parallel to or in the same plane of the horizon

______ 3. at right angles to the horizon; straight up and down

______ 4. a line on which numbers can be written or visualized

______ 5. the horizontal (↔) axis on a coordinate plane

______ 6. the vertical (↕) axis on a coordinate plane

______ 7. the horizontal and vertical number lines used in a rectangular graph or coordinate grid system as a fixed reference for determining the position of a point

A. axes (of a graph)

B. coordinate grid or system

C. horizontal (↔)

D. number line

E. vertical (↕)

F. x-axis

G. y-axis

Practice

Use the list below to write the correct term for each definition on the line provided.

coordinates	**quadrant**
ordered pair	***x*-coordinate**
origin	***y*-coordinate**
parallel	

____________________ 1. the intersection of the x-axis and y-axis in a coordinate plane, described by the ordered pair (0,0)

____________________ 2. numbers that correspond to points on a graph in the form (x, y)

____________________ 3. any of four regions formed by the axes in a rectangular coordinate system

____________________ 4. the location of a single point on a rectangular coordinate system where the digits represent the position relative to the x-axis and y-axis
Example: (x, y) or (3, 4)

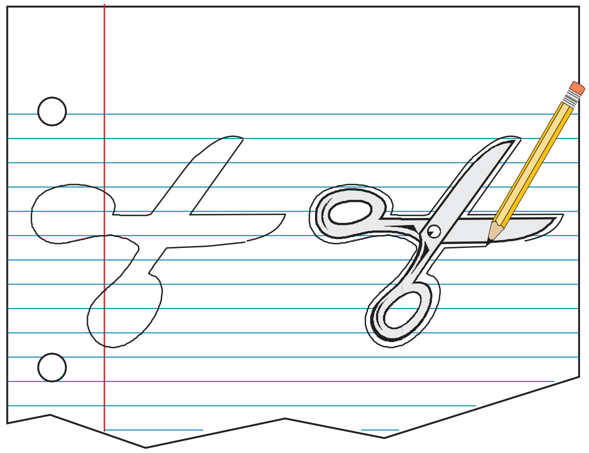
____________________ 5. the first number of an ordered pair

____________________ 6. being an equal distance at every point so as to never intersect

____________________ 7. the second number of an ordered pair

Transformations

Whenever you move, shrink, or enlarge a figure, you make a *transformation* of that figure. Supposed you trace a figure on a piece of paper. Then you move the figure and trace it again. You have just made a transformation of the first figure.

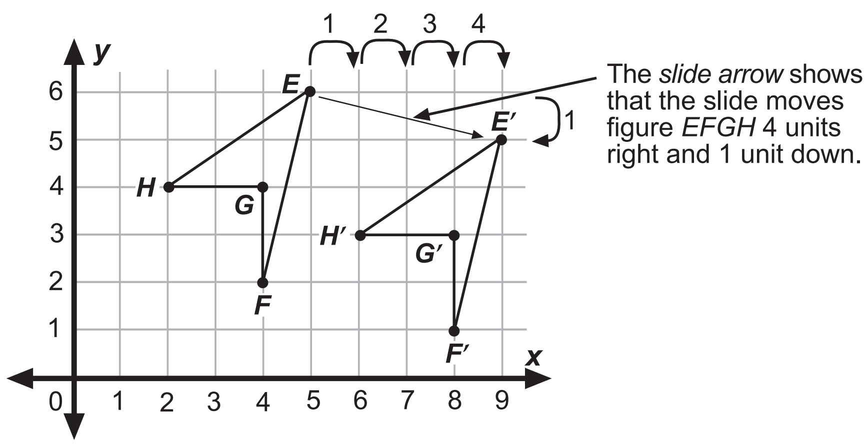

transformation

What are some of the ways you can move a figure?

Slide or Translation

You can slide a figure by moving it along a surface. The slide may be in a vertical (↕), horizontal (↔), or diagonal (↘) direction. A *slide* is also called a *translation*.

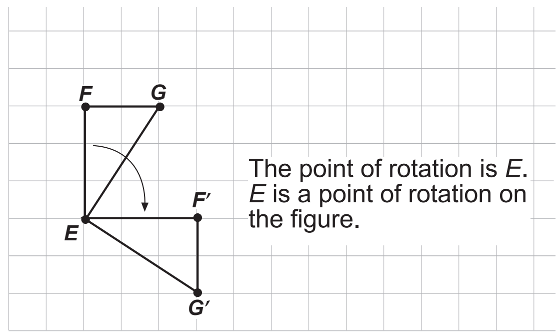
In a slide or translation, every point in the figure slides the *same distance* and in the *same direction*. Use a *slide arrow* to show the direction and the distance of the movement. The slide arrow has its endpoint on one point of the first figure and its arrow on the corresponding point in the second figure.

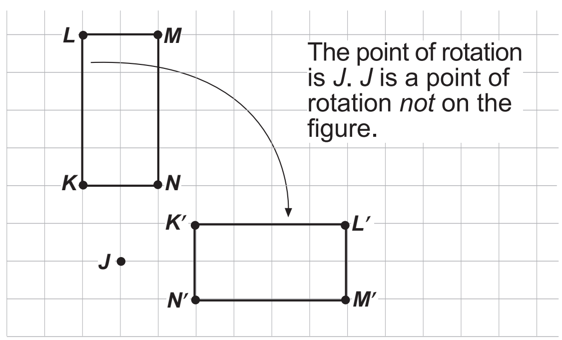


The figure to be moved is labeled with letters *EFGH*. It was moved 4 units to the right and 1 unit down. The transformed image (the second figure) is labeled with the same letters and a prime sign (′). You read the second figure as *E* prime, *F* prime, *G* prime, *H* prime.

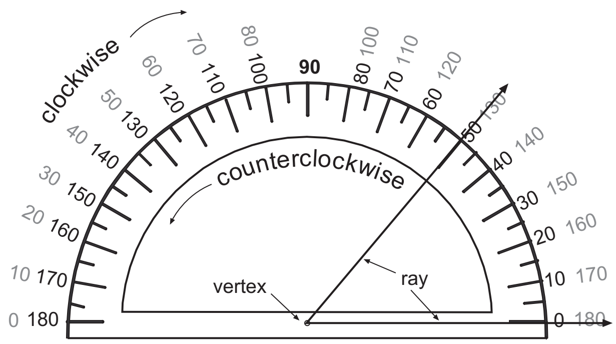
Turn or Rotation

When something rotates, it turns. You can rotate a figure by turning the figure around a point. The point can be on the figure or it can be some other point. This point is called the *turn center*, or *point of rotation*.





Measuring Angle of Turn or Rotation around a Point. A protractor is used to measure the angle of rotation around a point. Protractors are marked from 0 to 180 degrees in a clockwise manner as well as a counterclockwise manner. We see 10 and 170 in the same position. We see 55 and 125 in the same position. If we estimate the size of the angle before using the protractor, there is no doubt which measure is correct.



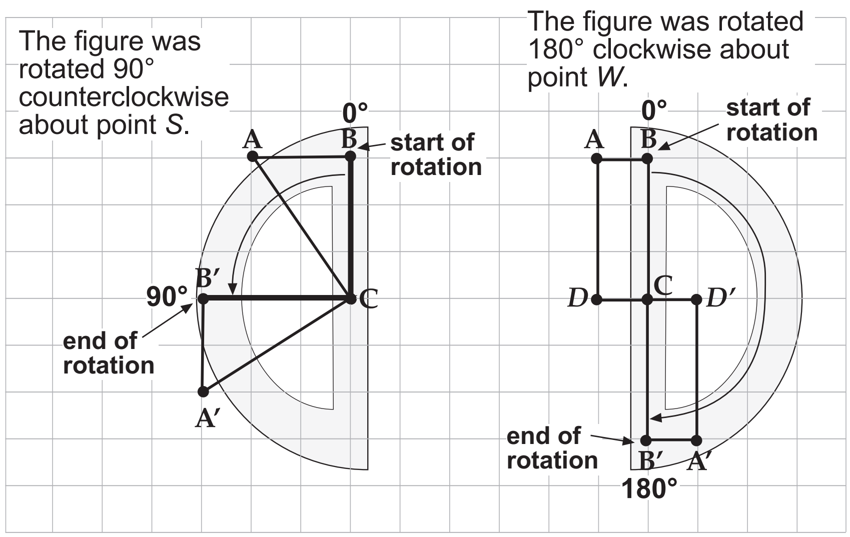
When using a protractor, make sure the vertex is lined up correctly and that one *ray* (⟶) passes through the zero measure. A straightedge is often helpful to extend a ray for easier reading of the measure.

The figures on the following page illustrate rotation around a point. Each figure can be rotated around a point. The image produced by the rotation is congruent to the original figure. A protractor is used to measure the angle of rotation.

- Place the center of the protractor on the point of the rotation.
- Line up the 0 degree mark with the start of the rotation.
- Use a straightedge to extend the line for easier reading of the measure.
- Rotate the figure and mark the place the rotated figure (the new image) matches the original figure.
- Extend the line with a straightedge and note the degree on the protractor.

The point of rotation represents the vertex of the angle. The ray through 0 degrees lines up with a point on the original figure. The ray through its corresponding point on the image passes through the measure of the *angle of rotation*.

Note that the measure of the angle of rotation for the first figure below is 90 degrees and the second figure is 180 degrees.

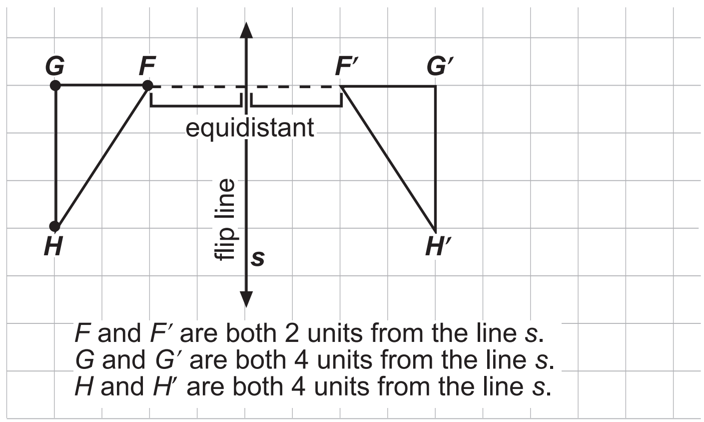


When you rotate a figure, you can describe the rotation by giving the direction (clockwise or counterclockwise) and the angle that the figure is rotated around the point of rotation. The amount of rotation is expressed in the number of degrees it was rotated from its starting point.

Flip or Reflection

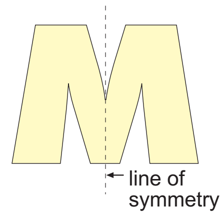
The reflection you see in a mirror is the reverse image of what you are looking at. In geometry, a reflection is a transformation in which a figure is flipped over a line. That is why a *reflection* is also called a *flip*. You flip the figure over a line.

Each point in a reflection image is **equidistant** (the same distance) from the line of reflection, called a *flip line*, as the corresponding point in the original figure.



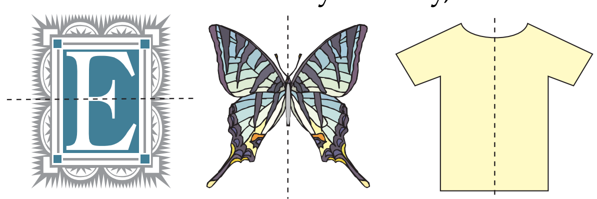
Symmetry

If a figure can be folded along a line so that it has two parts that are congruent and match exactly, that figure has **line symmetry**. *Line symmetry* is often just called **symmetry**. The *fold line* is called the **line of symmetry**.

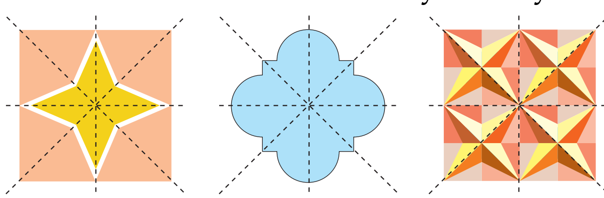


A figure can have no lines of symmetry,

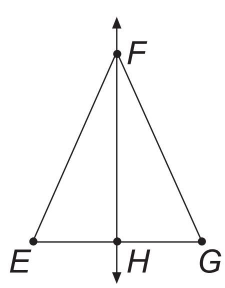
one line of symmetry,



or more than one line of symmetry.



Suppose triangle *EFG* below is reflected over line *FH*. The image of *E* is *G*. The image of *G* is *E*. And the image of *F* is *F* itself. The entire triangle corresponds with its reflection image. The triangle is symmetric with respect to line *FH*. It has line symmetry.



Types of Symmetry: Reflectional, Rotational, and Translational

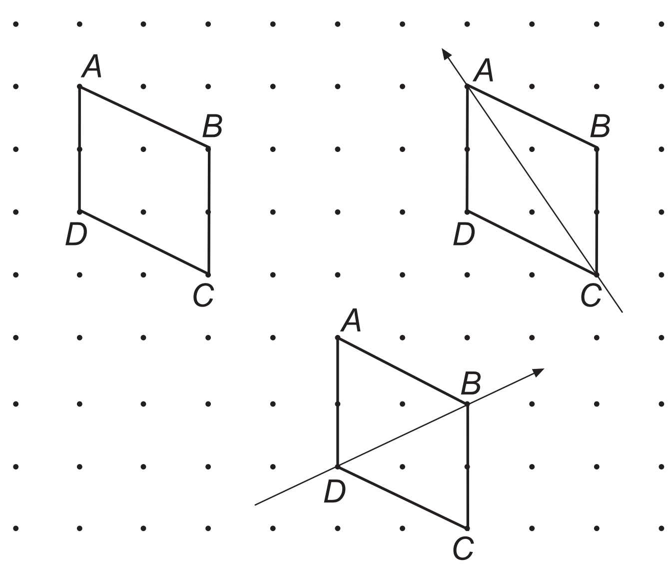
- A figure can have **reflectional symmetry**. If a figure has at least one line of symmetry, a figure has *reflectional symmetry*. Once split, one side is the *mirror image* or *reflection* of the other. It is even possible to have more than one line of reflectional symmetry.

- A figure can have **rotational symmetry**, also called *turn symmetry*. If a figure can *rotate* about its center point to a position that appears the same as the original position, it has *turn* or *rotational symmetry*. The amount of rotation is usually expressed in degrees.

- A figure can have **translational symmetry**. If a figure can *slide* on a **plane** (or *flat surface*) without turning or flipping and have its opposite sides stay congruent, it has *translational symmetry*. The distance and direction of the slide are important.

Which figures on the previous page do you think have reflectional symmetry? Which have turn or rotational symmetry? Which one could have translational symmetry?

Remember: Lines of symmetry do *not* have to be horizontal (↔) or vertical (↕). Lines of symmetry can also be diagonal (⤡).



Let's continue to explore symmetry with the practices that follow.

Practice

Complete the following.

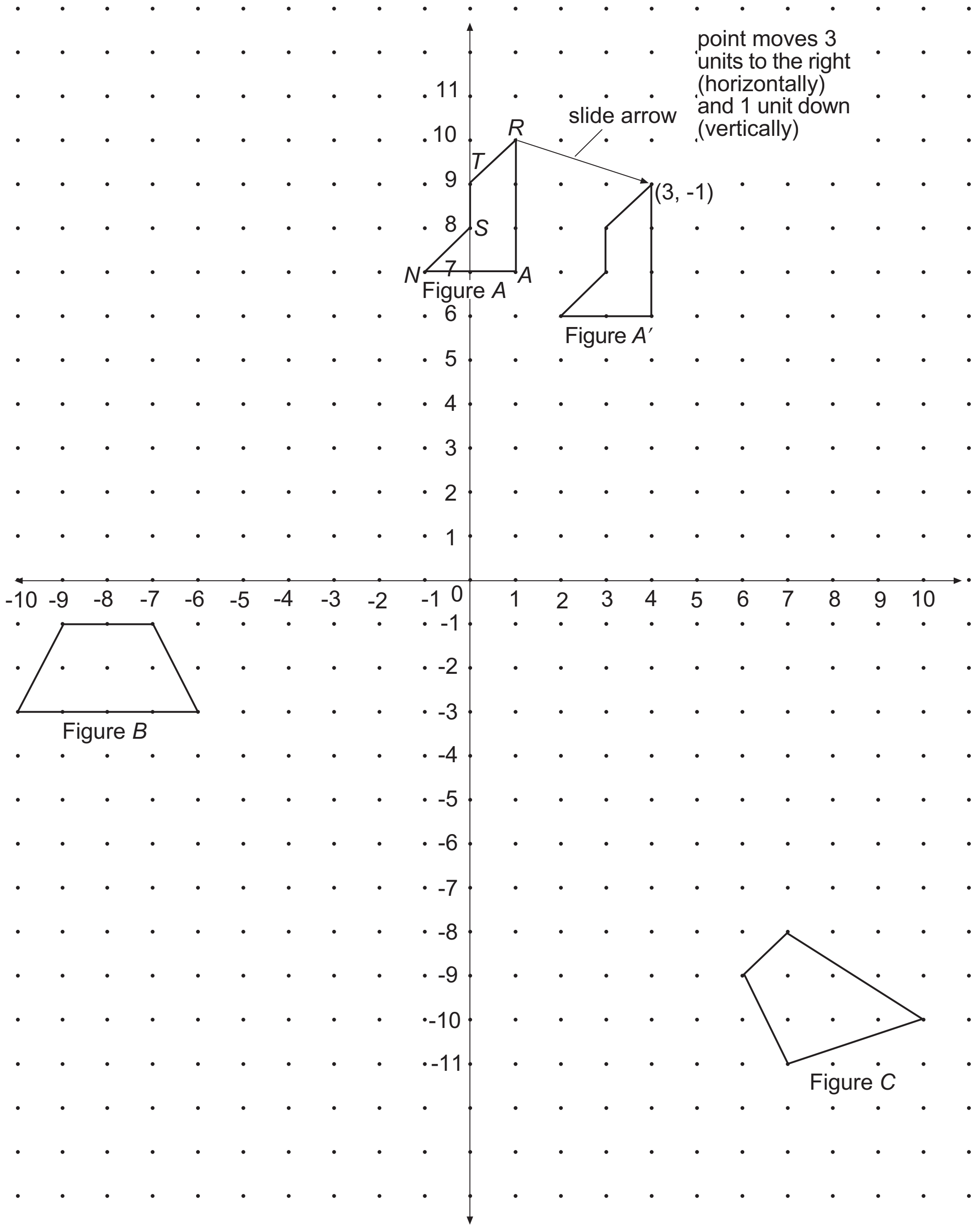
To describe a *translation*, or *slide*, a slide arrow is used to connect to show the direction of the slide. An ordered pair is used to indicate the number of units the point moves horizontally ($\leftrightarrow$) and vertically ($\updownarrow$).

- If the first member of the ordered pair is positive, the horizontal movement is to the right, and if it is negative, the horizontal movement is to the left.
- If the second member of the ordered pair is positive, the vertical movement is up, and if it is negative, the vertical movement is down.

Use the figures on the following page for numbers 1-3.

In the example on the following page, a *slide arrow* connects corresponding points on Figures *A* and *A′* (read as *A* prime). That point moves three units to the right (horizontally) and one unit down (vertically). The ordered pair describing the slide would be (3, -1). This ordered pair applies to each of the points on Figure *A*.

1. Locate and sketch one additional slide for Figure *A*.
2. Locate and sketch two successive slides for Figure *B* if the ordered pair describing the slide is (1, 4).
3. Locate and sketch two successive slides for Figure *C* if the ordered pair describing the slide is (-5, 1).



point moves 3 units to the right (horizontally) and 1 unit down (vertically)
slide arrow
R
T
S
N
A
(3, -1)
Figure A
Figure A′
Figure B
Figure C

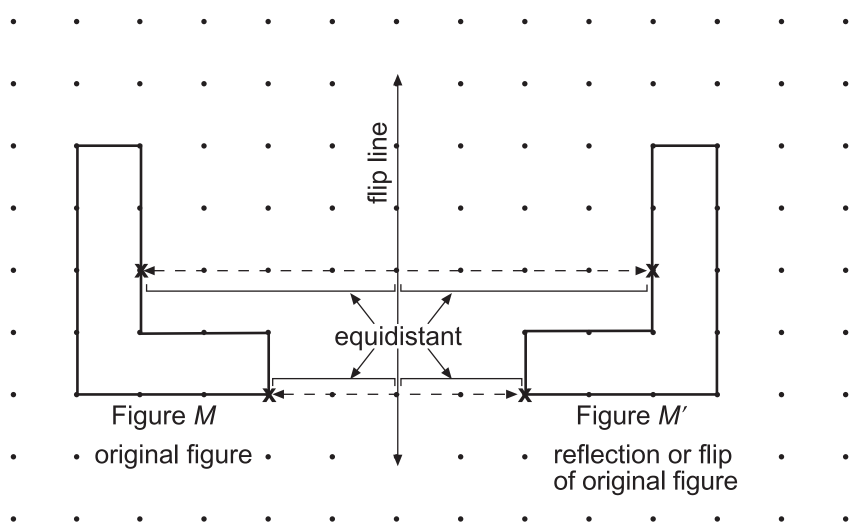
4. Use the sheet of dot paper below to create your own design. Provide the description for translating, or sliding it. Produce a minimum of two slides.

5. Find one example of the use of translation in the real world. Attach a sample or make a sketch and record your source.

Practice

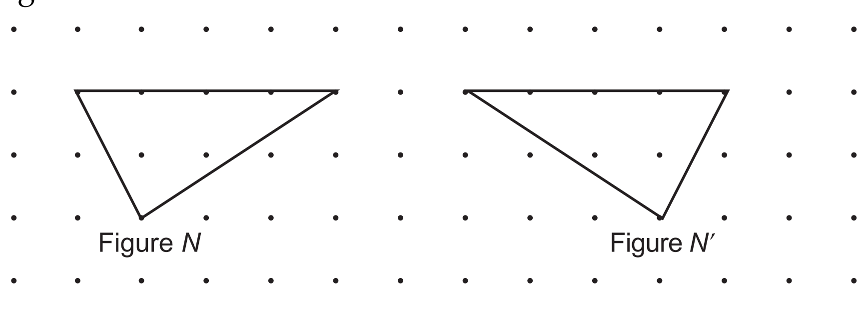
To describe a *reflection*, or *flip*, a line of reflection called a *flip line* is provided. The distance between a point on the original figure to the corresponding point on the reflected figure is *equidistant*, or the same distance, from the flip line. A line segment drawn between these two corresponding points will be *perpendicular* (⊥) to the flip line. We can therefore say the flip line is a *perpendicular bisector* (see page 194) of a line segment connecting any point on the original figure to its corresponding point on the reflected figure.

In the example below, Figure *M* has been flipped over the line shown. The line segment connecting a point on Figure *M* to its matching point on Figure *M′* is perpendicular to the flip line.

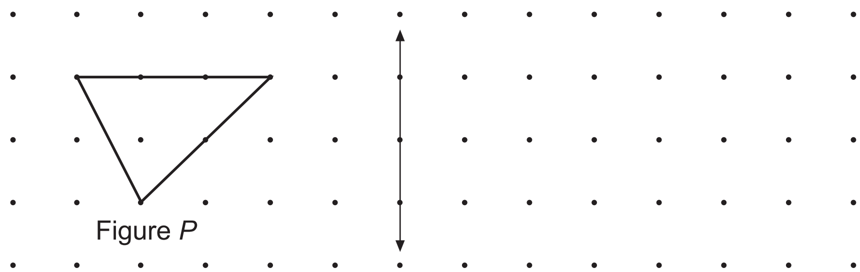


reflection and corresponding points

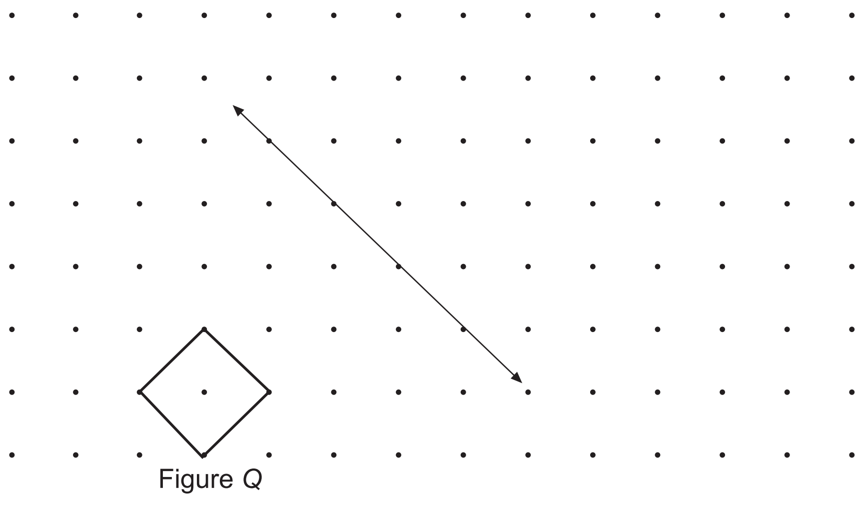
1. Draw the flip line over which Figure *N* was flipped producing Figure *N′*.



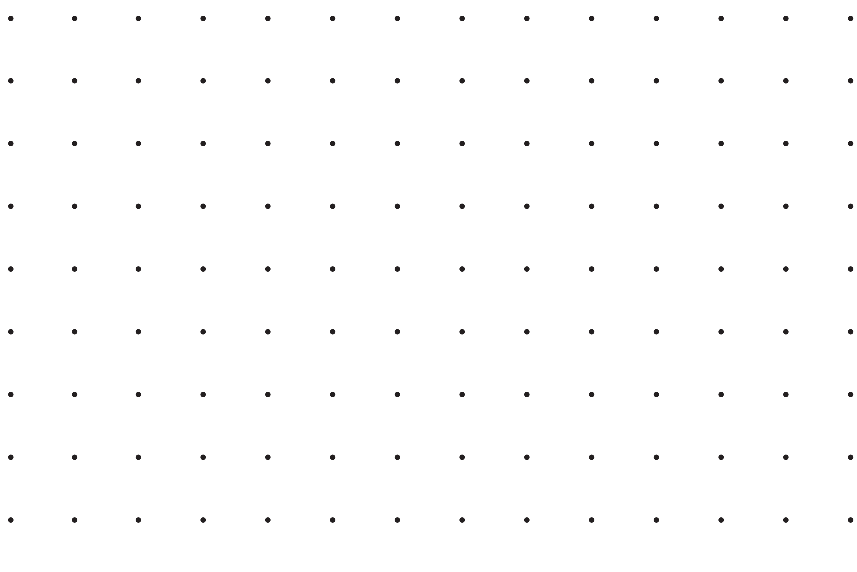
2. Flip Figure *P* over the flip line provided.



3. Flip Figure *Q* over the flip line provided.



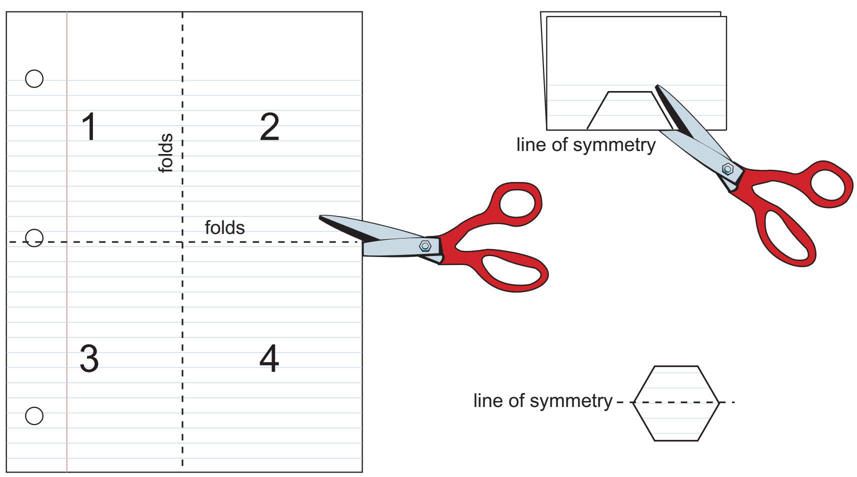
4. Create a design of your own and flip it over a line of your choosing.



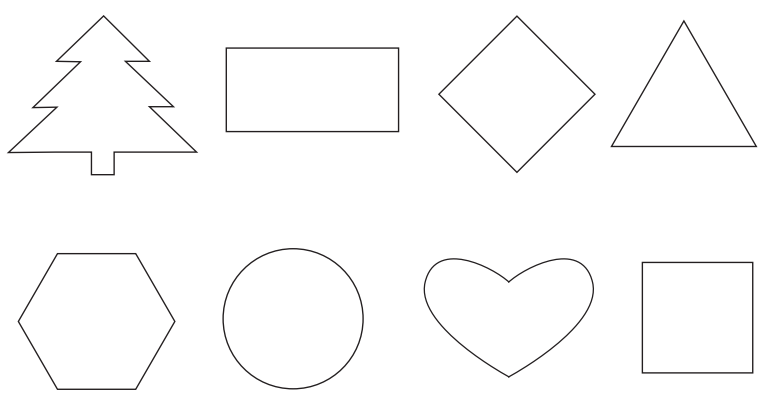
5. Find an example of flips or reflections in the real world. Share your findings with the class.

6. Fold a piece of paper into fourths. Cut along the folds. Now fold each of the four separate pieces in half. The crease, where the paper was folded, represents a *line of symmetry*.

   Choose any four of the eight designs below or other designs of your own. Cut half of the design out to produce the whole design. Unfold the cut out design. Each of your designs should have *reflection symmetry* or line symmetry.



(**Remember:** A figure has line symmetry when it can be folded along a line so that the two halves match exactly.)

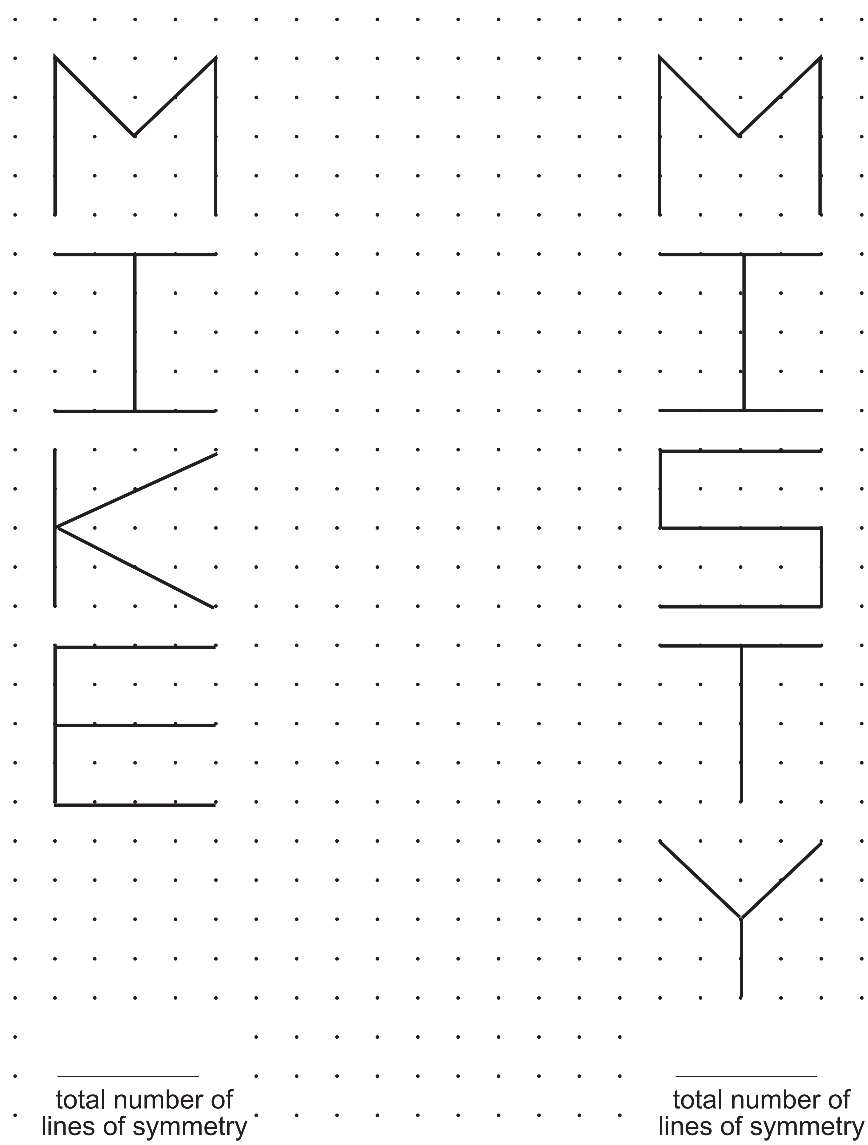


7. See the letters below. Mike and Misty find the examples of line symmetry in the letters of their names to be interesting. Mike claims that his name has fewer letters but more lines of symmetry than Misty's. Find the total number of lines of symmetry in the letters of their names.

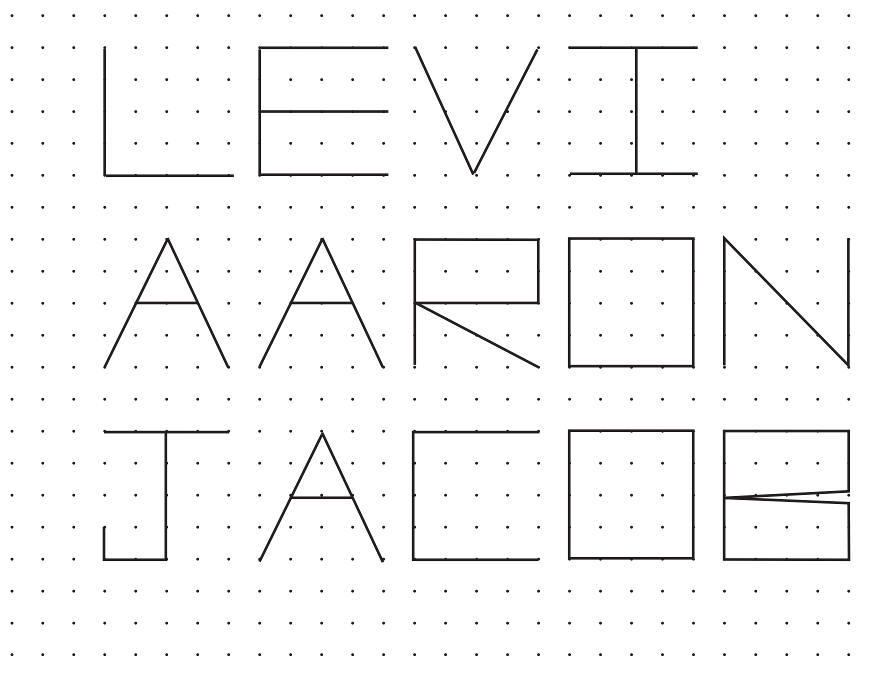
Write the number of lines of symmetry for each name below. Then explain whether or not Mike is correct.

Explanation: ____

____



8. The names of three students in Mike's class are Levi, Aaron, and Jacob. Mike prints the three names as illustrated below and asks his students to determine which letters have 1 line of symmetry, 2 lines, and so on. Sketch all lines of symmetry for each letter, if any exist. Complete the chart.



Letters and Lines of Symmetry

Number of Lines of Symmetry	Letters
0	
1	
2	
3	
4	
5	

9. Which of the names in number 8 had the least number of lines of symmetry? How many lines of symmetry did it have?

10. Write your first, middle, and last name on square dot paper. Sketch the lines of symmetry, if any, in each letter. Compare your name with the names of classmates.

Practice

Use the list below to write the correct term for each definition on the line provided.

equidistant	**plane**
line of symmetry	**symmetry**
perpendicular bisector	**transformation**

________________________ 1. when a line can be drawn through the center of a figure such that the two halves are congruent

________________________ 2. a flat surface

________________________ 3. a line that divides a figure into two congruent halves that are mirror images of each other

________________________ 4. a line that divides a line segment in half and meets the segment at right angles

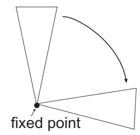
________________________ 5. equally distant

________________________ 6. an operation on a geometric figure by which another image is created

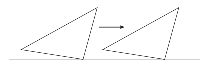
Practice

Match each definition with the correct **transformation**. *Write the letter on the line provided.*

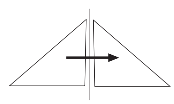
______ 1. a transformation of a figure by *turning* it about a center point or axis; also called a *turn*



______ 2. to move along in constant contact with the surface in a vertical, horizontal, or diagonal direction; also called a *translation*



______ 3. a transformation that produces the mirror image of a geometric figure; also called a *reflection*



A. flip

B. rotation

C. slide

Match each definition with the correct **type of symmetry**. *Write the letter on the line provided.*

______ 4. when a figure can slide on a plane (or flat surface) without turning or flipping and with opposite sides staying congruent

______ 5. when a figure has at least one line which splits the image in half, such that each half is the mirror image of the other; also called *line symmetry* or *mirror symmetry*

______ 6. when a figure can be turned less than 360 degrees about its center point to a position that appears the same as the original position; also called *turn symmetry*

A. reflectional symmetry

B. rotational symmetry

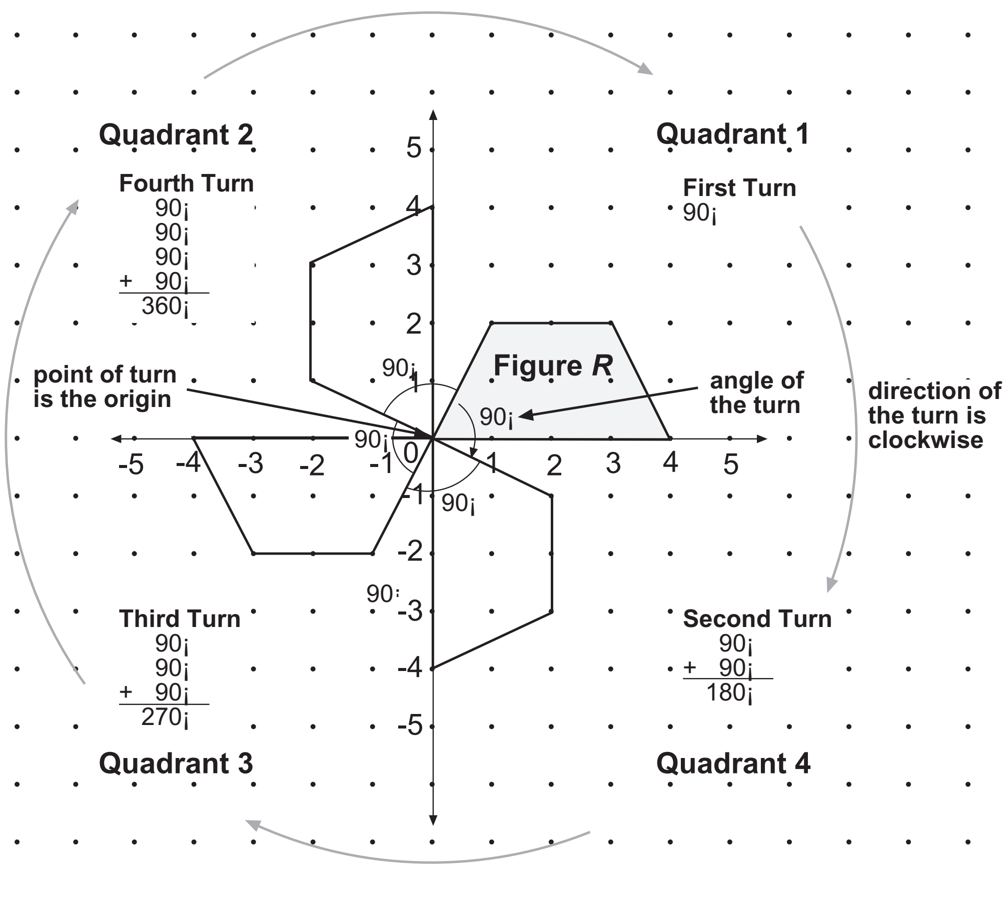
C. translational symmetry

Practice

Complete the following.

In order to describe a *turn* or *rotation*, the *point* of the turn, the *direction* of the turn, and the *angle* of the turn are provided. In the example below, Figure *R* has been turned about the origin in a clockwise movement 90 degrees, 180 degrees, and 270 degrees. The angle of rotation is said to be 90 degrees since that is the smallest angle of turn possible. The rotation could also be achieved in a counterclockwise direction.

It should be noted that if one rotation of 90 degrees was required for Figure *R*, the direction of turn would determine whether the image Figure *R′* was located in Quadrant 4 (clockwise turn) or Quadrant 2 (counterclockwise turn).



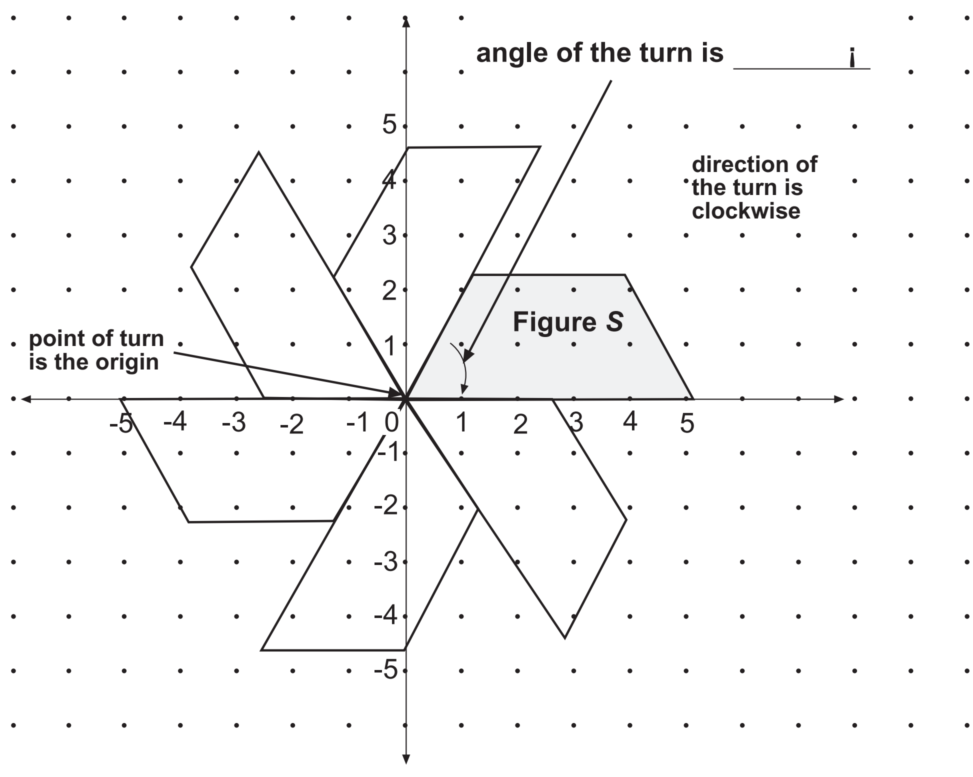
1. For Figure *S*, the point of turn is the origin. The direction of the turn is clockwise. What is the angle of turn? ______________________

   Explain how you determined the angle of turn. ______________

   ____

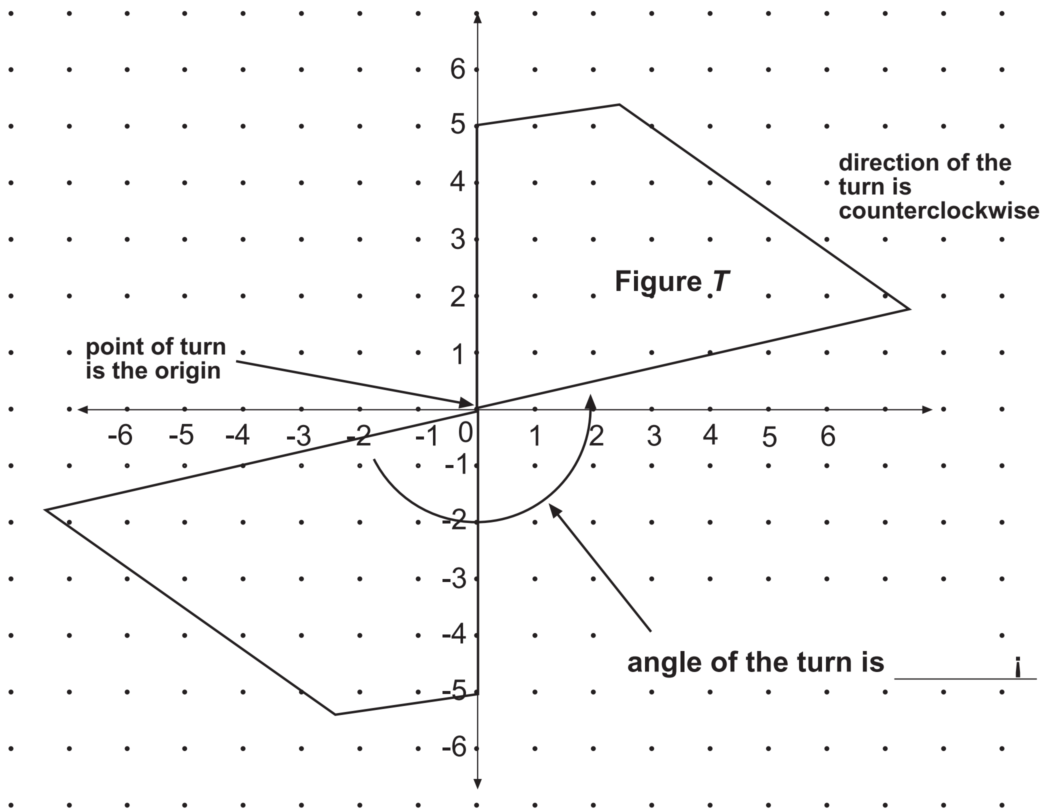
   ____

   ____



2. Figure T has been turned counterclockwise around the origin.

   What is the angle of turn for Figure T? ________________________



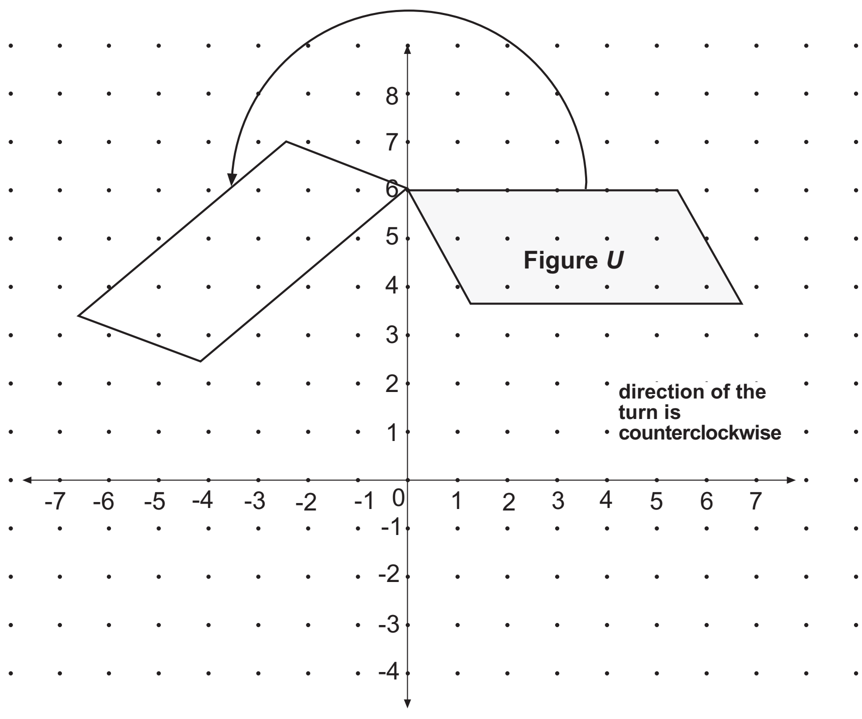
3. Figure *U* has been turned counterclockwise around the point (0,6). Which of the following estimates would be reasonable for the angle of turn?

   a. 60 to 100 degrees

   b. 100 to 140 degrees

   c. 140 to 180 degrees

   d. 180 to 220 degrees



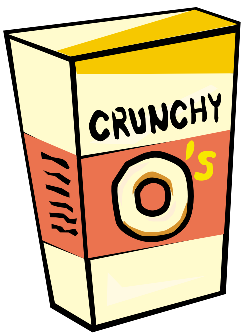
4. Design a figure and sketch at least one rotation of it. Specify the point around which the turn occurs, the angle of turn, and the direction of turn.

5. Find an example of rotations in the real world. Make a sketch and record your source. Share your findings with the class.

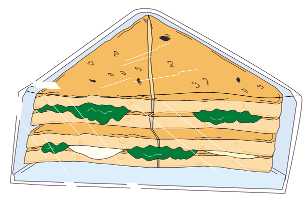
Lesson Four Purpose

- Understand the basic properties of, and relationships pertaining to, regular and irregular geometric shapes in three dimensions. (C.1.3.1)

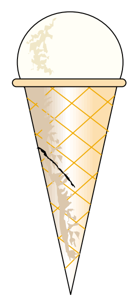
Regular and Irregular Three-Dimensional Shapes



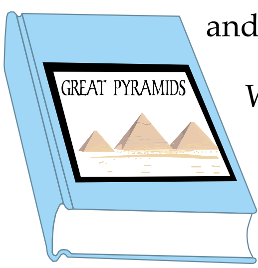
Three-dimensional shapes are encountered in daily living. Our mornings might begin with cereal from a **prism**, a **rectangular prism**.



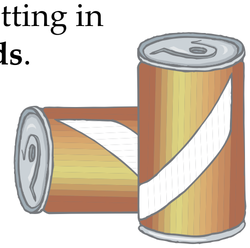
Our lunch may consist of a sandwich cut along a *diagonal* (⤡), with the two resulting triangular sandwiches stacked and packaged in a **triangular prism**.



A scoop of ice cream might represent a **sphere** and might be placed on top of a **cone**.



We might read a novel with a setting in Egypt and read about **pyramids**.



We might recycle empty cans which are in the shape of **cylinders**.

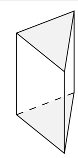
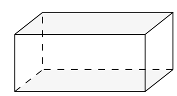
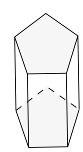
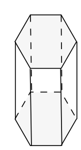
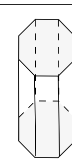
All of these three-dimensional shapes are **solid figures** that enclose part of space. *Solid figures* with flat surfaces are called **polyhedrons**. *Prisms* and *pyramids* are *polyhedrons*.

- A prism is a polyhedron with two parallel congruent *bases* and other **faces** that are all parallelograms.
- A pyramid is a polyhedron with a single base and other faces that are all triangles.
- A prism and a pyramid are identified by their bases.

Types of Prisms

There are many different kinds of prisms. A prism is named according to the shape of its bases.

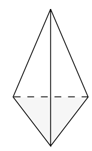
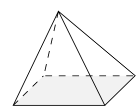
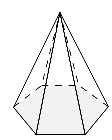
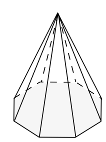
Prisms

Name of Polyhedron	Shape of Bases	Number of Lateral Faces	Example
triangular prism	**triangle**	**3**	
rectangular prism	**rectangle**	**4**	
pentagonal prism	**pentagon**	**5**	
hexagonal prism	**hexagon**	**6**	
octagonal prism	**octagon**	**8**	

Types of Pyramids

A pyramid is named according to the shape of its base. There are many different kinds of pyramids.

Pyramids

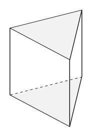
Name of Polyhedron	Shape of Bases	Number of Lateral Faces	Example
triangular pyramid	triangle	3	
square pyramid	square	4	
hexagonal pyramid	hexagon	6	
octagonal pyramid	octagon	8	

Practice

Complete the following.

Your task is to use geometric terms to describe each of the following three-dimensional solids. Mike provides the name of the three-dimensional *solid figure* and an illustration of it. Misty uses geometric terms to describe the triangular *prism* for you.

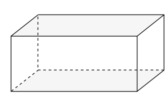
Mike provided an example of the triangular prism.

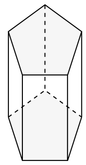
triangular prism 

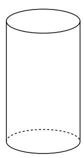
Then Misty described the triangular prism using geometric terms.

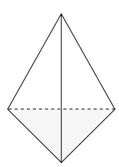
> The triangular prism is a 3-dimensional solid with two triangular bases that are congruent and parallel. It has 3 rectangular lateral faces. The total number of faces is 5.

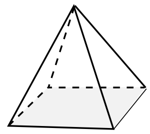
Now it is your turn to describe the following three-dimensional solids using geometric terms like Misty.

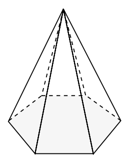
1. Rectangular prism 

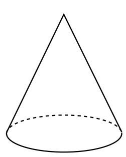
2. Pentagonal prism 

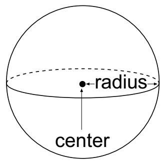
3. Cylinder 

4. Triangular pyramid 

5. Square pyramid 

6. Hexagonal pyramid 

7. Cone 

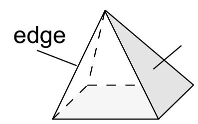
8. Sphere 

Practice

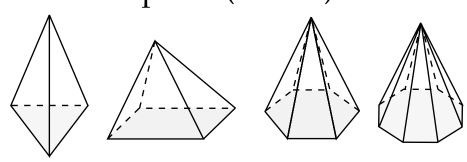
Use the list below to write the correct term for each definition on the line provided.

cone	**polyhedron**	**solid figures**
cylinder	**prism**	**sphere**
face	**pyramid**	**three-dimensional**

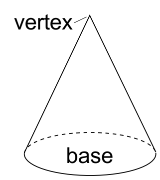
____________________ 1. one of the plane surfaces bounding a three-dimensional figure



____________________ 2. a three-dimensional figure (polyhedron) with a single base that is a polygon and whose faces are triangles and meet at a common point (vertex)

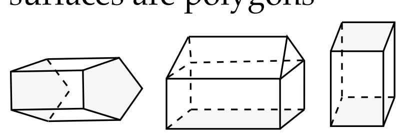


____________________ 3. a three-dimensional figure with one circular base in which a curved surface connects the base to the vertex

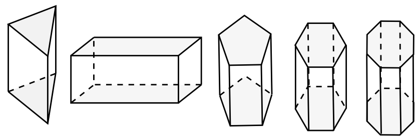


____________________ 4. existing in three dimensions; having length, width, and height

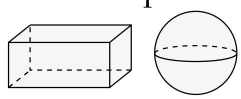
____________________ 5. a three-dimensional figure in which all surfaces are polygons



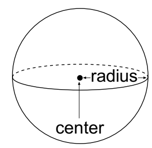
______________________ 6. a three-dimensional figure (polyhedron) with congruent, polygonal bases and lateral faces that are all parallelograms



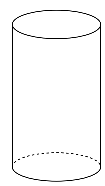
______________________ 7. three-dimensional figures that completely enclose a portion of space



______________________ 8. a three-dimensional figure in which all points on the surface are the same distance from the center



______________________ 9. a three-dimensional figure with two parallel congruent circular bases



Practice

Match each definition with the correct term. Write the letter on the line provided.

_____ 1. a triangle with at least two congruent sides and two congruent angles

_____ 2. an angle whose measure is exactly 180°

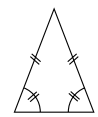
_____ 3. an angle whose measure is exactly 90°

_____ 4. an angle with a measure of more than 90° but less than 180°

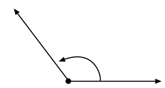
_____ 5. the opposite angles formed when two lines intersect

_____ 6. two angles, the sum of which is exactly 180°

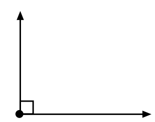
A. isosceles triangle



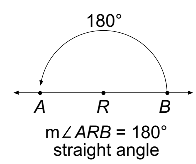
B. obtuse angle



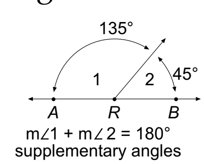
C. right angle



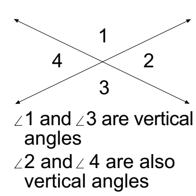
D. straight angle



E. supplementary angles



F. vertical angles



Practice

Use the list below to write the correct term for each definition on the line provided.

adjacent sides	**line of symmetry**	**perpendicular (⊥)**
congruent (≅)	**opposite sides**	**reflectional symmetry**
hexagon	**parallel (||)**	**rotational symmetry**
intersection		

______________________ 1. the point at which two lines meet

______________________ 2. being an equal distance at every point so as to never intersect

______________________ 3. forming a right angle

______________________ 4. when a figure has at least one line which splits the image in half, such that each half is the mirror image or reflection of the other; also called *line symmetry* or *mirror symmetry*

______________________ 5. when a figure can be turned less than 360 degrees about its center point to a position that appears the same as the original position; also called *turn symmetry*

______________________ 6. sides that are *directly across* from each other

______________________ 7. a line that divides a figure into two congruent halves that are mirror images of each other

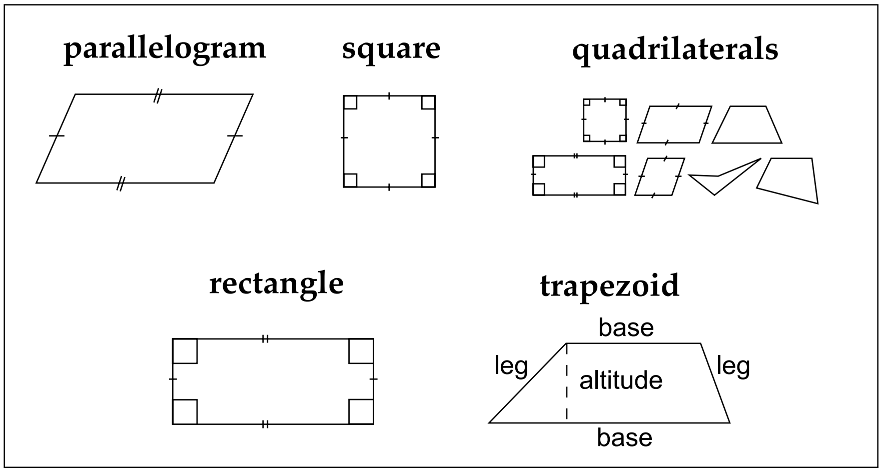
______________________ 8. sides that are *next to* each other and share a common vertex

______________________ 9. figures or objects that are the same shape and the same size

______________________ 10. a polygon with six sides

Practice

Use the **shapes** *of each of the* **figures** *below to correctly answer the following.*



Write **True** *if the statement is correct. Write* **False** *if the statement is* **not** *correct. If the statement is* **false***, rewrite it to make it true on the lines provided.*

__________ 1. A square, rectangle, and parallelogram have two opposite sides that are parallel and congruent.

____

____

__________ 2. A trapezoid has two pairs of opposite sides and both are parallel and congruent.

____

____

__________ 3. A square, rectangle, parallelogram, and trapezoid are all quadrilaterals.

____

____

__________ 4. The sum of all angle measures is 360° for all quadrilaterals.

____

____

__________ 5. All quadrilaterals have two opposite sides that are parallel and congruent.

____

____

__________ 6. Parallelograms and trapezoids are the only quadrilaterals where adjacent sides must be perpendicular.

____

____

__________ 7. All sides of a square are congruent.

____

____

Unit 4: Creating and Interpreting Patterns and Relationships

This unit emphasizes how patterns of change and relationships are used to describe and summarize information with algebraic expressions or equations to solve problems.

Unit Focus

Number Sense, Concepts, and Operations

- Associate standard numerals with numbers expressed with exponents. (A.1.3.1)
- Understand that numbers can be expressed in a variety of equivalent forms. (A.1.3.4)
- Understand and use exponential notation. (A.2.3.1)
- Add, subtract, multiply, and divide whole numbers and decimals to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculators. (A.3.3.3)
- Use concepts about numbers, including primes, factors, and multiples, to build number sequences. (A.5.3.1)

Algebraic Thinking

- Describe a wide variety of patterns. (D.1.3.1)
- Create and interpret tables, graphs, equations, and verbal descriptions to explain cause-and-effect relationships. (D.1.3.2)
- Represent and solve real-world problems with algebraic expressions and equations. (D.2.3.1)
- Use algebraic problem-solving strategies to solve real-world problems. (D.2.3.2)

Inventory Specialist

- counts items or checks computer printout of items on hand
- prepares reports on items in stock and items out of stock
- determines when to buy items and how many to buy based on prepared reports

Vocabulary

Study the vocabulary words and definitions below.

base (of an exponent) .... the number that is used as a factor a given number of times
Example: In 2^3, 2 is the base and 3 is the exponent.

chart .... see *table*

common multiple .... a number that is a multiple of two or more numbers
Example: 18 is a common multiple of 3, 6, and 9.

consecutive .... in order
Example: 6, 7, 8 are consecutive whole numbers and 4, 6, 8 are consecutive even numbers.

equation .... a mathematical sentence that equates one expression to another expression
Example: $2x = 10$

even number .... any whole number divisible by 2
Example: 2, 4, 6, 8, 10, 12 …

exponent (exponential form) ...... the number of times the base occurs as a factor
Example: 2^3 is the exponential form of 2 x 2 x 2. The numeral two (2) is called the *base*, and the numeral three (3) is called the *exponent*.

expression .... a collection of numbers, symbols, and/or operation signs that stands for a number
Example: $4r^2$; $3x + 2y$; $\sqrt{25}$

factor .... a number or expression that divides exactly another number
Example: 1, 2, 4, 5, 10, and 20 are factors of 20.

function .... a relation in which each value of x in a given set is paired with a unique value of y; can be described by a rule, a table, or a graph

integers .... the numbers in the set
{… , -4, -3, -2, -1, 0, 1, 2, 3, 4, …}

multiples .... the numbers that result from multiplying a given number by the set of whole numbers
Example: the multiples of 15 are 0, 15, 30, 45, 60, 75, etc.

odd number .... any whole number not divisible by 2
Example: 1, 3, 5, 7, 9, 11 …

pattern (relationship) .... a predictable or prescribed sequence of numbers, objects, etc.; also called a *relation* or *relationship*; may be described or presented using manipulatives, tables, graphics (pictures or drawings), or algebraic rules (functions)
Example: 2, 5, 8, 11...is a pattern. Each number in this sequence is three more than the preceding number. Any number in this sequence can be described by the algebraic rule, $3n - 1$, by using the set of counting numbers for n.

perfect square .... a number whose square root is a whole number
Example: 25 is a perfect square because $5 \times 5 = 25$.

power (of a number) .... an exponent; the number that tells how many times a number is used as a factor
Example: In 2^3, 3 is the power.

prime number .... any whole number with only two factors, 1 and itself
Example: 2, 3, 5, 7, 11, etc.

product .... the result of a multiplication
Example: In $6 \times 8 = 48$, 48 is the product.

relationship (relation) .... see *pattern*

rule .... a mathematical expression that describes a pattern or relationship, or a written description of the pattern or relationship

square (of a number) .... the result when a number is used as a factor twice
Example: 25 is the square of 5
$5^2 = 5 \times 5 = 25$

sum .... the result of an addition
Example: In $6 + 8 = 14$, 14 is the sum.

table (or chart) .... an orderly display of numerical information in rows and columns

value (of a variable) .... any of the numbers represented by the variable

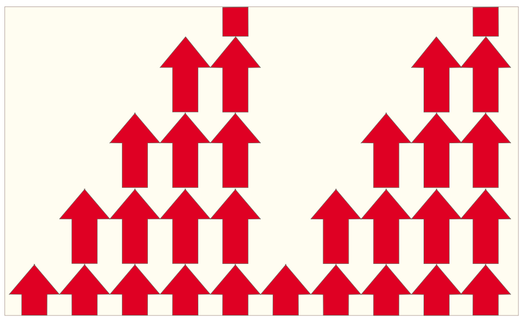
variable .... any symbol that could represent a number

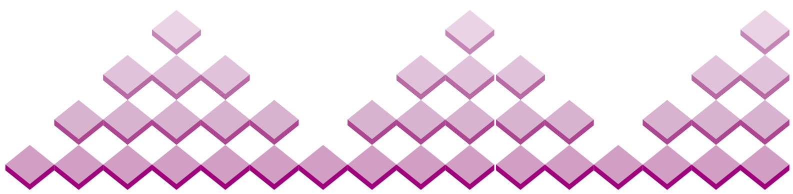
whole number .... any number in the set {0, 1, 2, 3, 4 …}

Unit 4: Creating and Interpreting Patterns and Relationships

Introduction

As you continue to develop your algebraic thinking and reasoning, you will have many opportunities to describe, to analyze, and to generalize a wide variety of *patterns* and *relationships*. You will become more comfortable and confident in using *expressions* and *equations* to represent and interpret situations. Algebraic thinking is empowering in problem solving. This unit should help strengthen your ability to think algebraically.





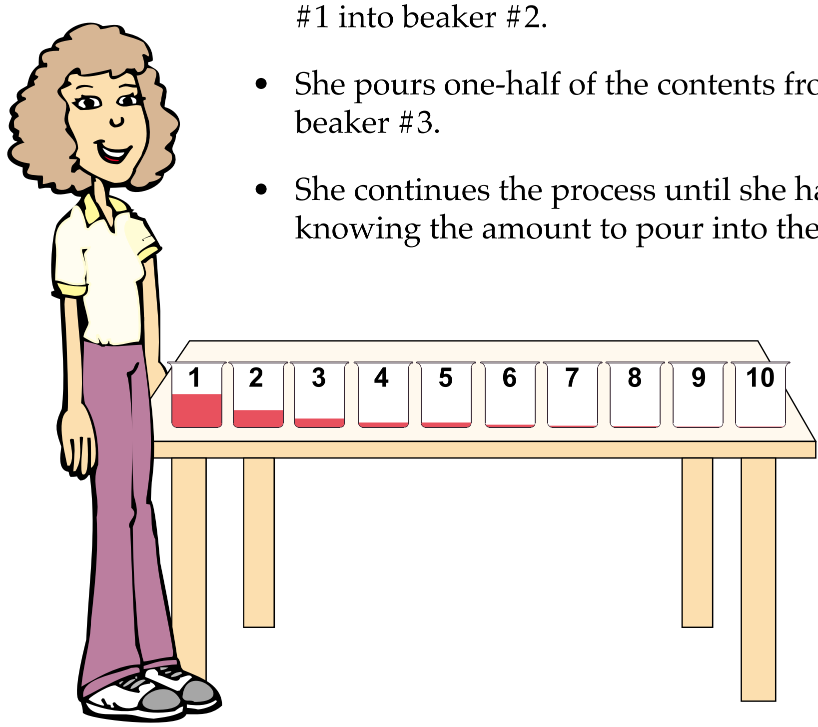
Lesson One Purpose

- Associate standard numerals with numbers expressed with exponents. (A.1.3.1)
- Understand that numbers can be represented in a variety of equivalent forms. (A.1.3.4)
- Understand and use exponential notation. (A.2.3.1)
- Describe a wide variety of patterns. (D.1.3.1)
- Create and interpret tables and verbal descriptions. (D.1.3.2)

Discovering Patterns and Relationships

Misty is preparing a lesson for you with a focus on **patterns** and **relationships**. She goes to the science lab and gets a set of 10 beakers of equal size.

- She fills beaker # 1 with colored water.
- She pours one-half of the colored water from beaker #1 into beaker #2.
- She pours one-half of the contents from beaker #2 into beaker #3.
- She continues the process until she has difficulty knowing the amount to pour into the next beaker.



She steps back and looks at the set of 10 beakers. She thinks students can see enough of the pattern to visualize what the 10^{th} beaker would look like if she had completed the task she set out to do. Misty thinks her lesson may be visually powerful in helping you see a pattern in progress that is to be continued. The next practices will deal with patterns in progress.

Practice

Complete the following.

1. Complete the **table** below that Misty has set up for you. (**Optional:** With your teacher's permission, you may use 10 clear plastic cups or 10 beakers and some water, with or without coloring added, to do the activity.)

   Misty used the information she gathered from her activity on the previous page.

Fractional Part of Beaker Filled

beaker number	start	1	2	3	4	5	6	7	8	9	10
fractional part of beaker filled at end of pour	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$						$\frac{1}{1024}$

2. Suppose you start with 100 milliliters (ml) of colored water. You pour one-half of it into beaker #1. Then you pour one-half of the 50 ml into beaker #2 and so on.

   Complete the table for the number of milliliters in each beaker. You may stop when you reach less than one milliliter.

Millimeters in Beakers

beaker number	start	1	2	3	4	5	6	7	8	9	10
number of milliliters (ml) in beaker at end of pour	100	50									

Practice

Complete the following.

Misty would like to choose a number that, when repeatedly divided by two, results in **whole numbers**. She decides to consider numbers she gets as **products** by repeatedly multiplying by 2. She finds these numbers are **powers** of 2 as well as **multiples** of 2.

Remember:

- The **expression** 2^3 can be read as two to the third power.
- The 2 is called the **base**. The 3 is called the **exponent**.
- The exponent tells us how many times to use the base as a **factor**.
- In the expression 2^3, we are to use 2 as a factor three times, 2 x 2 x 2.
- The value of the expression 2^3 is 8 because 2 x 2 x 2 = 8.

Complete the list Misty has started.

1. $2^1 = 2$
2. $2^2 = 2 \times 2 =$ ____________
3. $2^3 = 2 \times 2 \times 2 =$ ____________
4. $2^4 = 2 \times 2 \times 2 \times 2 =$ ____________
5. $2^5 = 2 \times 2 \times 2 \times 2 \times 2 =$ ____________

6. $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 =$ ____________

7. $2^7 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 =$ ____________

8. $2^8 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 =$ ____________

Misty decides to have you start with 256 milliliters (ml) of colored water. You will pour one-half of it into beaker #1. Then you will pour one-half of the 128 ml from beaker #1 into beaker #2 and so on.

9. Complete the table that would result from this experiment.

10. Will you pour less than 1 ml of water into any beaker during this experiment? ____________

    If so, when does this first occur? ____________________________

Millimeters of Colored Water in Beakers

beaker number	start	1	2	3	4	5	6	7	8	9	10
number of milliliters (ml) of colored water in beaker at end of pour	256	128									

Misty would like to start with enough milliliters of colored water to have 1 ml in beaker #10 at the end of the experiment.

11. How much water will be required to start the experiment?

____

12. Complete the table to demonstrate the accuracy of your answer.

Millimeters of Colored Water in Beakers

beaker number	start	1	2	3	4	5	6	7	8	9	10
number of milliliters (ml) of colored water in beaker at end of pour											

Practice

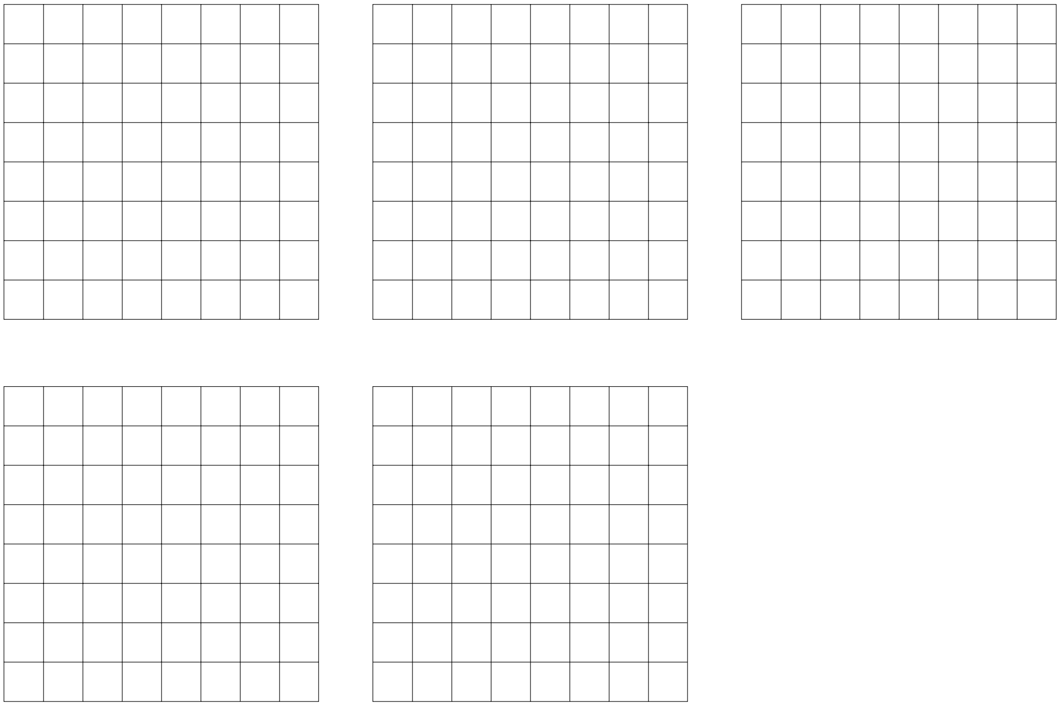
Complete the following.

1. If given the set of numbers: {2, 4, 8, 16, 32, …}, what would be the next number in the set? ____________ Why? __________________

   ____

   ____

2. If given the set of numbers: $\{\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32} \ldots\}$, what would be the next number in the set? ____________



3. On the 8 by 8 grids provided above, shade $\frac{1}{2}$ of the first, $\frac{1}{4}$ of the second, and continue until you have shaded $\frac{1}{32}$ of the fifth grid.

(**Remember:** $\frac{1}{32}$ is equal to $\frac{2}{64}$.)

4. Which would you prefer to use to model a pattern of successive reductions of $\frac{1}{2}$, the beakers of colored water (from the previous practice) or the shaded grids? Explain why.

   Answer: ____

   Explanation: ____

   ____

   ____

   ____

   ____

   ____

Practice

Complete the following.

You will use numbers in *exponential form* in future mathematics classes. Look for patterns and practice using exponential form.

Remember:

- Exponential form is a way of writing numbers using *exponents*.
- 2^5 is in exponential form of 2 x 2 x 2 x 2 x 2.

exponent

$$2^5 = 2 \times 2 \times 2 \times 2 \times 2$$

base

Write the **next number** *in the* **pattern**.

1. 3; 9 ; 27; 81; ____________
2. 3^1; 3^2; 3^3; 3^4; ____________
3. 4; 16; 64; 256; ____________
4. 4^1; 4^2; 4^3; 4^4; ____________
5. 5; 25; 125; 625; ____________
6. 5^1; 5^2; 5^3; 5^4; ____________
7. 6; 36; 216; 1,296; ____________
8. 6^1; 6^2; 6^3; 6^4; ____________
9. 7; 49; 343; 2,401; ____________

10. 7^1; 7^2; 7^3; 7^4; ____________

11. 8; 64; 512; 4,096; ____________

12. 8^1; 8^2; 8^3; 8^4; ____________

13. 9; 81; 729; 6,561; ____________

14. 9^1; 9^2; 9^3; 9^4; ____________

15. 10; 100; 1,000; 10,000; ____________

16. 10^1; 10^2; 10^3; 10^4; ____________

Practice

Complete the following.

You should be careful not to confuse *powers of a number* with *multiples of a number*. This task will be a mix of problems to encourage you to look carefully for a variety of patterns.

Remember:

- Powers of a number are the result of multiplying a number by itself a given number of times.
  Example: $3^3 = 3 \times 3 \times 3 = 27$.
- Multiples of a number are the result of multiplying a number by the set of *whole numbers*.
  Example: The multiples of 6 are 0, 6, 12, 18, 24, 30, 36, 42, etc.

1. 2; 4; 8; 16; ______ ; ______ ; ______ ; ______

   What is the 8^{th} number in this set? ______

2. 11; 121; 1,331; ______

   What is the 4^{th} number in this set? ______

3. 5; 25; 125; ______ ; ______

   What is the 5^{th} number in this set? ______

4. 100; 200; 300; 400; ______

   What is the 5^{th} number in this set? ______

Practice

Use patterns to solve the following problems. The first three figures have been drawn.

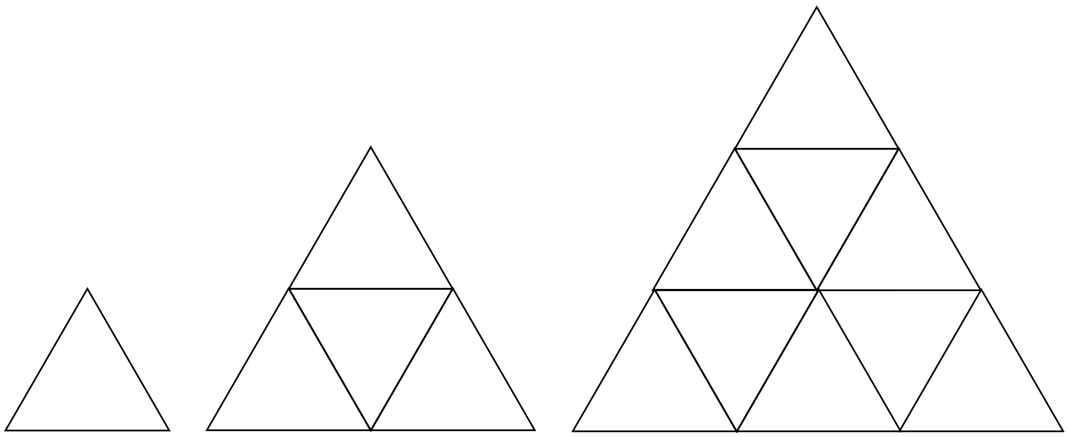


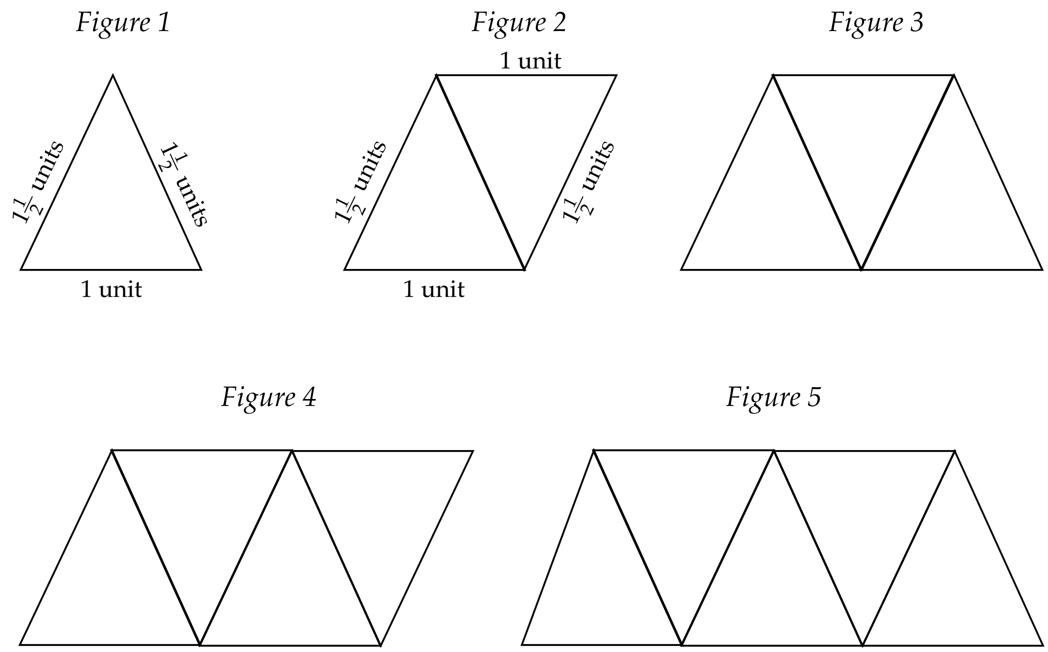
Figure 1 *Figure 2* *Figure 3*

each triangle = 1 unit triangle

1. Complete the table below.

Triangle Units Formed by Equilateral Triangles

Figure	Number of Unit Triangles Required to Make Figure
one	1
two	4
three	
four	
five	



each base = 1 unit
each side = $1\frac{1}{2}$ units

2. Complete the table below.

Perimeter of Figures Formed by Equilateral Triangles

Figure	Perimeter
one	4 units
two	5 units
three	
four	
five	

Figure 1

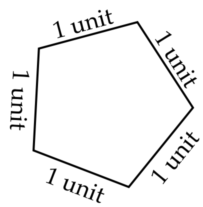


Figure 2

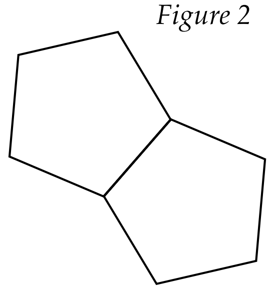


Figure 3

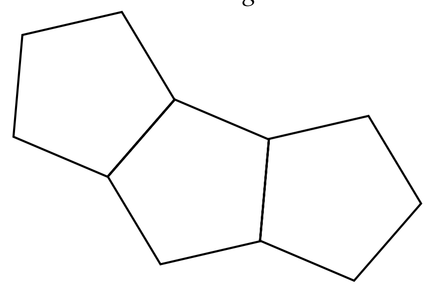


Figure 4

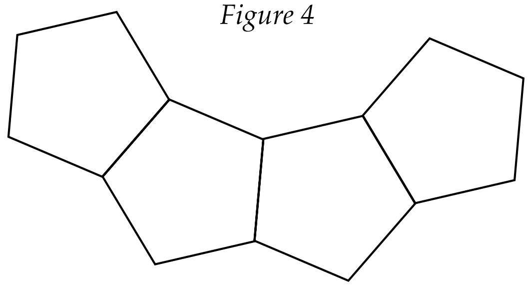


Figure 5
(You may illustrate below.)

each side = 1 unit

3. Complete the table below.

Perimeter of Figures Formed by Pentagons

Figure	Perimeter
one	5 units
two	8 units
three	
four	
five	

Practice

Use the list below to complete the following statements.

base	**multiples**
exponent	**pattern**
exponential form	**power**
expression	**product**
factor	

1. The ________________________ 8^3 can be read eight to the third

   ________________________ .

   The three is the ________________________ .

   The eight is the ________________________ .

2. The exponent tells how many times to use the base as a

   ________________________ .

3. 8 x 8 x 8 can be written in ________________________ as 8^3.

4. The ________________________ of 5 are 0, 5, 10, 15, 20, 25, 30, 35, etc.

5. In the multiplication problem 5 x 8 is 40, 40 is the

   ________________________ .

6. $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}$ is a ________________________ or a predictable sequence of numbers. Each number in the sequence is one-half of the previous number.

Practice

Complete the following.

When Misty shared her lesson plans with Mike, he noticed some patterns he could build on in another lesson. His lesson will focus on one basic question. That question is provided below for you.

Mike's one basic question:
How can you use *patterns* to tell me the *units digit* for the 4th power of 6, the 5th power of 7, the 99th power of 2, or any power of a given number I choose?

units digit
↓
624
↑
ones place

- First, when solving Mike's problem, remember that the *units digit* in the number 624 is 4 because 4 is in the units place, or *ones place*.
- Second, complete the tables for each power and look for patterns.
- Third, use the patterns to answer the questions provided.
- Fourth, write a paragraph in response to Mike's question.

1. Complete the following table. Each column represents consecutive powers for the number N shown at the top of the column.

Table of Exponents

N	1	2	3	4	5	6	7	8	9	10
N^1	1	2	3	4	5	6	7	8	9	10
N^2	1	4	9	16	25	36	49	64	81	100
N^3	1	8	27							
N^4	1	16	81							
N^5	1	32								
N^6	1	64								
N^7	1									
N^8	1									

2. What is the units digit in the 4^{th} power of 1, the 6^{th} power of 1, the 48^{th} power of 1? ____________________

3. What is the units digit in the 2^{nd} power of 5, the 7^{th} power of 5, the 200^{th} power of 5? ____________________

4. What is the units digit in the 3^{rd} power of 6, the 9^{th} power of 6, the 51^{st} power of 6? ____________________

5. What is the units digit in the 3^{rd} power of 11, the 9^{th} power of 11, the 69^{th} power of 11? ____________________ (**Hint:** The table on page 277 can help even though it only contains powers of 1-10.)

6. What is the units digit in the 2^{nd} power of 241, the 3^{rd} power of 241? ____________________

7. What other base has a pattern similar to bases of 1, 5, and 6 when raised to any power? ____________________

8. Underline the units digit in each of the following powers of 2.
   2; 4; 8; 16; 32; 64; 128; 256; 512; 1,024; 2,048; 4,096

9. What is the units digit in the 4^{th} power of 2, the 8^{th} power of 2, the 12^{th} power of 2, the 16^{th} power of 2, the 20^{th} power of 2, the 400^{th} power of 2? ____________________

10. What is the units digit in the 3^{rd} power of 2, the 7^{th} power of 2, the 11^{th} power of 2, the 55^{th} power of 2? ____________________

11. What is the units digit in the 2^{nd} power of 2, the 6^{th} power of 2, the 98^{th} power of 2? ____________________

12. What is the units digit in the 1^{st} power of 2, the 5^{th} power of 2, the 9^{th} power of 2, the 101^{st} power of 2? ____________________

13. Underline the units digit in each of the following powers of 3.

    3; 9; 27; 81; 243; 729; 2,187

14. What is the units digit in the 4^{th} power of 3, the 8^{th} power of 3, the 12^{th} power of 3? ____________________

15. What is the units digit in the 3^{rd} power of 3, the 7^{th} power of 3, the 35^{th} power of 3? ____________________

16. What is the units digit in the 2^{nd} power of 3, the 6^{th} power of 3, the 50^{th} power of 3? ____________________

17. What is the units digit in the 1^{st} power of 3, the 5^{th} power of 3, the 29^{th} power of 3? ____________________

18. Use the same strategy for powers of 7 and 8.

    a. What is the units digit in the 4^{th} power of 7, the 23^{rd} power of 7, the 50th power of 7, the 99^{th} power of 7? ____________________

    b. What is the units digit in the 4^{th} power of 8, the 15^{th} power of 8, the 23^{rd} power of 8, the 25^{th} power of 8? ____________________

19. The pattern for the units digit for powers 4 and 9 is different from the others.

a. What is the units digit for the 6^{th} power of 4, the 9^{th} power of 4?

____

b. What is the pattern for powers of 4? ____________________

c. What is the units digit for the 8^{th} power of 9, the 9^{th} power of 9?

____

d. What is the pattern for powers of 9? ____________________

Go back to Mike's one basic question at the beginning of the practice.

20. How can you use patterns to tell the units digit for the value of any base to any power? Answer in paragraph form.

____

____

____

____

____

____

____

____

____

____

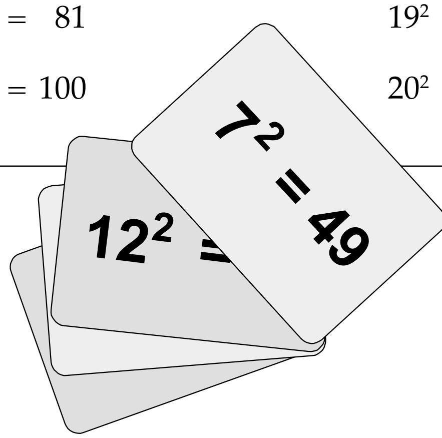
Perfect Squares

Mike has taken a lot of courses in mathematics since he was in middle school, and he has often used **perfect squares**. A *perfect square* is a number whose square root is a whole number. A perfect square is a product of an **integer** multiplied by itself. He thinks his students already know the **squares of the numbers** of 1-10. He will now challenge them to learn the squares of the numbers 11-20.

Mike and Misty often use calculators but not for basic facts they have committed to memory. Their instant recall is much faster than a calculator for this purpose. The list of these perfect squares is printed below for you. Making a set of flash cards to practice these basic facts will help you in the future.

Perfect Squares

$1^2 = 1$	$11^2 = 121$
$2^2 = 4$	$12^2 = 144$
$3^2 = 9$	$13^2 = 169$
$4^2 = 16$	$14^2 = 196$
$5^2 = 25$	$15^2 = 225$
$6^2 = 36$	$16^2 = 256$
$7^2 = 49$	$17^2 = 289$
$8^2 = 64$	$18^2 = 324$
$9^2 = 81$	$19^2 = 361$
$10^2 = 100$	$20^2 = 400$



Practice

Match each definition with the correct term. Write the letter on the line provided.

______ 1. a predictable or prescribed sequence of numbers, objects, etc.

______ 2. a collection of numbers, symbols, and/or operation signs that stands for a number

______ 3. an orderly display of numerical information in rows and columns

______ 4. the numbers that result from multiplying a given number by the set of whole numbers

______ 5. the result of a multiplication

______ 6. the number that is used as a factor a given number of times

______ 7. the number of times the base occurs as a factor

______ 8. a number or expression that divides exactly another number

______ 9. a number whose square root is a whole number

______ 10. the numbers in the set {… , -4, -3, -2, -1, 0, 1, 2, 3, 4, …}

______ 11. any number in the set {0, 1, 2, 3, 4 …}

A. base (of an exponent)

B. exponent (exponential form)

C. expression

D. factor

E. integers

F. multiples

G. pattern (relationship)

H. perfect square

I. product

J. table (or chart)

K. whole number

Lesson Two Purpose

- Understand and use exponential notation. (A.2.3.1)
- Add, subtract, multiply, and divide whole numbers and decimals to solve problems using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Create and interpret tables, graphs, equations, and verbal descriptions to explain cause-and-effect relationships. (D.1.3.2)

Guess My Rule

Mike and Misty are preparing a series of Guess My Rule problems for students.

In each table, one column will give **values** for a number represented by the **variable** x. A **rule** will be applied to the number, and the result will appear in the second column of values represented by y.

An example of this might be as follows.

X	*Y*
3	6
4	7
5	8
6	9
7	?

Mike thinks some students will take a quick look at the first pair of values (3, 6) and think the rule is to *double* the value of x. Those students might then give an answer of 14.

He knows some students will look at more than the first pair. They will see that 4 was not doubled because the corresponding value for y is 7, not 8.

They will see the same is true for 5 and for 6.

Students will need to recognize that each value for y is 3 more than the corresponding value for x.

The rule for this **function** or *relationship* would be as follows.

$y = 3 + x$
or
$y = x + 3$

When x is 7, y would be 3 more than 7 or 10.

When we are using the letter *x* to represent a number, we should not use x for a multiplication sign.

We can write 3 x 5 as follows.

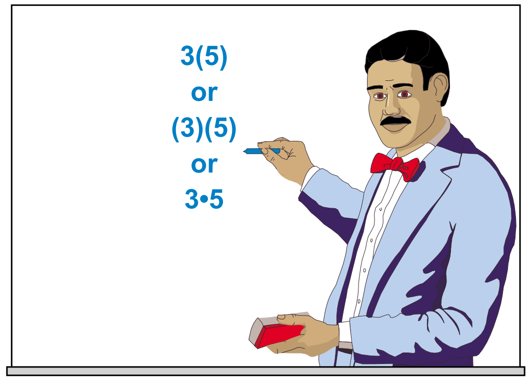
3(5)

or

(3)(5)

or

3•5



Notice on the last problem that the dot is placed in a *center position* between the two numbers.

Practice

Find and apply a **rule** *for each of Mike's following tables.* **Explain** *the rule in words* **or write** *it as an* **equation** *in the form of* ***y*** **=** ______ .

1.

X	Y
4	2
5	2.5
6	3
7	3.5
8	?

Mike's rule is ____ .

2.

X	Y
3	8
4	9
5	10
6	11
7	?

Mike's rule is ____ .

3.

X	Y
4	1
5	2
6	3
7	4
8	?

Mike's rule is ____ .

4.

X	Y
2	12
4	?
6	36
8	?
10	60

Mike's rule is ______________________________________ .

5.

X	Y
3	9
5	?
7	49
9	?
11	121

Mike's rule is ______________________________________ .

6.

X	Y
5	20
10	?
15	60
20	80
25	?

Mike's rule is ______________________________________ .

7.

X	*Y*
2	24
3	36
4	48
5	?
6	?

Mike's rule is ____ .

Practice

Find and apply a **rule** *for each of Mike's following tables.* **Explain** *the rule in words* **or write** *it as an* **equation** *in the form of* ***y*** **=** ______ .

1.

X	Y
1	3
2	6
3	?
4	12
5	?

Mike's rule is ____ .

2.

X	Y
1	?
2	72
3	108
4	144
5	?

Mike's rule is ____ .

3.

X	Y
1	16
2	?
3	48
4	64
5	80

Mike's rule is ____ .

4.

X	Y
5	10,000
10	20,000
15	?
20	40,000
25	?

Mike's rule is ____ .

5.

X	Y
1	5,280
2	10,560
3	15,840
4	?
5	?

Mike's rule is ____ .

6.

X	Y
1	$1.59
2	$3.18
3	$4.77
4	?
5	?

Mike's rule is ____ .

Practice

Each **table** *in the* **practice** *on pages 285-287 was created from one of the following statements. Match each statement with the correct* **table**. *Write the* **table number** *on the line provided.*

______ A. The number of 8-ounce cups in a quart is 4.

______ B. My brother is 3 years younger than I am.

______ C. A recipe for chocolate frosting calls for 6 cups of powdered sugar for each cup of cocoa.

______ D. The number of inches in a foot is 12.

______ E. A recipe for cornbread calls for a half cup milk for each cup of meal.

______ F. My sister is 5 years older than I am.

______ G. The area of a square can be found by multiplying the length of a side times itself, or by squaring the length of a side.

Practice

Each **table** *in the* **practice** *on pages 288-289 was created from one of the following statements. Match each statement with the correct* **table**. *Write the* **table number** *on the line provided.*

______ A. The number of feet in a yard is 3.

______ B. The number of feet in a mile is 5,280.

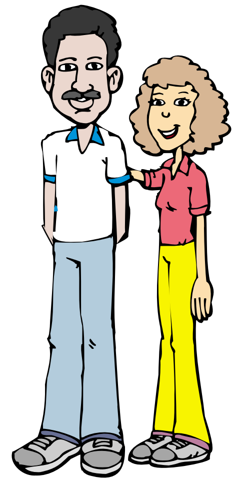
______ C. The number of inches in a yard is 36.

______ D. The cost of one gallon of fuel is $1.59.

______ E. The number of pounds in a ton is 2,000.

______ F. The number of ounces in a pound is 16.

Numbers in a Pattern



Misty and Mike often think alike. They believe that the fact they have been good friends since childhood has something to do with this. They decide to see how well Mike can tell Misty what she is thinking as she begins numbers in a *pattern*.

Misty says,

> "The set of numbers I am thinking of begins with 1, 4, … ."

Mike replies,

> "The next two numbers in your pattern could be 16 and 64 since each is 4 times greater than the previous number."

For example: 1, 4, 16, 64, … because

1	x	4	=	4
4	x	4	=	16
16	x	4	=	64

Mike thinks a bit more and continues,

> "The next two numbers could also be 7 and 10 because each number is 3 more than the previous number."

For example: 1, 4, 7, 10, … because

1	+	3	=	4
4	+	3	=	7
7	+	3	=	10

Misty says,

> "Each of your ideas would result in patterns, but neither is what I was thinking. The third number in my pattern would be 9 and the fourth would be 16."

Mike replies,

> "Your set of 1, 4, 9, 16,… is adding 3, then 5, then 7, so
>
> - I'll add 9 to 16 and get 25.
> - I'll add 11 to 25 and get 36.
> - I'll add 13 to 36 and get 49."

For example: 1, 4, 9, 16, …

$$\begin{aligned} 1 + 3 &= 4 \\ 4 + 5 &= 9 \\ 9 + 7 &= 16 \\ 16 + 9 &= 25 \\ 25 + 11 &= 36 \\ 36 + 13 &= 49 \end{aligned}$$

Misty says,

> "Wow and double wow! That is the set I was thinking of, but I was thinking of the square of the numbers, 1, 2, 3, 4, …
>
> Your rule works as well as mine. It produces the same set of numbers in my pattern! I'll bet that won't happen again soon!"
>
> *For example*: 1, 4, 9, 16, … because

$$\begin{aligned} 1^2 &= 1 \\ 2^2 &= 4 \\ 3^2 &= 9 \\ 4^2 &= 16 \end{aligned}$$

Practice

Complete the following.

Mike and Misty are starting more *patterns*. Your task is to *continue their patterns* as directed.

Misty and Mike hope you realize that when you are looking for *patterns to continue a series of numbers*, that it is important to *check and re-check the rule* you believe is being followed to *create the set*.

1. a. (2, 5, ______ , ______ , ______)

   Each number is 3 *more than* the previous number.

   b. (2, 5, ______ , ______ , ______)

   Each number is 1 *less than* 3 times the previous number.

2. a. (3, 6, ______ , ______ , ______)

   Each number is 3 *more than* the previous number.

   b. (3, 6, ______ , ______ , ______)

   Each number is *twice* the previous number.

3. a. (4, 12, ______ , ______ , ______)

   Each number is 8 *more than* the previous number.

   b. (4, 12, ______ , ______ , ______)

   Each number is 3 *times* the previous number.

4. a. (2, 6, ______ , ______ , ______)

Each number is 4 *more than* the previous number.

b. (2, 6, ______ , ______ , ______)

The second number is 4 *more than* the first. The third number is 6 *more than* the second. The fourth number is 8 *more than* the third, and so on.

5. a. (6, 9, ______ , ______ , ______)

Each number is 3 *more than* the previous number.

b. (6, 9, ______ , ______ , ______)

Each number is a **consecutive** multiple of 3.

6. a. (1, 3, ______ , ______ , ______)

The numbers are *consecutive* **odd numbers**.

b. (1, 3, ______ , ______ , ______)

Each number is 2 *more than* the previous number.

7. a. (1, 1, ______ , ______ , ______)

Each number is the *same* as the previous number.

b. (1, 1, ______ , ______ , ______)

The third number will be the **sum** of the first and second. The fourth number will be the *sum* of the second and third, and so on.

(**Remember:** *Consecutive* means in order.)

Practice

Match each definition with the correct term. Write the letter on the line provided.

______ 1. any of the numbers represented by the variable

______ 2. a relation in which each value of x in a given set is paired with a unique value of y

______ 3. any symbol that could represent a number

______ 4. a mathematical expression that describes a pattern or relationship, or a written description of the pattern or relationship

______ 5. in order

______ 6. a mathematical sentence that equates one expression to another expression
Example: $2x = 10$

______ 7. any whole number not divisible by 2
Example: 1, 3, 5, 7, 9, 11 …

______ 8. the result of an addition

A. consecutive

B. equation

C. function

D. odd number

E. rule

F. sum

G. value (of a variable)

H. variable

Lesson Three Purpose

- Represent and solve real-world problems with algebraic expressions and equations. (D.2.3.1)

Using Algebraic Expressions and Equations

For each *equation* provided in this lesson, an explanation is given. You will use the equation to answer the questions.

For example:

$C = 5p$

In the equation above, C represents cost of movie and p represents how many people are going.

- What is the cost if 4 people go to the movie?

  Answer:

  $C = 5(4)$
  The cost equals \$5.00 times 4 people.

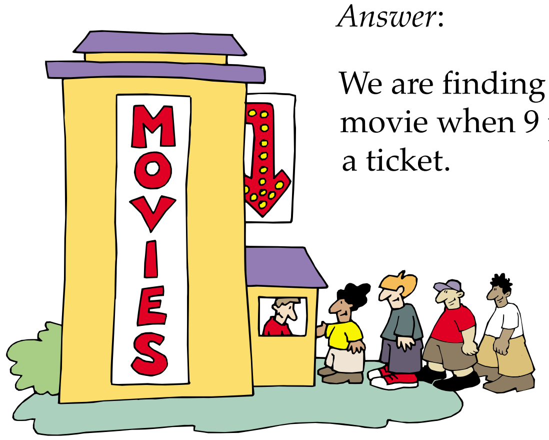
  $C = 20$
  The cost equals \$20.00.

  Therefore, the cost for 4 people to go to the movie is \$20.00.

- If we use the equation to solve a problem and write $C = 5(9)$, what problem are we solving?

  Answer:

  We are finding the total cost of admission for the movie when 9 people go and each pays \$5.00 for a ticket.



Practice

Solve the following.

1. $B = E + 3$

   In the equation above, B represents Beth's age and E represents Edwina's age.

   a. What is Beth's age if Edwina is 11? ____________________________

   b. If we use the equation to solve a problem and write

      $B = 13 + 3$, what problem are we solving? ____________________

      ____

      ____

   c. If we use the equation to solve a problem and write $20 = E + 3$,

      what problem are we solving? _________________________________

      ____

      ____

   d. What does the 3 represent in the equation? ___________________

      ____

      ____

2. $S = 8h$

   In the equation above, S represents the salary and h represents the number of hours worked by an employee.

   a. If the employee worked 20 hours, what was the salary? ________

b. If we use the equation to solve a problem and write $S = 8(15)$, what problem are we solving? ______________________________

____

____

c. If we use the equation to solve a problem and write $96 = 8h$, what problem are we solving? ______________________________

____

____

d. What does the 8 represent in the problem? ___________________

____

____

3. $C = p + 0.07p$

In the equation above, C represents the total cost of an item, p represents the original price of the item, and $0.07p$ represents the amount of sales tax on the item.

a. If the original price of an item is \$20.00 what is the *total cost* of the item? ____

b. If we are using the equation to solve a problem and write $C = 15 + 0.07(15)$, what problem are we solving? _____________

____

____

c. If we are using the equation to solve a problem and write $107 = p + 0.07p$, what problem are we solving? ________________

____

____

d. What does the 0.07 represent in the problem? ________________

____

____

4. $F = \frac{9}{5}C + 32$

In the equation above, F represents the temperature in Fahrenheit, and C represents the temperature in Celsius.

a. If the temperature is 15 degrees Celsius, what is the temperature in Fahrenheit? ________________________________

b. If we are using the equation to solve a problem and write $F = \frac{9}{5}(20) + 32$, what problem are we solving? ________________

____

____

c. If we are using the equation to solve a problem and write $77 = \frac{9}{5}C + 32$, what problem are we solving? ________________

____

____

d. What does the 32 represent in the equation? ________________

____

____

5. $A = lw$

In the equation above, A represents the *area* of a rectangle, l represents the *length*, and w represents the *width*.

a. If the length (l) is 4 units and the width (w) is 3 units, what is the area (A)? ______________________________

b. If we are using the equation to solve the problem and write $A = 5(8)$, what problem are we solving? ______________________________

____

____

c. If we are using the equation to solve a problem and write $40 = 4w$, what problem are we solving? ______________________________

____

____

d. If we are using the equation to solve a problem and write $36 = 3(l)$, what problem are we solving? ______________________________

____

____

Practice

Solve the following.

1. $Y = \frac{I}{36}$

   In the equation above, Y represents the number of yards, and I represents the number of inches.

   a. What is the number of yards if the number of inches is 144? ___

   b. If we are using the equation to solve a problem and write $Y = \frac{108}{36}$, what problem are we solving? ______________________

   ____

   ____

   c. If we are using the equation to solve a problem and write $6 = \frac{I}{36}$, what problem are we solving? ______________________

   ____

   ____

   d. What does the 36 represent in the equation? __________________

   ____

   ____

2. $P = 4s$

   In the equation above, P represents the *perimeter* of a square, and s represents the length of one *side* of the square.

   a. What is the perimeter (P) if a side (s) length is 7? ______________

b. If we are using the equation to solve a problem and write $P = 4(3)$, what problem are we solving? ____________________

____

____

c. If we are using the equation to solve a problem and write $20 = 4s$, what problem are we solving? ____________________

____

____

d. What does the 4 represent in the equation? ________________

____

____

3. $F = 5{,}280m$

In the equation above, F represents the number of feet and m represents the number of miles.

a. What is the number of feet if the number of miles is 4? ________

b. If we are using the equation to solve a problem and write $F = 5{,}280(3)$, what problem are we solving? ________________

____

____

c. If we are using the equation to solve a problem and write $52{,}800 = 5{,}280m$, what problem are we solving? ______________

____

____

d. What does the 5,280 represent in the equation? ______________

____

____

4. $C = m + 0.15m$

In the equation above, C represents the cost of a meal eaten in a restaurant, and m represents the price of the meal without a tip.

a. What is the cost if the price of the meal without a tip is $10.00?

____

b. If we are using the equation to solve a problem and write $C = 8 + 0.15(8)$, what problem are we solving? _______________

____

____

c. If we are using the equation to solve a problem and write $13.80 = m + 0.15m$, what problem are we solving? ____________

____

____

d. What does the 0.15 represent in the equation? _______________

____

____

5. $D = rt$

In the equation above, D represents the *distance*, r represents the *rate of speed*, and t represents the *length of time*.

a. What is the distance (D) traveled when the rate of speed (r) is 60 miles per hour and the length of time (t) is 3 hours? ___________

b. If we are solving a problem and we write $D = 50(6)$, what problem are we solving? ___________________________________

____

____

c. If we are solving a problem and we write $200 = 50t$, what problem are we solving? ___________________________________

____

____

d. If we are solving a problem and we write $625 = 10r$, what problem are we solving? ___________________________________

____

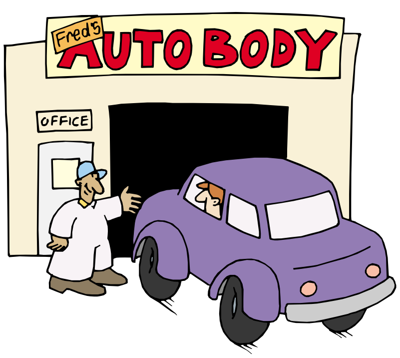
____

Lesson Four Purpose

- Understand and use exponential notation. (A.2.3.1)
- Add, subtract, multiply, and divide whole numbers and decimals to solve real-world problems using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Use concepts about numbers, including primes, factors, and multiples, to build number sequences. (A.5.3.1)
- Use algebraic problem-solving strategies to solve real-world problems. (D.2.3.2)

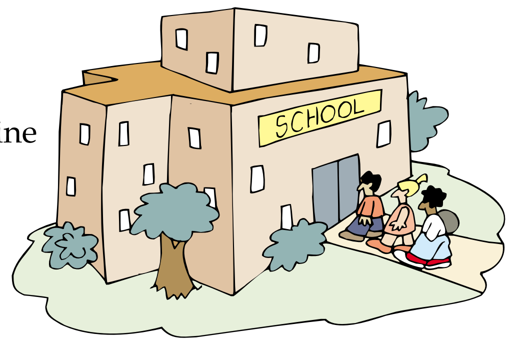
Problem Solving

In this lesson you will use algebraic problem-solving strategies to solve real-world problems.



Practice

Answer the following.



At the Martin Luther King Middle School, a "Welcome" communication line exists for the sixth grade. When a new sixth grader enrolls at the school, the name of a sixth grader is drawn from a box, and that student is called to the office to meet the new student and to take the student on a tour of the building.

The second day, two more names are drawn, and those students carry out a specific task.

The third day, four names are drawn, and those students carry out a specific task. This continues until the student has been in school for six school days.

Continue the pattern to show how many names will be drawn and which task might be most appropriate for that number of people if the tasks are as follows:

a. Tour the school library and learn about its many resources.

b. Review the school handbook to learn school rules and policies.

c. Share information about clubs and other extracurricular activities

d. Share a table in the cafeteria at lunchtime.

e. Speak to the new student in the hallway, and call the new student by name.

Complete the **chart** *below.*

Sixth Grade "Welcome" Communication Line

Day	Number of Names Drawn	Task for Students Whose Names are Drawn
1	1	Tour of school
2		
3		
4		
5		
6		

Practice

Write **True** *if the statement is correct. Write* **False** *if the statement is* not *correct.*

Marci is curious about a lot of things in math and often explores using a calculator. She has recently learned about *exponents* in her math class and has made a *table* to help her look for *patterns*. Study the following table and refer to it as needed to determine whether each statement below is **True** or **False**.

Table Exponents

N	1	2	3	4
N^1	1	2	3	4
N^2	1	4	9	16
N^3	1	8	27	64
N^4	1	16	81	256
N^5	1	32	243	1,024

__________ 1. Any number to the first *power* is that same number. The first power means we are using one *factor* of that number.
For example, $2^1 = 2$ and $10^1 = 10$.

__________ 2. The number 1 to any power is 1.
For example, 1^5 means we are using five factors of 1 (1 x 1 x 1 x 1 x 1).

__________ 3. A number to the third power means we are using three factors of that number.
For example, (n x n x n) or (2 x 2 x 2) or (8 x 8 x 8).

__________ 4. When a number greater than 1 is raised to various powers, the *greater the power*, the *greater the value* of the *expression*.
For example, 2^5 is greater than (>) 2^3 because the *value* of the expression 2^5 is 32, and 32 is greater than the value of the expression 2^3, or 8.

__________ 5. The value of the number 2, raised to any power greater than or equal to ($\geq$)1, will be an **even number**.

__________ 6. The value of any even number greater than 1, raised to any power greater than or equal to 1, will be an even number.

__________ 7. The value of the number 3, raised to any power greater than or equal to 1, will be an *odd number.*

__________ 8. The value of any odd number, raised to any power greater than or equal to 1, will be an odd number.

__________ 9. As the powers of 2 increase by 1, the value doubles.

__________ 10. As the powers of 2 decrease by 1, the value is halved.

__________ 11. The value of 2^0 must therefore be 1.

__________ 12. As the powers of 3 increase by 1, the value triples.

__________ 13. As the powers of 3 decrease by 1, the value is one-third as much.

__________ 14. The value of 3^0 must therefore be 1.

__________ 15. The value of 4^0 is 1.

__________ 16. Exactly two of the above statements are true.

Practice

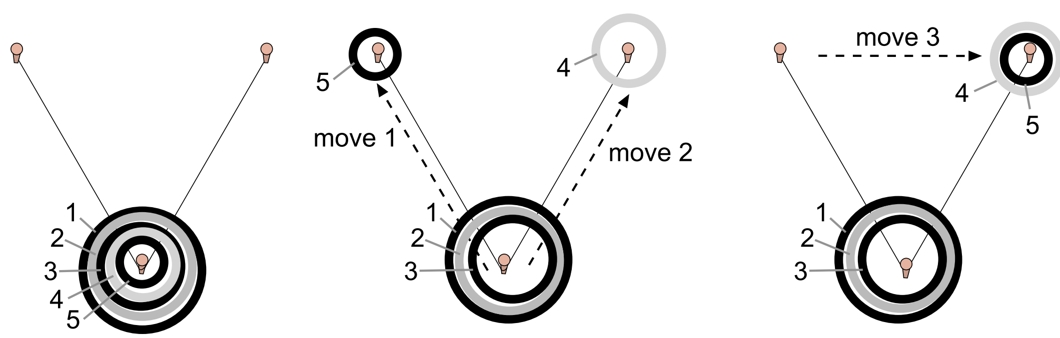
Answer the following.

Mike and Misty have made a game for their students to play.

They will make the game a bit more difficult each day. Play their game and answer the questions provided. Place your answers on the table below.

The Tower of V

At the two upper points and at the lower point of the V, a total of 3 pegs are located. On the peg at the lower point, 5 rings are stacked with the largest on the bottom and the smallest on the top.



- You are to move the rings to one of the pegs at the upper points of the V.
- You may move one ring at a time.
- You may not place a larger ring on a smaller ring.
- You are to determine the smallest number of moves possible to accomplish the task.

You may use the patterns provided on page 313 to trace and then cut out to play the **Tower of V Game**.

Record the number of moves on the lines provided *and* in The Tower of V table below.

1. Using 2 rings, what is the smallest number of moves possible? ______
2. Using 3 rings, what is the smallest number of moves possible? ______
3. Using 4 rings, what is the smallest number of moves possible? ______
4. Using 5 rings, what is the smallest number of moves possible? ______
5. What prediction would you make for the smallest number of moves

   required for 6 rings? ______ 7 rings? ______

The Tower of V

Number of Rings	Number of Moves Required
2	
3	
4	
5	
6	
7	

6. Look at your completed table above. What relationship, if any, exists between the smallest number of moves and powers of two? Give an example.

____

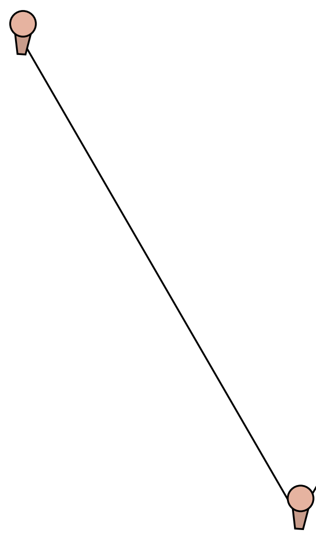
____

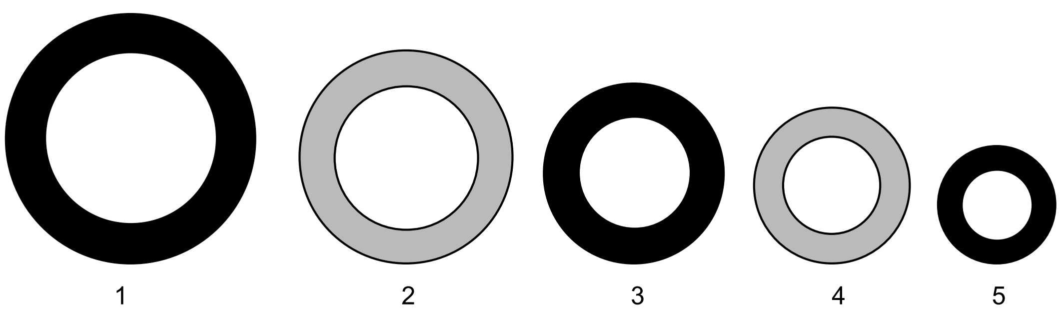
____

____

____

Circle patterns to cut out and place on V.





Practice

Answer the following.

A man wins $1,000,000. He decides to give it to charity, but he does not want to give all of it at one time.

- The first year he will give one-half of the $1,000,000 to charity.
- During the second year, he will give one-half of the $500,000 left to charity.
- He will continue this process until he has less than $500, at which time he will give all of what is left to charity.

Complete the chart on the following page to determine how many years will be required for his annual donation to fall below the $500 mark. **Round each annual donation to the nearest whole cent.**

Annual Charity Donations

Year	Amount in Account	Amount Given to Charity
1	$1,000,000	$500,000
2	$500,000	$250,000
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		

Practice

Answer the following.

Multiples and Prime Numbers

1	**2**	**3**	4	**5**	6	**7**	8	9	10
11	12	**13**	14	15	16	**17**	18	**19**	20
21	22	**23**	24	25	26	27	28	**29**	30
31	32	33	34	35	36	**37**	38	39	40
41	42	**43**	44	45	46	**47**	48	49	50
51	52	**53**	54	55	56	57	58	**59**	60
61	62	63	64	65	66	**67**	68	69	70
71	72	**73**	74	75	76	77	78	**79**	80
81	82	**83**	84	85	86	87	88	**89**	90
91	92	93	94	95	96	**97**	98	99	100

1. In the chart above, place a small 2 in the upper left corner of any *multiple* of 2 greater than 2 and less than or equal to 100.

2. In the chart above, place a small 3 in the upper right corner of any multiple of 3 greater than 3 and less than or equal to 100.

3. In the chart above, place a small 5 in the lower left corner of any multiple of 5 greater than 5 and less than or equal to 100.

4. In the chart above, place a small 7 in the lower right corner of any multiple of 7 greater than 7 and less than or equal to 100.

We can now see that the numbers in bold print are **prime numbers**. And we can see *prime factors*, less than or equal to 7, of the numbers from 1 to 100.

Remember:

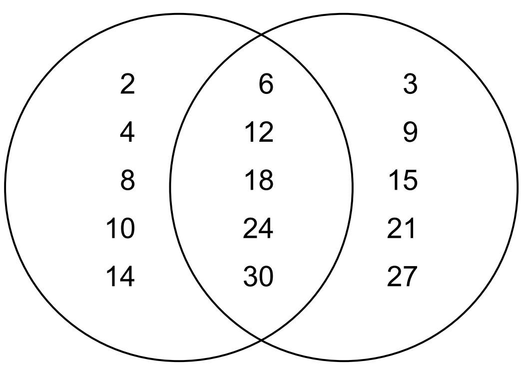
- A *prime number* is any whole number with two and only two factors, 1 and itself, such as 2, 3, 5, 7, 11, etc.
- A *prime factor* is any factor of a number which is a prime number.
- Every *natural number* is either prime or can be expressed as a product of primes except for the number 1.

Remember: *Natural numbers* are the counting numbers in the set {1, 2, 3, 4, 5, …}.

Use this information to complete the following Venn diagrams. Place at least *5 numbers in each part* of the Venn diagram. *Then write a statement* to describe the pattern that you see of **common multiples** in each Venn diagram.

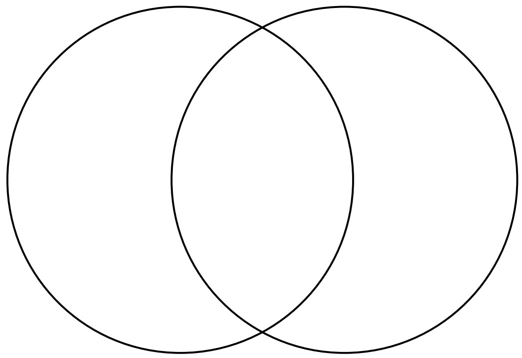
The first Venn diagram is *completed for you*.

5. Multiples of 2 and 3



Pattern: Common multiples of 2 and 3 are multiples of 6. In the chart on page 317 all multiples of 6 have 2s and 3s in the corners.

6. Multiples of 2 and 5



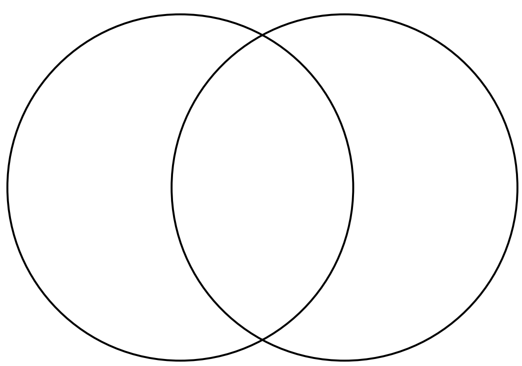
Pattern: ____

____

____

____

7. Multiples of 3 and 5



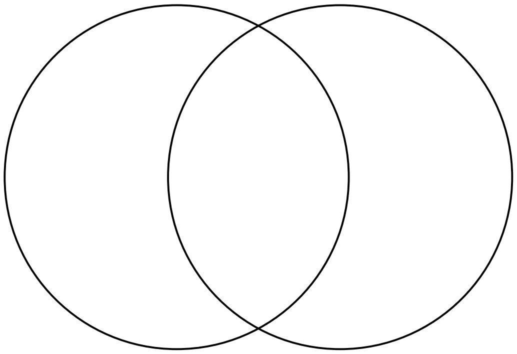
Pattern: ____

____

____

____

8. Multiples of 2 and 7



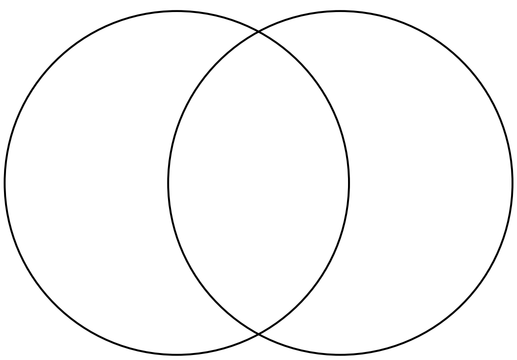
Pattern: ____

____

____

____

9. Multiples of 3 and 7



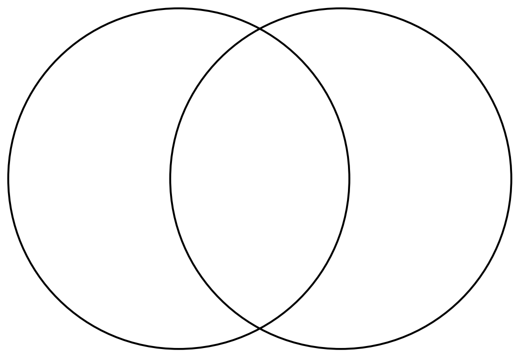
Pattern: ____

____

____

____

10. Multiples of 5 and 7



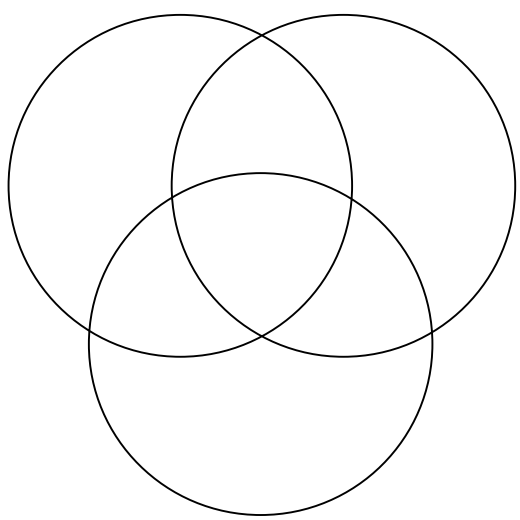
Pattern: ______________________________

____

____

____

11. Multiples of 2, 3, and 5



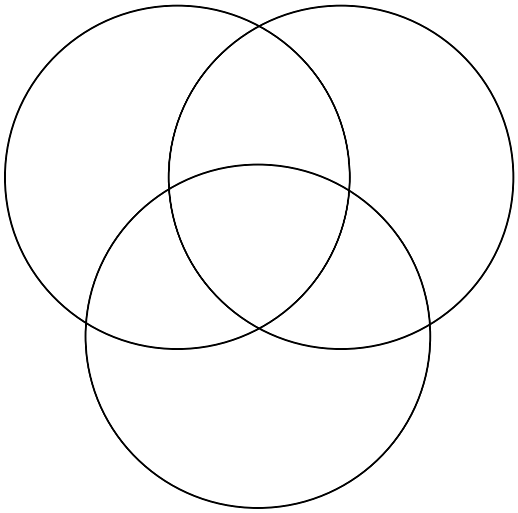
Pattern: ______________________________

____

____

____

12. Multiples of 2, 3, and 7



Pattern: ____

____

____

____

13. Explain why there is no number less than 100 that is a common multiple of 2, 3, 5 and 7.

Explanation: ____

____

____

____

____

Practice

Answer the following.

Hint: The Venn diagrams you completed in the previous practice will be helpful in answering problems 1-8.

1. One light blinks every 2 seconds, and another blinks every 3 seconds. How many times in one minute will both lights blink at the same time?

   ____

2. One light blinks every 2 seconds, and another blinks every 5 seconds. How many times in one minute will both lights blink at the same time?

   ____

3. One light blinks every 2 seconds, and another blinks every 7 seconds. How many times in one minute will both lights blink at the same time?

   ____

4. One light blinks every 3 seconds, and another blinks every 5 seconds. How many times in one minute will both lights blink at the same time?

   ____

5. One light blinks every 3 seconds, and another blinks every 7 seconds. How many times in one minute will both lights blink at the same time?

   ____

6. One light blinks every 2 seconds, a second light blinks every 3 seconds, and a third light blinks every 5 seconds. How many times in one minute will all three lights blink at the same time?

____

7. The school cafeteria serves pizza every 2nd day and hamburgers every 3rd day. In a month of 20 school days, how many days will the cafeteria serve pizza and hamburgers?

____

8. The school cafeteria serves ice cream every 3rd day and cookies every 5th day. In a school year of 180 days, how many days will the cafeteria serve ice cream and cookies?

____

9. Mrs. McGarity is purchasing 24 square yards of carpet for a rectangular-shaped room that is 12 feet wide. Assuming that the area of the room is 24 square yards, what is the length of the room?

Remember: Area equals length times width or $A = l(w)$ and that 1 yard = 3 feet.

____

10. The school library has set up a new policy regarding fines for overdue books. The fine starts at one cent the first day and doubles each day the book is late. What will be the amount of the fine if a book is returned 10 days late?

____

11. Bahia and Ora went fishing and each caught one fish. Bahia's fish weighed twice as much as Ora's, and the total weight of the two fish was six pounds. How much did Ora's fish weigh?

____

Practice

Use the list below to write the correct term for each definition on the line provided.

common multiple	**factor**	**prime number**
equation	**odd number**	**table**
even number	**pattern**	**value (of a variable)**
expression	**power (of a number)**	

________________________ 1. any of the numbers represented by the variable

________________________ 2. a collection of numbers, symbols, and/or operation signs that stands for a number

________________________ 3. a number or expression that divides exactly another number

________________________ 4. an exponent; the number that tells how many times a number is used as a factor

________________________ 5. a predictable or prescribed sequence of numbers, objects, etc.

________________________ 6. an orderly display of numerical information in rows and columns

________________________ 7. a number that is a multiple of two or more numbers

________________________ 8. any whole number with only two factors, 1 and itself

________________________ 9. any whole number not divisible by 2

________________________ 10. any whole number divisible by 2

________________________ 11. a mathematical sentence that equates one expression to another expression

Unit 5: Probability and Statistics

This unit emphasizes how statistical methods, measures of central tendency, and probability concepts are used to gather and analyze data to solve problems.

Unit Focus

Numbers Sense, Concepts, and Operations

- Understand that numbers can be represented in a variety of equivalent forms, including fractions, decimals, and percents. (A.1.3.4)
- Add, subtract, multiply, and divide whole numbers, decimals, and fractions to solve real-world problems, using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)

Measurement

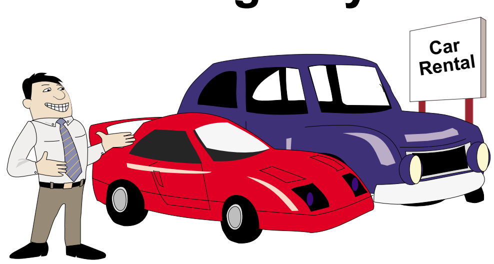
- Solve problems involving units of measure and convert answers to a larger or smaller unit. (B.2.3.2))

Data Analysis and Probability

- Collect, organize, and display data in a variety of forms, including tables, charts, and bar graphs, to determine how different ways of presenting data can lead to different interpretations. (E.1.3.1)
- Understand and apply the concepts of range and central tendency (mean, median, and mode). (E.1.3.2)
- Analyze real-world data by applying appropriate formulas for measures of central tendency and organizing data in a quality display, using appropriate technology, including calculators and computers. (E.1.3.3)

- Compare experimental results with mathematical expectations of probabilities. (E.2.3.1)
- Collect, organize, and display data in a variety of forms, including tables, line graphs, charts, and bar graphs, to determine how different ways of presenting data can lead to different interpretations. (E.3.3.1)

Car Rental Agency Manager



- notes the condition of each vehicle
- determines total cost of rentals based on different rates and types of charges
- schedules and keeps track of vehicles' dates of loan and dates of return

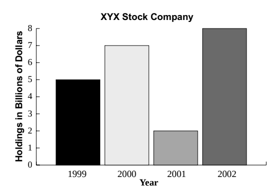
Vocabulary

Study the vocabulary words and definitions below.

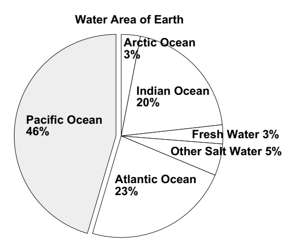
ascending order .... moving from lower to higher

axes (of a graph) .... the horizontal and vertical number lines used in a rectangular graph or coordinate grid system as a fixed reference for determining the position of a point; (singular: *axis*)

bar graph .... a graph used to compare quantities in which lengths of bars are used to compare numbers



circle graph .... a graph used to compare parts of a whole; the whole amount is shown as a circle, and each part is shown as a percent of the whole



cube .... a rectangular prism that has six square faces

data .... information in the form of numbers gathered for statistical purposes

data display .... different ways of displaying data in tables, charts, or graphs
Example: pictographs; circle graphs; single, double, or triple bar and line graphs; histograms; stem-and-leaf plots; and scatterplots

decimal number .... any number written with a decimal point in the number
Example: A decimal number falls between two whole numbers, such as 1.5 falls between 1 and 2. Decimal numbers smaller than 1 are sometimes called decimal fractions, such as five-tenths is written 0.5.

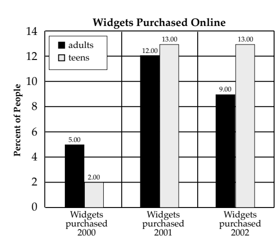
descending order .... moving from higher to lower

degree (°) .... common unit used in measuring angles

denominator .... the bottom number of a fraction, indicating the number of equal parts a whole was divided into
Example: In the fraction $\frac{2}{3}$ the denominator is 3, meaning the whole was divided into 3 equal parts.

difference .... the result of a subtraction
Example: In $16 - 9 = 7$, 7 is the difference.

double bar graph .... a graph used to compare quantities of *two* sets of data in which length of bars are used to compare numbers



equation .... a mathematical sentence that equates one expression to another expression
Example: $2x = 10$

estimation .... the use of rounding and/or other strategies to determine a reasonably accurate approximation without calculating an exact answer

even number .... any whole number divisible by 2
Example: 2, 4, 6, 8, 10, 12 …

experimental or empirical probability .... the likelihood of an event happening that is based on *experience and observation* rather than theory

factor .... a number or expression that divides exactly another number
Example: 1, 2, 4, 5, 10, and 20 are factors of 20.

fraction .... any number representing some part of a whole; of the form $\frac{a}{b}$
Example: One-half written in fractional form is $\frac{1}{2}$.

graph .... a drawing used to represent data
Example: bar graphs, double bar graphs, circle graphs, and line graphs

greatest common factor (GCF) .... the largest of the common factors of two or more numbers
Example: For 6 and 8, 2 is the greatest common factor.

likelihood .... the chance that something is likely to happen; see *probability*

maximum .... the largest amount or number allowed or possible

mean (or average) .... the arithmetic average of a set of numbers

median .... the middle point of a set of ordered numbers where half of the numbers are above the median and half are below it

minimum .... the smallest amount or number allowed or possible

mode .... the score or data point found most often in a set of numbers

multiples .... the numbers that result from multiplying a given number by the set of whole numbers
Example: the multiples of 15 are 0, 15, 30, 45, 60, 75, etc.

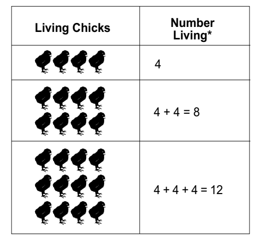
numerator .... the top number of a fraction, indicating the number of equal parts being considered
Example: In the fraction $\frac{2}{3}$, the numerator is 2.

odd number .... any whole number *not* divisible by 2
Example: 1, 3, 5, 7, 9, 11 …

outcome .... a possible result of a probability experiment

percent (%) .... a special-case ratio in which the second term is always 100
Example: The ratio is written as a whole number followed by a percent sign, such as 25% which means the ratio of 25 to 100.

pictograph .... a graph used to compare data in which picture symbols represent a specified number of items



prime number .... any whole number with only two factors, 1 and itself
Example: 2, 3, 5, 7, 11, etc.

probability .... the ratio of the number of favorable outcomes to the total number of outcomes

product .... the result of a multiplication
Example: In 6 x 8 = 48, 48 is the product.

quotient .... the result of a division
Example: In 42 ÷ 7 = 6, 6 is the quotient.

random .... by chance, with no outcome any more likely than another

range (of a set of numbers) .... the difference between the highest (H) and the lowest value (L) in a set of data; sometimes calculated as H - L + 1

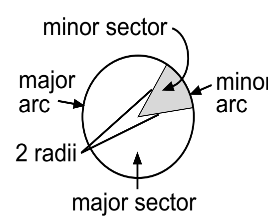
ratio .... the quotient of two numbers used to compare two quantities
Example: The ratio of 3 to 4 is $\frac{3}{4}$.

rounded number .... a number approximated to a specified place
Example: A commonly used rule to round a number is as follows.

- If the digit in the first place after the specified place is 5 or more, *round up* by adding 1 to the digit in the specified place ($\underline{4}61$ rounded to the nearest hundred is 500).
- If the digit in the first place after the specified place is less than 5, *round down* by *not* changing the digit in the specified place ($\underline{4}41$ rounded to the nearest hundred is 400).

scales .... the numeric values assigned to the axes of a graph

sector .... a part of a circle bounded by two radii and the arc or curve created between any two of its points



stem-and-leaf plot .... a way of organizing data to show their frequency

Number of Goals Scored

Stem	Leaves
1	5 9
2	1 3 7 7 7
3	0 1 3 4 4 5 6 7
4	2 3 6 7 8

Key: **2** | 3 represents 23.

sum .... the result of an addition
Example: In 6 + 8 = 14,
14 is the sum.

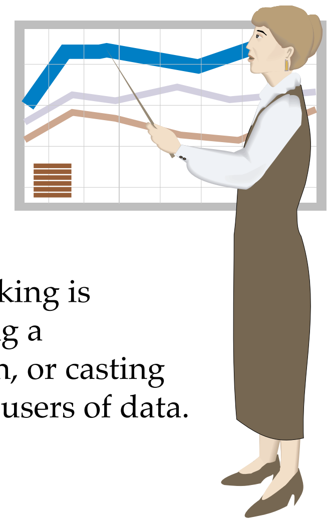
table (or chart) .... an orderly display of numerical information in rows and columns

theoretical probability .... the likelihood of an event happening that is based on *theory* rather than on experience and observation

whole number .... any number in the set {0, 1, 2, 3, 4 …}

Unit 5: Probability and Statistics

Introduction



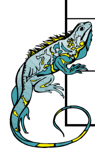
This unit emphasizes how statistical methods, measures of central tendency, and probability concepts are used to gather and analyze data to solve problems. The study of statistics and probability can be interesting and entertaining as well as important in dealing with everyday life. The ability to analyze data to support decision making is empowering. Whether playing a game, making a purchase, choosing a medical treatment option, or casting a vote, data analysis can help us become wise users of data.

Lesson One Purpose

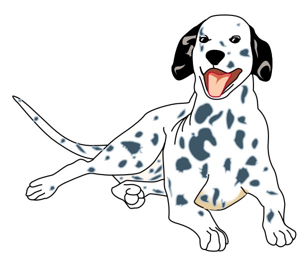
- Understand that numbers can be represented in a variety of equivalent forms including fractions, decimals, and percents. (A.1.3.4)
- Add, subtract, multiply and divide whole numbers and decimals using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Organize and display data in a variety of forms. (E.1.3.1)
- Understand and apply the concepts of range and central tendency (mean, median, and mode). (E. 1.3.2)
- Analyze real-world data by applying appropriate formulas for measure of central tendency and organizing data in a quality display, using appropriate technology, including calculators and computers. (E.1.3.3)

Using Mean, Median, Mode, and Range

Five families in a neighborhood are surveyed to determine the number of pets. The results are as follows.



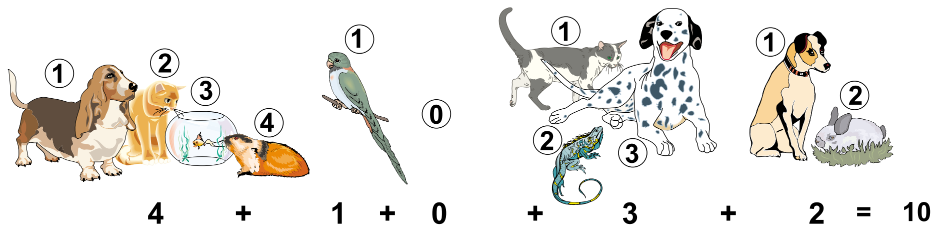
Family	Number of Pets
A	4
B	1
C	0
D	3
E	2



We can think of the **mean** number of pets for the families as being the number of pets of each family—*if* each family has the same number of pets. To do this, each family would need to swap around until they all had the same number of pets. Family A might give 2 of its pets to Family C. Family D might give 1 of its pets to Family B. Each family would then have 2 pets.

The *mean* is an *average* of a set of numbers. The mean is found by adding all the values in a set and then dividing the **sum** by the number of values.

The number of pets $4 + 1 + 0 + 3 + 2 = 10$.



If 10 (the sum of the pets) is divided by 5 (the number of families), the **quotient** is 2. The mean number of pets for the five families is 2.

$10 \div 5 = 2$

The **median** is the numerical value that marks the *middle* of an ordered set of **data**. We can put the number of pets in **ascending order** *from lower to higher*.

ascending order

0 1 2 3 4

We see that the middle of the ordered set is 2.
The *median* number of pets for the five families is 2.

We can put the number of pets in **descending order** *from higher to lower*.

descending order

4 3 2 1 0

We see that the middle of the ordered set is 2. The median number of pets for the five families is 2.

The numerical *value that occurs most frequently* in a data set is the **mode**. Since no number occurs more frequently than another in this data set, there is *no* mode.

The **range** is reported in one of two ways.

- The range is reported as ranging from the *lowest value to the highest*.

  The range of the number of pets is 0 to 4.

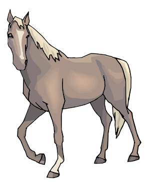
- The range is reported as the **difference** between the *highest and lowest values*.

  The range of the number of pets is 4.

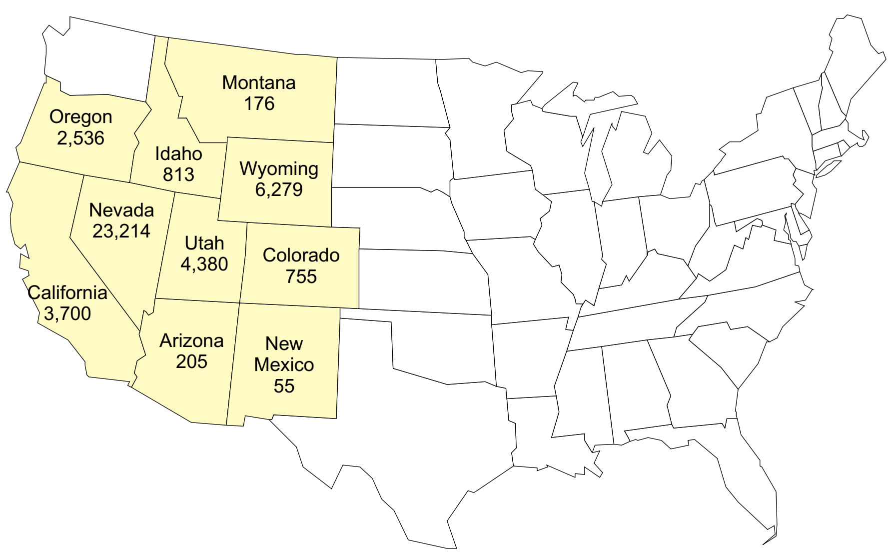
Practice

Answer the following.

The data below was included in a news article about wild horses in the United States. The news article reported on where wild horses are most *prevalent*, or most likely to be found, in the United States. Use the data to complete the following statements.



State	Approximate Number of Wild Horses
Arizona	205
California	3,700
Colorado	755
Idaho	813
Montana	176
Nevada	23,214
New Mexico	55
Oregon	2,536
Utah	4,380
Wyoming	6,279



Ten states where wild horses are the most prevalent.

Use the **data** *on the* **previous page** *to complete the following.*

1. The number of wild horses in these states ranges from a low of ____________ to a high of ____________ . Sometimes the range is reported this way. Sometimes it is reported as the positive difference in these numbers. If the low of ____________ is subtracted from the high of ____________ , the difference is ____________ .

2. The number, or numbers, appearing most frequently in a set of data is called the *mode*. A mode ____________ (does, does not) exist for this data set.

3. a. To find the mean of a data set, we find the sum of all entries and divide the sum by the number of entries. Fill in the missing entries.

$$\frac{205 + 3{,}700 + 755 + \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_ + \_\_\_\_\_}{10}$$

   b. Find the mean number of wild horses in these 10 states. **Round your answer to the nearest whole number**.

   Answer: ________________________

4. a. To find the median of a set of data, put the data in order from smallest to largest *or* from largest to smallest.

   - If there is an **odd number** of numbers in a data set, and the data is arranged from smallest to largest or from largest to smallest, the *number in the middle* is the median.
   - If the number of numbers in the data set is an **even number**, and the data is arranged from smallest to largest or from largest to smallest, the *mean, or average, of the two numbers in the center* is the median.

Place the data about wild horses on the lines provided either from smallest to largest *or* largest to smallest.

_____ , _____ , _____ , _____ , _____ , _____ , _____ , _____ , _____ , _____

b. Our data set has an even number of entries. Find the median for this data set. **Round your answer to the nearest whole number.**

Answer: ________________________

Practice

Use the list below to write the correct term for each definition on the line provided.

ascending order	**median**
data	**mode**
descending order	**quotient**
difference	**range**
mean	**sum**

________________ 1. the result of a subtraction

________________ 2. the result of a division

________________ 3. the result of an addition

________________ 4. moving from higher to lower

________________ 5. moving from lower to higher

________________ 6. the arithmetic average of a set of numbers

________________ 7. information in the form of numbers gathered for statistical purposes

________________ 8. the middle point of a set of ordered numbers where half of the numbers are above the median and half are below it

________________ 9. the difference between the highest (H) and the lowest value (L) in a set of data

________________ 10. the score or data point found most often in a set of numbers

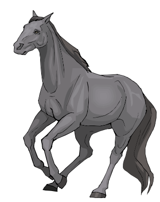
Practice

Answer the following using the **data** *from the* **practice** *on page 340.*

1. In the first practice, the news report chose to order the states in the table alphabetically. They could have ordered them by the states with *greatest* number of wild horses to the *least* number of wild horses. Or they could have ordered from *least to greatest*.

   Make a **table** ordering the states with *either* the greatest to least or least to greatest number of wild horses.

State	Approximate Number of Wild Horses



2. Which of the three **data display** options do you think would be best for an article about wild horses—alphabetical, greatest to least, or least to greatest?

Answer: ____________________________

Why? ____

____

____

____

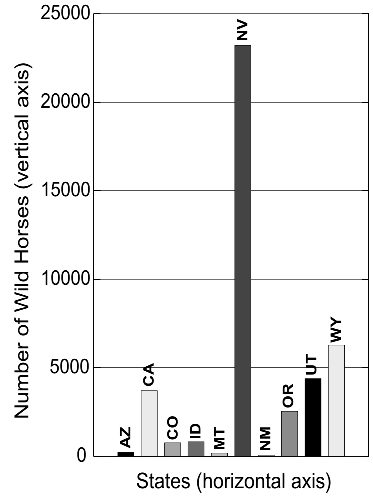
____

Practice

Answer the following using the **data** *from the* **practice** on page 340.

News reports often include **graphs** to display the data provided. The questions in the practice below will explore the effectiveness of a graph for this data.

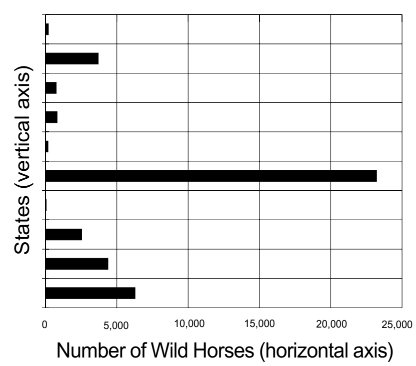
A **bar graph** is a graph used to compare quantities. It uses lengths of bars to compare numbers. A bar graph made with the data from the previous practice could be made in two different ways.



- The names of the states could be placed on the horizontal (↔) axis with bars representing the numbers in vertical positions.

  or

- The names of the states could be placed on the vertical (↕) axis with bars representing the numbers in horizontal positions.



To display data on a graph, **scales** are used. *Scales* are the numeric values assigned to the **axes of a graph**.

1. If the scale used for lengths of bars was 50 to accommodate New Mexico's 55 wild horses, how many 50s would need to be represented for a bar representing the number of wild horses in Nevada?

   Answer: ____________

   The number will not be a whole number. How should we deal with the remainder? Explain in words.

   Explanation: ____

   ____

2. If the scale used for lengths of bars was 100 to accommodate New Mexico's 55 wild horses, how many 100s would need to be represented for a bar representing the number of wild horses in Nevada?

   Answer: __________

3. If the scale used for lengths of bars was 1,000 to more conveniently represent the number of wild horses in Nevada, what impact would this have on the bars for New Mexico, Montana, and Arizona? Explain in words.

   Explanation: ____

   ____

   ____

   ____

4. A news reporter might have a general rule that says the greater the range in the data, the less effective a bar graph is likely to be. Explain why this rule *does* or *does not* make sense.

   Explanation: ____

   ____

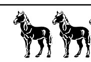
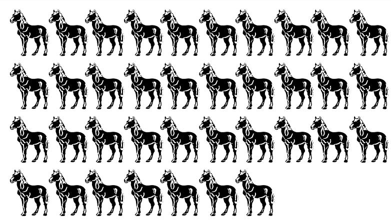
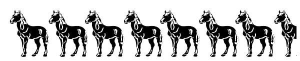
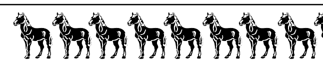
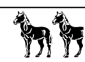
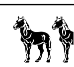
   ____

   ____

Practice

Answer the following using the **data** *from the* **practice** *on page 340.*

A **pictograph** is another type of graph used to compare data. The pictograph uses *picture symbols* to represent a specified number of items.

Approximate Number of Wild Horses	States
	**Arizona**
	**California**
	**Colorado**
	**Idaho**
	**Montana**
	**New Mexico**

1. A pictograph could use a picture of a wild horse to represent a certain number of horses from the practice on page 340. How would this be similar to the bar graph? Explain in words.

   Explanation: ____

   ____

2. When the range of numbers is great, would using a pictograph instead of a bar graph solve the problem of choosing a scale?

   Answer: __________

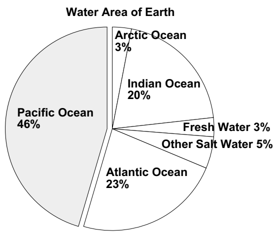
   Explanation: ____

   ____

   ____

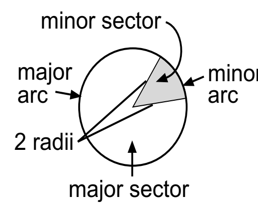
Practice

Answer the following using the **data** *from the* **practice** *on page 340.*



A **circle graph** is often used to *show and compare* parts of a whole. The whole amount is shown as a circle. Each part of the circle is shown as a **percent (%)** of the whole. To the left is a circle graph of the areas of water of Earth.

To make a circle graph for the 10-state wild horse population in the practice on page 340, the whole circle would represent 100% of the wild horse population. Each **sector** or part of the circle would represent the percentage of wild horses in a given state. A *sector* of a circle is bounded by two *radii* and the *arc* of curve created between any two of its points. To determine the percentage of wild horses in each state some arithmetic would be necessary.



The fractional part of the wild horse population can be expressed as a **fraction** or **decimal**. For Arizona, the fractional part can be represented as $\frac{205}{42113}$ or 0.005. We know that 205 of the 42,113 wild horses are in Arizona. The equivalent decimal can be found by dividing 205 by 42,113 and rounding to the desired place.

$$\frac{205}{42113} \qquad\qquad 42113\,\overline{)\,205.0000}^{\;.0048} = .005$$

fractional part of wild horses in Arizona

equivalent decimal

To write the percent equivalent of the decimal, we multiply the decimal number by 100

or

use the equivalent decimal and move the decimal point two places to the right, then add a percent sign (%).

$$\begin{array}{r} .005 \\ \times\ 100 \\ \hline 000 \\ 000 \\ 005 \\ \hline 00.500 \end{array}$$

.005 = 0.5%

To find the number of **degrees (°)** in the circle to represent a state, we multiply the fractional part of the wild horse population by 360 since there are 360 degrees in a circle. We can use the fraction, the decimal, or the percent when multiplying.

- multiplying the fractional part

$\frac{205}{42113}$ x 360

$\frac{205}{42113}$ x $\frac{360}{1}$ = $\frac{73800}{42113}$

1.75 = 1.8

- multiplying the decimal part time 360 degrees

0.005 x 360 =

$$\begin{array}{r} 0.005 \\ \times\ 360 \\ \hline 0000 \\ 0030 \\ 0015 \\ \hline 001.800 \end{array} = 1.8$$

- multiplying the percent times 360 degrees (using paper and pencil) is the same as above because

0.5% = .005, so the problem is the same

.005 x 360 = 1.8

When making a circle graph for the 10-state wild horse population, we know a certain fractional part or percent of the total wild horse population is in each state. We also know a fractional part or percent of the circle must represent the wild horse population in that state.

First, you will use the numbers of wild horses in each of the 10 states from the practice on page 340 to complete the two tables on the following page. Then you will use the information from your two tables to complete a circle graph on page 353.

1. Complete the table. The table has been started for you.
   - Round the *decimals* to the nearest *thousandth*.
   - Round the *percents* to the nearest *tenth*.
   - Round the *number of degrees* to the nearest *tenth*.

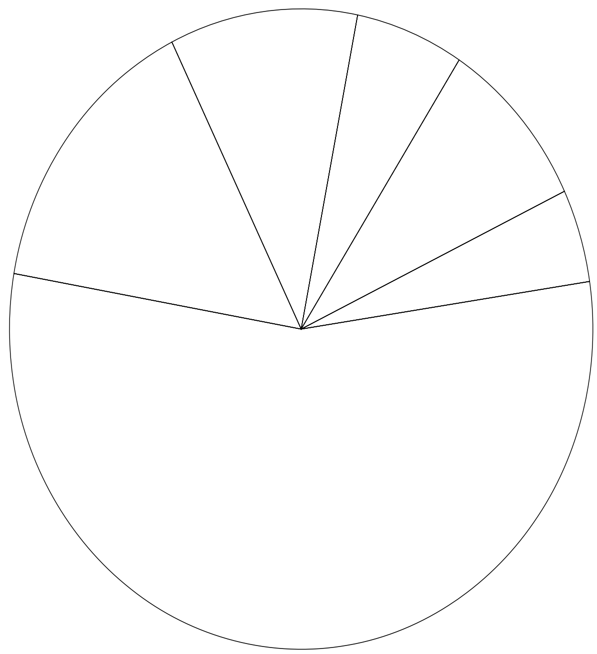
State	Fractional Part	Decimal	Percent	Percent of 360 Degrees
Arizona	$\frac{205}{42113}$	0.005	0.5%	$\frac{205}{42113}$ x 360 = 1.8 or 0.005 x 360 = 1.8 or 0.5% of 360 = 1.8
California	$\frac{3700}{42113}$	0.088	8.8%	$\frac{3700}{42113}$ x 360 = 32 or 0.088 x 360 = 32 or 8.8% of 360 = 32
Colorado				
Idaho				
Montana				
Nevada				
New Mexico				
Oregon				
Utah				
Wyoming				

2. To draw a *sector*, or *part of the circle determined by an angle*, measuring 0.5 degrees for New Mexico's wild horse population will be difficult. Therefore, we will have to group some states into one sector. First, we might let five sectors represent each of the five of the states with more than 1,000 wild horses. Then we can let one additional sector represent the five states with less than 1,000 wild horses each.

Use your data from the previous page to complete the table below.

State(s)	**Number of Degrees in Circle Graph**
Nevada	
Wyoming	
Utah	
California	
Oregon	
Idaho, Colorado,	
Arizona, Montana, New Mexico	

3. Below, a properly divided circle graph is provided. Give the graph a *title* and add the *names of the states* in the correct sectors.



4. In your opinion, would it be helpful to include this graph in the news article about wild horses? Why or why not?

____

____

____

____

____

____

Practice

Match each definition with the correct term. Write the letter on the line provided.

______ 1.	a number approximated to a specified place	A. even number
______ 2.	any number in the set {0, 1, 2, 3, 4 …}	B. graph
______ 3.	any whole number *not* divisible by 2	C. odd number
______ 4.	any whole number divisible by 2	D. rounded number
______ 5.	an orderly display of numerical information in rows and columns	E. table
______ 6.	a drawing used to represent data	F. whole number

Practice

Answer the following using the **data** *from the* **practice** *on page 340.*

These questions will deal with statements that a reader might make after reading the data from the news article in the practice on page 340. Users of data sometimes make *valid* or sound assumptions. Sometimes they make *invalid* or unsound assumptions. This may result in proper use *or* misuse of statistical data. Think about this as you decide which of the statements are *true* and which are *false*.

Write **True** *if the statement is correct. Write* **False** *if the statement is not correct. If the statement is* **false**, *tell why.*

__________ 1. Most wild horses in the world are found in the western part of the United States.

____

____

__________ 2. Most wild horses living in the United States are found in the western part of the United States.

____

____

__________ 3. The further west you go in the United States, the greater the number of wild horses.

____

____

__________ 4. Washington is the only state on the west coast with *no* wild horses.

____

____

__________ 5. There are *no* wild horses east of Colorado.

____

____

__________ 6. More than half of the wild horses in the 10 states where they are most prevalent in the United States are found in Nevada.

____

____

__________ 7. The number of wild horses reported in each state is likely the result of actually counting each and every horse.

____

____

__________ 8. The number of wild horses reported in each state is likely the result of a survey sent to residents in the state about how many wild horses they have seen in their area.

____

____

__________ 9. The number of wild horses reported in each state is likely the result of an **estimate** based upon scientific measures.

____

____

Bonus item

__________ 10. Wild horses *cannot* live where temperatures may fall below zero degrees Fahrenheit or rise above 100 degrees Fahrenheit.

____

____

Practice

Answer the following.

Mike and Misty have a friend who is a Tallahassee police officer. In March 2001, the officer compared salaries of Tallahassee police officers with officers in other Florida cities of similar size.

See the table below of the data collected to compare *annual* (yearly) salaries of police officers in four Florida cities of similar size. Use the table below to answer the following.

City	Population	Minimum Annual Salary	Maximum Annual Salary
Tallahassee	145,610	$29,400	$43,700
Ft. Lauderdale	148,871	$37,100	$52,600
Hollywood	127,680	$35,200	$53,300
Pembroke Pines	120,081	$38,700	$54,600

1. Find the range:

   a. for the population of the four cities. ______________________

   b. for the **minimum** annual salary. ______________________

   c. for the **maximum** annual salary. ______________________

2. Write a sentence:

   a. explaining how the population of Tallahassee may justify comparison with the other three cities for salary of police officers.

   ____

   ____

b. explaining how a user might ask for data on any other cities of comparable size.

____

____

____

____

3. Find the mean:

   a. for the population of the four cities. ________________________

   b. for the minimum annual salary. ___________________________

   c. for the maximum annual salary. ___________________________

4. Write a sentence:

   a. comparing the population of Tallahassee to the mean for the four cities.

   ____

   ____

   b. explaining how a user of the data might ask for data on salaries for other comparable cities.

   ____

   ____

   c. comparing Tallahassee's minimum and maximum salaries to the minimum and maximum mean.

   ____

   ____

   ____

Practice

Answer the following.

In the research from the previous practice, the officer also determined the number of years it takes an officer in each of the four cities to reach the maximum salary after beginning at the minimum salary.

1. In Ft. Lauderdale, an officer begins at $37,100. At the end of 5 years, the officer earns $52,600. Assume the salary increases the same amount annually (yearly) for each of the 5 years.

   Use the following **equation** to determine the annual salary increase per year. Then complete the table to determine the salary after 1 year, 2 years, 3 years, 4 years, and 5 years.

   $\frac{(52600 - 37100)}{5}$ = ___________ amount of salary increase per year

2. In Hollywood, an officer begins at $35,200. At the end of 7 years, the officer earns $53,302. Assume the salary increases the same amount annually for each of the 7 years.

   Use the following equation to determine the annual salary increase per year, then complete the table to determine the salary after 1 year, 2 years, 3 years, 4 years, 5 years, 6 years, and 7 years.

   $\frac{(53302 - 35200)}{7}$ = ___________ amount of salary increase per year

3. In Pembroke Pines, an officer begins at $38,700 per year. At the end of 8 years, the officer earns $54,604. Assume the salary increases the same amount annually for each of the eight years.

   Use the following equation to determine the annual salary increase per year, then complete the table to determine the salary after 1 year, 2 years, 3 years, 4 years,
   5 years, 6 years, 7 years, and 8 years.

   $\frac{(54604 - 38700)}{8}$ = ___________ amount of salary increase per year

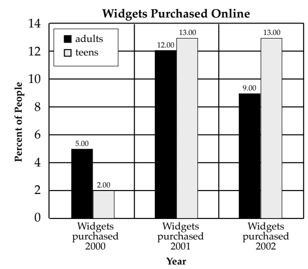
4. In Tallahassee, an officer begins at $29,400 per year and at the end of 17 years, the officer earns $43,697. Assume the salary increases the same amount annually for each of the 17 years.

Use the following equation to determine the annual salary increase per year, then complete the table to determine the salary after 1 year, 2 years, 3 years, 4 years, 5 years, 6 years, 7 years, and on up through 17 years.

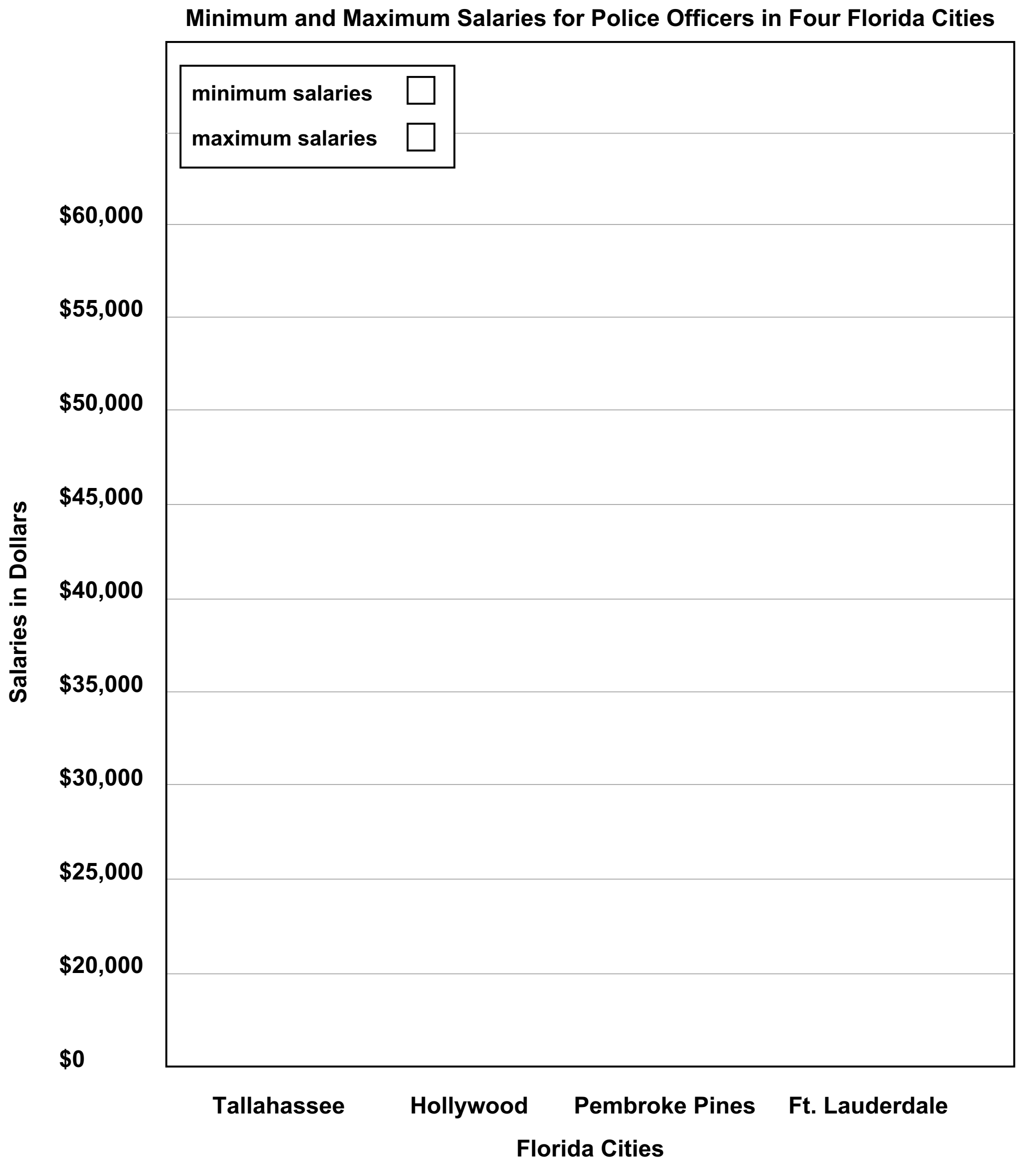
$\frac{(43697 - 29400)}{17}$ = ____________ amount of salary increase per year

Year	Ft. Lauderdale	Hollywood	Pembroke Pines	Tallahassee
1	$37,100	$35,200	$38,700	$29,400
2				
3				
4				
5				
6	$52,600			
7	$52,600			
8	$52,600	$53,302		
9	$52,600	$53,302	$54,604	
10	$52,600	$53,302	$54,604	
11	$52,600	$53,302	$54,604	
12	$52,600	$53,302	$54,604	
13	$52,600	$53,302	$54,604	
14	$52,600	$53,302	$54,604	
15	$52,600	$53,302	$54,604	
16	$52,600	$53,302	$54,604	
17	$52,600	$53,302	$54,604	
18	$52,600	$53,302	$54,604	$43,697

5. *Bar graphs* use lengths of bars to *compare quantities* of data about different things at a given time. **Double bar graphs** are used to compare quantities of *two sets of data*. In the double bar graph on the right, different colored bars illustrate data on purchases by adults and teens.



Use the data on the previous page to make the *double bar graph* below. Remember to indicate in the key what color of bar you used for minimum salaries and what color of bar you used for maximum salaries.



6. Pretend you are Mike and Misty's police officer friend in Tallahassee.

   a. Write at least three statements you might use if you had the opportunity to speak before the authorities responsible for setting salaries of police officers in Tallahassee.

      ____

      ____

      ____

      ____

      ____

      ____

   b. If you accompanied one or more of your statements with a display, would you use one of the tables, the bar graph, or a combination? Why or why not?

      ____

      ____

      ____

      ____

      ____

      ____

Practice

Use the list below to write the correct term for each definition on the line provided.

axes (of a graph)	**equation**	**percent (%)**
bar graph	**estimation**	**pictograph**
circle graph	**fraction**	**scales**
decimal number	**maximum**	**sector**
degree (°)	**minimum**	

______________________ 1. the largest amount or number allowed or possible

______________________ 2. the smallest amount or number allowed or possible

______________________ 3. common unit used in measuring angles

______________________ 4. any number representing some part of a whole; of the form $\frac{a}{b}$

______________________ 5. any number written with a decimal point in the number

______________________ 6. a special-case ratio in which the second term is always 100

______________________ 7. a graph used to compare parts of a whole; the whole amount is shown as a circle, and each part is shown as a percent of the whole

______________________ 8. a graph used to compare data in which picture symbols represent a specified number of items

______________________ 9. the numeric values assigned to the axes of a graph

_______________ 10. the horizontal and vertical number lines used in a rectangular graph or coordinate grid system as a fixed reference for determining the position of a point

_______________ 11. a graph used to compare quantities in which lengths of bars are used to compare numbers

_______________ 12. a part of a circle bounded by two radii and the arc or curve created between any two of its points

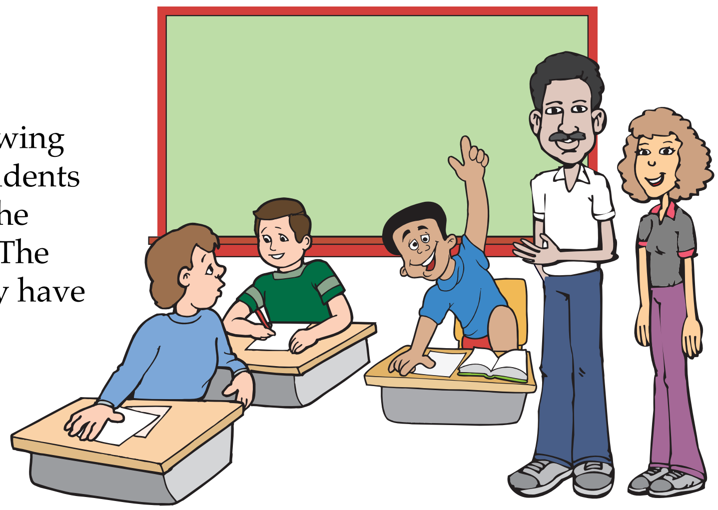
_______________ 13. the use of rounding and/or other strategies to determine a reasonably accurate approximation without calculating an exact answer

_______________ 14. a mathematical sentence that equates one expression to another expression

Lesson Two Purpose

- Add, subtract, multiply and divide whole numbers and decimals using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Organize and display in a variety of forms. (E.1.3.1)
- Compare experimental results with mathematical expectations of probabilities. (E.2.3.1)

Probability



Mike and Misty are reviewing **probability** with their students before making plans for the school's annual carnival. The students share things they have learned in the past about probability, including the following.

- If an event is *impossible*, the probability is 0 (zero).
- If an event is *certain*, the probability is 1 (one).
- If an event is *not* impossible but it is *not* certain, the probability is greater than (>) 0 but less than (<) 1.
- Probability describes the **likelihood** that an event will occur. For example, if a bag contains 2 blue blocks and 3 red blocks and one is **randomly** drawn, the probability of drawing a blue block is $\frac{2}{5}$.
- When we experiment by tossing a coin 30 times to determine the number of *heads* (H) and *tails* (T), we are finding **experimental** or **empirical probability**. Such experiments can help us predict what might happen over the long run. If 13 of the 30 tosses of the coin were heads, the *experimental probability* of getting heads would be $\frac{13}{30}$.

Experimental probability is written as a *fraction*. The number of times a *favorable* **outcome** occurs is the **numerator**. The total number of *trials* is the **denominator**. The fraction represents the **ratio** of the number of heads to the number of tosses.

$$\frac{13}{30} = \frac{\text{number of times favorable outcomes occurred}}{\text{total number of trials}}$$

- When we analyze a situation mathematically, we are finding the **theoretical probability**. If we are analyzing what happens when we toss two coins, we determine that there are four possible *outcomes*.

  HH, HT, TH, TT

  The *theoretical probability* of getting two heads is $\frac{1}{4}$. The number of times the *favorable outcome* may occur is the *numerator*. The number of *possible outcomes* is the *denominator*. The fraction represents the *ratio* of the number of favorable outcomes (HH) to the total number of outcomes.

$$\frac{1}{4} = \frac{\text{number of favorable outcomes}}{\text{total number of possible outcomes}}$$

Practice

Answer the following.

1. Fifty-two cards numbered 1-52 are placed in a box. One card is randomly selected. What is the probability that the number on the card is a **prime number** and is a **multiple** of 5? (**Hint:** First list prime numbers and then find any multiples of 5.)

   Answer: ______________________________

   Explain how you got your answer. ______________________________

   ____

   ____

   ____

   ____

2. Aretha has four pairs of earrings in a box. Assume the earrings are the same size and shape. If she randomly selects one earring and then another, what is the probability that the two will be from the same pair? (**Hint:** Make an organized list of possible outcomes.)

   Answer: ______________________________

   Explain how you got your answer. ______________________________

   ____

   ____

   ____

   ____

3. Problem number 2 could easily be an experiment. Place 2 blue blocks, 2 red blocks, 2 yellow blocks, and 2 green blocks in a bag. (You could substitute colored squares of paper.)

- Draw one block from the bag. Do not put it back in the bag.
- Draw a second block.
- Keep a record of whether or not the colors matched.
- Put the two blocks back into the bag.
- Repeat the experiment 20 times.

Block Experiment Outcomes

number of trials	color of 1st block	color of 2nd block	match yes / no
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			
		total number of matches =	

How does your experimental probability compare with the theoretical probability you gave for your answer to problem number 2?

Explain. ____

____

____

____

____

4. Richard has seven pairs of socks, and each pair is a different color. If he randomly selects one sock and then another, what is the probability that the two will be from the same pair?

Answer: ____

Explain how you got your answer. ______________________________

____

____

____

Practice

Match each definition with the correct term. Write the letter on the line provided.

______ 1. the top number of a fraction, indicating the number of equal parts being considered	A. denominator
______ 2. the bottom number of a fraction, indicating the number of equal parts a whole was divided into	B. experimental or empirical probability
______ 3. by chance, with no outcome any more likely than another	C. likelihood
______ 4. the quotient of two numbers used to compare two quantities	D. numerator
______ 5. a possible result of a probability experiment	E. outcome
______ 6. the chance that something is likely to happen	F. random
______ 7. the likelihood of an event happening that is based on *theory* rather than on experience and observation	G. ratio
______ 8. the likelihood of an event happening that is based on *experience and observation* rather than theory	H. theoretical probability

Practice

Answer the following.

Four cards numbered 5, 10, 15, and 20 are placed in a box. Two cards are randomly selected. Before responding to the questions below, do this experiment 18 times. Keep a record of the two cards drawn each time on the table below. Put the two cards back in the box after each trial.

Card Outcomes

number of trials	number on 1st card	number on 2nd card	sum of the two numbers
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			

Your record can help you make a list of *all* possible outcomes on the following page. The list will also support your work in answering the questions on the following page and in explaining how you got your answer.

List all possible outcomes.

______________________ = __________

______________________ = __________

______________________ = __________

______________________ = __________

______________________ = __________

______________________ = __________

1. What is the probability that the *sum* of the two numbers on the cards is even?

   Answer: ______________________

   Explain how you got your answer. ______________________

   ____

   ____

   ____

2. What is the probability that the **greatest common factor (GCF)** of the sum of the two numbers is 5?

   Answer: ______________________

   Explain how you got your answer. ______________________

   ____

   ____

   ____

3. What is the probability that one number is a **factor** of the other?

Answer: ____________________

Explain how you got your answer. ____________________

____

____

____

4. What is the probability that both numbers are *multiples* of 5?

Answer: ____________________

Explain how you got your answer. ____________________

____

____

____

5. What is the probability that neither number is *prime*?

Answer: ____________________

Explain how you got your answer. ____________________

____

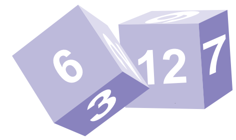
____

____

Practice

Answer the following.

Timor has one *number* **cube** or one die with the numbers 1-6 on the *faces* and a second number cube with the numbers 7-12 on the faces. Complete the table below for possible sums to answer the following.



Sum Game Outcomes

+	1	2	3	4	5	6
7	8					
8		10			13	
9			12			
10			13	14		
11	12				16	
12						18

(**Hint:** If you want to experiment with this problem, you could use a washable marker to write the numbers 7-12 on one of the standard cubes or devise some other method for a 7-12 cube.)

1. What is the probability the sum of two numbers rolled will be

   greater than 20? ________________________

   Why? ____

   ____

2. What is the probability the sum of two numbers rolled will

   be 8? ________________________ 18? ________________________

3. What is the probability the sum of the two numbers rolled will be 9? ________________________ 17? ________________________

4. What is the probability the sum of the two numbers rolled will be 13? ________________________

5. What is the probability the sum of the two numbers rolled will be prime? ________________________

6. What is the probability the sum of the two numbers rolled will be greater than 13? ______________ Less than 13? ______________

7. What is the range of possible sums? ________________________

Practice

Answer the following.

One number cube has the odd numbers 1, 3, 5 with each appearing on two faces. A second number cube has the even numbers 2, 4, 6 with each appearing on two faces. The cubes are tossed.

Complete the tables below for possible sums and **products** to answer the questions on the following page.

Sum Game Outcomes

+	1	1	3	3	5	5
2		3				
2				5		
4						
4						9
6						11
6					11	

Product Game Outcomes

x	1	1	3	3	5	5
2						
2						
4					20	
4				12		
6			18			
6						30

Use your **sum and product tables** *on the previous page to answer the following.*

1. What is the probability the *sum* of two numbers rolled will be odd?

 ____

2. What is the probability the *sum* of two numbers rolled will be prime? ____________

3. What is the probability the *product* of two numbers rolled will be prime? ____________

4. What is the probability the *product* of two numbers rolled will be a factor of 60? ____________

5. What is the probability the *product* of two numbers rolled will be greater (>) than 12? ____________

6. What is the probability the *sum* of two numbers rolled will be less than (<) 7? ____________

7. What is the probability the *sum* of two numbers rolled will be a factor of 99? ____________

Practice

Match each definition with the correct term. Write the letter on the line provided.

_______ 1. any whole number with only two factors, 1 and itself

_______ 2. the numbers that result from multiplying a given number by the set of whole numbers

_______ 3. the largest of the common factors of two or more numbers

_______ 4. a number or expression that divides exactly another number

_______ 5. a rectangular prism that has six square faces

_______ 6. the result of a multiplication

A. cube

B. factor

C. greatest common factor (GCF)

D. multiples

E. prime number

F. product

Practice

Answer the following.

The students on Mike and Misty's teams are considering some games to sponsor for the annual school carnival. They would like a game that would be fairly easy to set up and that would not take long for a player to play. They want players to win often enough to make others want to play the game.

The school will sell tickets for the carnival at a cost of 10 cents each.

- Each game will cost one ticket to play.
- The school plans to spend one-half the money from the sale of tickets on prizes.
- The other half of the money will be used to purchase cold-water drinking fountains for students to use.

Consider each of the following games. Complete the table of information on page 383. Team A's Game One is analyzed for you. You will analyze the others.

Team A's Suggestions for Three Types of Games

Team A's Game One

- A coin will be tossed.
- If it is heads, the player wins.

Analysis of Team A's Game One

- If one coin is tossed, there are two possible outcomes. They are as follows.

  heads
  or
  tails

- For each ticket bought at 10 cents, the probability of winning is $\frac{1}{2}$.

- If five cents is spent on prizes for each ticket,

  and we expect one win per two tickets,

  the value of a prize for this game could be 10 cents.

- A player might expect to win once when playing this game twice, but he also may win both times or he may *not* win at all.

Team A's Game Two

1. Flip two coins.

- If they both come up heads, the player wins.
- If they come up any other way, the player loses.

  a. If two coins are tossed, there are four possible outcomes. They are as follows.

  First Coin **Second Coin**

  ____________ , ____________ or

  ____________ , ____________ or

  ____________ , ____________ or

  ____________ , ____________

  b. Of these four outcomes, ____________ results in a win.

  c. The probability of winning is ____________ .

  d. A student purchases 4 tickets to use playing this game four times and thinks one win is likely. The cost of the four tickets is ____________ cents.

  e. The value of the prize for this game should be one-half or ____________ cents.

Team A's Game Three

2. Flip three coins.

- If they all three come up heads, the player wins.
- If they come up any other way, the player loses.

a. When three coins are tossed, eight outcomes are possible. They are as follows.

First Coin	Second Coin	Third Coin
____________	____________	____________
____________	____________	____________
____________	____________	____________
____________	____________	____________
____________	____________	____________
____________	____________	____________
____________	____________	____________
____________	____________	____________

b. Of these eight outcomes, ____________ results in a win.

c. The probability of winning is ____________ .

d. A student purchases 8 tickets to use playing this game eight times. The cost of the eight tickets is ____________ cents.

e. The value of the prize for this game should be ____________ cents.

3. Complete the table below based on your answers about Team A's Game Two and Game Three. Then answer numbers 4-6 on pages 384-386 before completing the rest of the table for Team B's Game Two and Three.

Team A s Game Two and Game Three

game	probability of winning	number of tickets likely needed to win	cost for prize	amount for water fountain fund
Team A, Game One	$\frac{1}{2}$	2	10 cents	10 cents
Team A, Game Two				
Team A, Game Three				
Team B, Game One	$\frac{1}{3}$	3	15 cents	15 cents
Team B, Game Two				
Team B, Game Three				

Team B's Suggestions for Three Types of Games

Team B's Game One

Team B's Game One is analyzed for you. You will analyze the others.

There will be two blue marbles (blue 1 and blue 2) and one red marble in a bag. The marbles will be the same size and will have a smooth finish. The player reaches into the bag and draws out two of the marbles.

- If they match, a prize is won.
- If they do *not* match, no prize is won.

Analysis of Team B's Game One

- The possible outcomes are as follows.

  blue 1, red
  or
  blue 1, blue 2
  or
  blue 2, red

- One of the three outcomes provides a winning combination of a match.
- The probability of winning is $\frac{1}{3}$.
- If a player purchases 3 tickets and plays the game three times, it appears likely that one win will occur.
- The prize could have a value of 15 cents.

Team B's Game Two

4. Four marbles will be placed in a bag. Two will be blue (blue 1 and blue 2), and two will be red (red 1 and red 2).

- The player will pull two marbles from the bag.
- The player wins if the two are the same color.

  a. The six possible outcomes are as follows.

  ____________ , ____________

  ____________ , ____________

  ____________ , ____________

  ____________ , ____________

  ____________ , ____________

  ____________ , ____________

b. The probability of winning is ____________ .

c. If a player purchases six tickets and plays six times, it is likely ____________ wins will occur.

d. The value of a prize would be ____________ cents.

Team B's Game Three

5. Five marbles will be placed in a bag. There will be two blue (blue 1 and blue 2), two red (red 1 and red 2), and one yellow. Two will be drawn.

- If the colors are the same, the player wins.

a. The 10 possible outcomes are as follows.

____________ , ____________

____________ , ____________

____________ , ____________

____________ , ____________

____________ , ____________

____________ , ____________

____________ , ____________

____________ , ____________

____________ , ____________

____________ , ____________

b. The probability of winning is ____________ .

c. If a player buys 10 tickets and plays the game 10 times,

_____________ wins are likely.

d. The value of a prize could be _____________ cents.

6. Go back and complete the rest of the table on page 383.

Lesson Three Purpose

- Add, subtract, multiply and divide whole numbers and decimals using appropriate methods of computing, such as mental mathematics, paper and pencil, and calculator. (A.3.3.3)
- Solve problems involving units of measures and convert answers to a larger or smaller unit. (B.2.3.2)
- Organize and display data in a variety of forms. (E.1.3.1)
- Understand and apply the concepts of range and central tendency (mean, median, and mode). (E. 1.3.2)
- Analyze real-world data by applying appropriate formulas of central tendency and organizing data in a quality display. (E.1.3.3)

Analyzing Data Using Range and Central Tendency—Mean, Median, and Mode

In this lesson, you will apply the concepts of range and central tendency to real-world data. Go back to Lesson One of this unit if you need a quick review of how to find the *range*, *mean*, *median*, and *mode*.

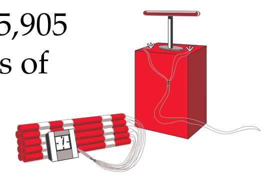
Practice

Answer the following.

1. In the fall of 2000, public and private elementary and secondary schools in the United States expected 53,000,000 students to enter their schools. If 2,120,000 classrooms were needed, what was the *mean* number of students per classroom?

   Answer: ______________________________

2. In March 2000, there were 5,905 holes made in the Seattle's Kingdome Seahawks' football stadium. Each of the 5,905 holes were stuffed with a mean number of 12 ounces of dynamite to bring it down. What was the *total* number of pounds of dynamite used? (**Remember:** 1 pound = 16 ounces.)



   Answer: ______________________________

3. a. Determine the *mean* cabin height for the four business jets listed in the following table.
   (**Remember:** 1 foot = 12 inches.)

   Answer: ______________________________

Business Jet Cabin Heights

Business Jet	Cabin Height
Learjet 31A	4 feet 5 inches
Learjet 60	5 feet 8 inches
Cessna Citation	4 feet 7 inches
GulfstreamV	6 feet 2 inches

   b. Determine the *range* in cabin heights for the four business jets.

   Answer: ______________________________

Practice

Answer the following.

1. Forty students on Team A had a mean score of 80 on a test. Sixty students on Team B had a mean score of 70 on the same test.

   To determine the mean score for the 100 students on Teams A and B, complete the following.

   a. If 40 students had a mean score of 80, the *total* of their scores

      was ____________ .

   b. If 60 students had a mean score of 70, the *total* of their scores

      was ____________ .

   c. The *total* of the scores for the 100 students was ____________ .

   d. The *mean* score for the 100 students was ____________ .

2. If Charles Schultz had drawn 250 more *Peanuts* comic strips during his 50-year career, the mean number of strips per year would have been 365.

   To determine how many comic strips Charles Schultz drew, complete the following.

   a. If the mean number of strips drawn was 365 for the 50-year period, the total number of strips would have been

      ____________ .

   b. He drew 250 ____________ (more, less).

   c. The total number of comic strips drawn by Charles Schultz was

      ____________ .

Stem-and-Leaf Plot

A **stem-and-leaf plot** is a way of organizing data to show their frequency. It is a quick way to picture the shape of the data while including the actual numerical values in the display or graph.

- The *stem* is the number to the left of a vertical (↕) line in the display. In the stem-and-leaf plot below, the stem represents the *tens digit* for each data entry.
- The *leaves* in the plot are to the right of the vertical line, and they represent the *final digit* in the number. The leaves in our plot below represent the *units digit* for each number.

Stem	Leaves
2	1 1 3 7 8

In this example, the data entries are 21, 21, 23, 27, and 28.

Practice

Answer the following.

A news article reported the time, in months, given to investigate jet accidents. The jet accidents involved commercial flights by United States carriers originating in the United States from 1964-96. The data is provided in the stem-and-leaf plot. Use the plot to complete the statements.

In this stem-and-leaf plot, the *stem* represents the *tens digit* and the *leaves* represent the *units digit*.)

Number of Months of Investigation

Stem	Leaves
0	5 6 6 6 7 7 7 7 8 8 8 8 8 8 9 9 9
1	0 0 0 0 0 1 1 2 2 2 3 3 3 4 4 4 4 4 5 5 5 5 5 6 7 7 8 9
2	1 1 3 7 8
3	0 4
4	9
5	4

1. The shortest time of investigation was 05 or 5 months and the longest time of investigation was ____________ months. The *range* was ________________________ .

2. The numbers appearing five or more times are as follows: 8, 10, ____________ , and ____________ . The *mode* for the data set is ____________ .

3. There were 54 investigations, and there are 54 data entries in the stem-and-leaf plot. To find the *median*, the numbers are first arranged in numerical order. This task ____________ (is, is not) already completed because of the format of the stem-and-leaf plot.

4. One-half of 54 is ____________ . There will be ____________ numbers in the first half of the data set and ____________ numbers in the last half of the data set.

5. The last number in the first half and the first number in the second half are the *middle* numbers of the numerically arranged data set. These numbers are ____________ and ____________ .

6. The *median* for the data set is ____________ .

7. The mean for the set of data is the total time of the investigations, in months, which is 792 divided by the number of investigations, which is 54. The *mean*, rounded to the nearest month, is ___________ .

8. If you wrote an article about the time given to these investigations for a news report and chose to state a "typical" amount of time, would you choose the mode, median, or mean? ____________

   Why? ____

   ____

   ____

   ____

9. If you wrote an article about the time given to these investigations for a news report and chose to state the *range*, would you choose to report it as the difference between the shortest and longest investigations, which is ____________ , or as a range from ____________ (shortest) to ____________ (longest)?

   Why? ____

   ____

   ____

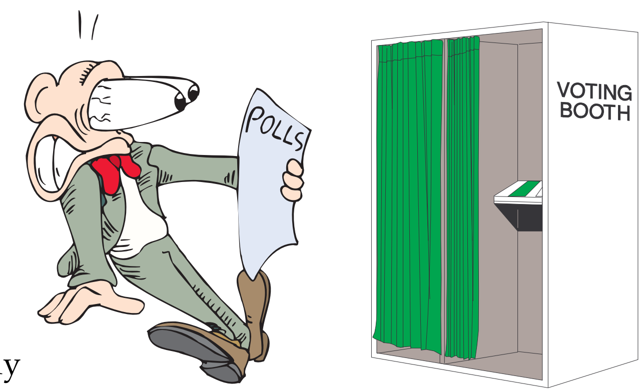
   ____

Lesson Four Purpose

- Collect and organize data in a variety of forms including tables and charts. (E.1.3.1)
- Analyze real-world data by applying appropriate formulas for measures of central tendency and organizing data in a quality display, using appropriate technology. (E.1.3.3)

Basing Decisions on Statistics

The results of surveys often impact decision making. Survey results are also used in advertising. Polls may be conducted prior to elections to determine which candidate is favored at a given time. Polls are also conducted to see how voters feel about a specific issue, such as a tax cut. We will study this area of statistics in this lesson.



Practice

Answer the following.

The following questions represent the first draft of questions a group of students have written for surveys they plan to conduct with people over the age of 18.

In each pair of questions, *circle the number and letter* of the one you prefer and explain why.

1a. Is your car large?

1b. How many people can be seated and accommodated with seat belts in your car?

Explanation: ____

____

____

2a. Do you exceed the speed limit when you drive?

2b. Do you drive fast?

Explanation: ____

____

____

3a. Should elementary or secondary students be required to wear uniforms to school?

3b. Should students be required to wear uniforms in elementary school? In secondary school?

Explanation: ____

____

____

4a. Do you prefer pizza or hamburgers?

4b. Which do you prefer? Pizza served piping hot with your choice of toppings? Hamburgers?

Explanation: ____

____

____

5a. What size pizzas do you buy? Small? Medium? Large?

5b. What are the diameter measures for the pizzas you buy?
6 inches? 7 inches? 8 inches? 9 inches? 10 inches? 11 inches?
12 inches? 13 inches? 14 inches? 15 inches? Larger than 15 inches?

Explanation: ____

____

____

Practice

Answer the following.

Write *two questions* to use on a survey to help you gather data about students in your school. Your questions should deal with the characteristics of a typical middle school student. For example, you may be interested in surveying some of the following topics.

- heights
- ages
- family size
- pets
- hobbies
- favorite beverage
- preferred type of music
- bedtimes
- preferred place in the home to study
- preferred type of calculator
- favorite sport
- participation in school sponsored extra-curricular activities

As you can see, the list could continue.

Make sure your questions are clear so that each person responding will interpret the question the same way.

The responses to your questions should allow you to organize, display, and analyze them.

1. ______________________________

____

____

2. ______________________________

____

____

____

Practice

Answer the following.

When your survey is conducted, it will *not* be convenient to ask each and every person in the school for a response unless your school is very small. It also will *not* be convenient to organize responses when there are a great many. The challenge is to find a sample that *accurately represents* the population of the school. Such a sample is called a *representative sample*.

Sometimes eliminating poor choices is helpful in making better ones.

For each survey question shown, a sample that is likely *not* to be representative is described. Explain why this sample would be a poor choice for a representative sample.

1. Which sport should be dropped from extra-curricular activities in our school since budget cuts require that one be eliminated?

   Sample for survey: During basketball practice today, the members of the boys' team and girls' team will be surveyed. They represent both sexes and all three grade levels in the school.

   Explanation: ____

   ____

   ____

2. Which of the following two cereals should the cafeteria serve for breakfast: Wholesome Wheat Flakes or Choco-Nut Crunch?

   Sample for Survey: A survey form will be placed in each teacher's box to be completed and returned to the office the next day. The teachers know which cereal is best for students.

   Explanation: ____

   ____

   ____

3. Which of the following locks do you prefer for lockers in the school: combination locks or padlocks requiring a key?

   Sample for Survey: A survey will be mailed to randomly selected parents of 30 students from each grade level. The survey is to be returned by mail to the school within 10 days. The parents are the most likely to purchase the locks and know which is best.

   Explanation: ____

   ____

   ____

4. Which of the following ways do you get to school each morning: walk; ride bike; ride bus; driven by parent, sibling, or friend; other?

   Sample for Survey: A survey will be taken in the front of the school near the area where parents pick up and drop off students on Mondays of each week for four weeks.

   Explanation: ____

   ____

   ____

5. Are the rules in our school fair?

   Sample for Survey: A survey will be given to each student sent to the office for breaking school rules each day for one week.

   Explanation: ____

   ____

   ____

6. Go back to the previous practice. Reread the two survey questions you wrote. What changes can you make to the two questions to make them better?

   Explanation: ____

   ____

   ____

7. Describe who you would choose for a representative sample from your school for your two survey questions. Explain why the sample is representative.

   Your choice of representative sample: ______________________

   ____

   ____

   Explanation: ____

   ____

   ____

Practice

Answer the following.

This assignment will count as 40% of your Unit Assessment grade.

Scoring

- survey questions .... 10 points
  (get teacher approval before next step)
- identification of representative sample .... 10 points
  (get teacher approval before next step)
- display .... 10 points
- written summary of analysis and
  class presentation .... 10 points
- total .... 40 points

Assignment

- Finalize your two survey questions using the practice on page 397 and number 6 on page 400. Get teacher approval.
- Finalize your choice of a representative sample using number 7 on page 400. Get teacher approval.
- Conduct your survey.
- Organize the responses.
- Display the results for each in a chart or graph. Tell why this display might be more appropriate for your data than another type of display.
- Write a summary of your analysis for each of the original questions.
- Make a presentation to the class.

Plan of Action

Complete the following.

1. Question #1: ____

   ____

   ____

2. Question #2: ____

   ____

   ____

3. Describe the representative sample for the two questions.________

   ____

   ____

   ____

   Why is the sample you chose representative for question #1? ___

   ____

   ____

   ____

   Why is the sample you chose also representative for question #2?

   ____

   ____

   ____

4. Describe your plan to organize the responses. ________________

____

____

____

5. Describe the chart or graph you chose to display your data and why.

____

____

____

6. Describe what you plan to emphasize in your summary about your analysis for each question's responses.

   Question #1: ____

   ____

   ____

   Question #2: ____

   ____

   ____

7. Use the information on the following pages to enhance your presentation.

Keep this chart handy as you prepare to do oral presentations for your class.

Prepare for Oral Presentations	
Practice	Read your presentation aloud several times to yourself in the mirror and to one or two friends or family members.
Breathe	Take a deep breath before you begin your presentation. Remember to stop and breathe during your presentation, too.
Eye Contact	Look at your audience. Vary your gaze in different directions.
Smile	Smiling before your speech and after your speech will help the audience feel connected to you.

Your display should do the following.

Design a Display to Relate and Enrich

- Enhance the presentation, not distract from it.
- Hold the audience's attention.
- Be easy to read and understand.
- Be interesting.

Use these guidelines to improve the delivery of your presentation.

Improve the Delivery of Your Presentation

- Speak clearly.
- Speak at a suitable volume—neither too loudly nor too softly.
- Speak at a suitable tempo or speed—neither too slowly nor too quickly.
- Make the pitch of your voice appropriate to what you are expressing—neither too high nor too low.

Use this chart to practice presenting your speech.

Characteristics of Good Oral Presentations

Elements	Characteristics	Definitions
Preparation	**1. Subject Knowledge**	• the presentation subject is thoroughly researched and the speaker is prepared for any questions that may be asked
	2. Organization	• the presentation material is arranged or put together in an orderly way—using index cards, outlines, or visual materials to keep presentation well paced and on track
	3. Audience Awareness	• the presentation is prepared for the type of audience receiving the information—speaking or writing is appropriate for and understood by the target audience
Speaking	**4. Enunciation**	• words are spoken clearly, without mumbling, making each sound distinct
	5. Pronunciation	• words are spoken according to a dictionary's pronunciation guide
	6. Volume	• the sound produced by the voice is not too loud or too soft; the sound changes during the presentation to match what is being described
	7. Tempo	• the speed at which words are spoken is not too fast or too slow; the speed may change to match what is being described
	8. Pitch	• the highness or lowness of the sound of the voice matches what is being described
	9. Expressiveness	• the presentation (or words) are communicated in a vivid and persuasive manner
	10. Complete Sentences	• the presentation uses a group or groups of words that present a complete thought
Body Language	**11. Eye Contact**	• the speaker looks directly into the eyes of one or more persons—communicates the speaker's confidence, alertness, and empathy with the audience
	12. Natural Gestures	• the speaker uses normal movement of the hands, head, or other body parts to express the speaker's thoughts or feelings—gestures should emphasize presentation points, not distract from them
	13. Good Posture	• the speaker carries or holds his body straight while sitting, standing, or walking—conveys confidence and readiness; slouching conveys the opposite—unreadiness, indifference

As you practice your presentation, use the chart below to rate yourself.

Presentation Evaluation

VOLUME	Too Loud	Loud & Clear	Too Quiet	Comments
TEMPO	Too Fast	Even Pace	Too Slow	Comments
PITCH	Too Low	Moderate Pitch	Too High	Comments
VISUAL DISPLAY	Too Few	Moderate Amount	Too Many	Comments
CONTENT	Unorganized	Organized & on Subject	Off Subject	Comments

Practice

Use the list below to write the correct term for each definition on the line provided.

bar graph	**mode**
data	**range**
mean	**stem-and-leaf plot**
median	

Number of Goals Scored

Stem	Leaves
1	5 9
2	1 3 7 7 7
3	0 1 3 4 4 5 6 7
4	2 3 6 7 8

Key: 2 | 3 represents 23.

____________________ 1. a way of organizing data to show their frequency

____________________ 2. the score or data point found most often in a set of numbers

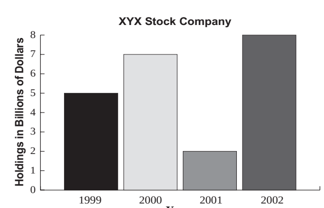
____________________ 3. the middle point of a set of ordered numbers where half of the numbers are above the median and half are below it

____________________ 4. the difference between the highest (H) and the lowest value (L) in a set of data

____________________ 5. the arithmetic average of a set of numbers

____________________ 6. information in the form of numbers gathered for statistical purposes

____________________ 7. a graph used to compare quantities in which lengths of bars are used to compare numbers



Practice

Match each definition with the correct term. Write the letter on the line provided.

______ 1. any whole number *not* divisible by 2

______ 2. a possible result of a probability experiment

______ 3. a rectangular prism that has six square faces

______ 4. any whole number divisible by 2

______ 5. the result of an addition

______ 6. the result of a multiplication

______ 7. the ratio of the number of favorable outcomes to the total number of outcomes

A. cube

B. even number

C. odd number

D. outcome

E. probability

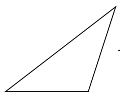
F. product

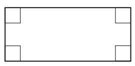
G. sum

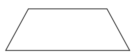
Appendices

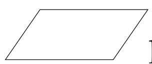
FCAT Mathematics Reference Sheet

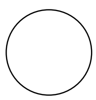
Formulas


triangle — area = $\frac{1}{2}bh$


rectangle — area = lw


trapezoid — area = $\frac{1}{2}h(b_1 + b_2)$


parallelogram — area = bh


circle — area = πr^2
circumference = $\pi d = 2\pi r$

Key

b = base

h = height

l = length

w = width

d = diameter

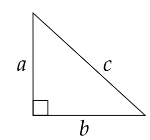
r = radius

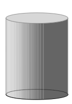
Use 3.14 or $\frac{22}{7}$ for π.

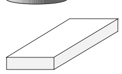
In a polygon, the sum of the measures of the interior angles is equal to *180 (n - 2)*, where *n* represents the number of sides.

Pythagorean Theorem

$c^2 = a^2 + b^2$




right circular cylinder — volume = $\pi r^2 h$ — total surface area = $2\pi rh + 2\pi r^2$


rectangular solid — volume = lwh — total surface area = $2(lw) + 2(hw) + 2(lh)$

Conversions

1 yard = 3 feet = 36 inches
1 mile = 1,760 yards = 5,280 feet
1 acre = 43,560 square feet
1 hour = 60 minutes
1 minute = 60 seconds

1 liter = 1000 milliliters = 1000 cubic centimeters
1 meter = 100 centimeters = 1000 millimeters
1 kilometer = 1000 meters
1 gram = 1000 milligrams
1 kilogram = 1000 gram

1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts

1 pound = 16 ounces
1 ton = 2,000 pounds

Index

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